Traders’ heterogeneity and bubble-crash patterns in experimental asset markets

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\textbf{A B S T R A C T}

We propose a heterogeneous agent model for experimental closed-book call markets with speculators, fundamental and noise traders. We provide structural estimates of the parameters of the model using new experimental data, which allow us to track individual behavior, cognitive reflection abilities, and accuracy of price forecasts. Based on the model’s predictions for individual behavior we identify different types of traders in the data. We find that fundamental traders and speculators have higher terminal wealth and perform better on a cognitive reflection test and price forecasting than noise traders. More importantly, we find that all three types of traders are important to understand the mechanics of bubbles and crashes. In the initial period, fundamental traders buy from noise traders. Next, speculators buy from fundamental traders during the boom. Finally, speculators generate the crash by selling to noise traders. Our model predicts smaller bubbles if the cash and asset endowments are higher, keeping the cash-to-asset ratio constant. Our theory has predictive power as we confirm this prediction with additional out-of-sample data.

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1. Introduction

There are several examples of bubbles: the Dutch tulip mania (1634–1637), the South Sea Company Bubble (1720), the Roaring Twenties stock-market bubble (1922–1929), the Dot-com bubble (1995–2000) and more recently, real-estate bubbles in the US as well as Europe and China. Bubbles generate price distortions that may lead to allocative inefficiencies and financial crises. Bubbles are a complex phenomenon, attracting economists to use theoretical models and empirical methods to study them. Laboratory experiments provide a useful tool to study bubbles empirically since they allow economists to control a variety of factors that are difficult to control in field environments (e.g., trading institutions, the fundamental value process and the dividend process).

Bubbles and crashes in experimental asset markets were first documented by Smith et al. (1988) (SSW) and proved to be a very robust result in experimental economics.\textsuperscript{1} Bubbles are attributed to a combination of factors including speculation,
subject confusion, lack of common knowledge of rationality and lack of rationality (e.g., Smith et al., 1988, 2000; Lei et al., 2001; Caginalp and Ilieva, 2005; Ackert and Kluger, 2006; Haruvy and Noussair, 2006; Kirchler et al., 2012; Moinas and Pouget, 2013; Akiyama et al., 2013; Cheung et al., 2014). Generally, a clear understanding of the mechanics of bubble formation is still missing. For instance, we do not have many models that help us to understand when and why bubbles start and crash (Brunnermeier, 2008).

In this paper we propose an explanatory theory based on a heterogeneous agent model which sheds light on the mechanics of bubble formation in experimental closed-book call markets and we provide evidence for its predictive power. We also collect new experimental data, which allow us to track individual behavior and to control for cognitive reflection abilities of subjects.

While the Smith et al. (1988) laboratory environment is much simpler than field asset markets, we think it can shed light on behavior in field markets. Existing experimental studies provide support for the existence of speculative and other behavioral biases among traders in laboratory asset markets (e.g., Smith et al., 1988; Lei et al., 2001; Caginalp and Ilieva, 2005; Haruvy and Noussair, 2006). These biases are also present in field markets and are conjectured to contribute to bubble formation more generally. Specifically, there is evidence that heterogeneous strategies and different levels of traders’ sophistication in actual markets may contribute to bubbles’ formation (e.g., DeLong et al., 1990a: Griffin et al., 2011). Furthermore, the Smith et al. (1988) environment is the prominent paradigm for the study of bubbles and factors affecting bubble formation in long-lived experimental asset markets. Many laboratory studies focus on this environment, but, due to the complexity of the environment, theoretical modeling has not been developed (with a couple of exceptions, e.g., Duffy and Unver, 2006; Haruvy and Noussair, 2006).

In this paper, we formalize behavioral biases that we believe play an important role in bubble formation. There are three classes of agents in the model: noise traders, fundamental traders and speculators. Our traders’ types are similar in spirit to the types introduced by DeLong et al. (1990a). However, the novelty of our approach is that we model traders’ types differently in order to account for the laboratory environment and to match the trading volume dynamics in addition to the price dynamics of the experimental data. Along the lines of Duffy and Unver (2006), noise traders are equally likely to be either buyers or sellers in each period, and their bid/ask price is determined by the previous period clearing price and a noise term. Fundamental traders tend to buy when the price is below and tend to sell when the price is above the fundamental value, forming price expectations adaptively. Speculators form their price expectations taking into account the presence of noise traders in the spirit of Level-1 traders. They buy when the price is expected to increase and sell otherwise, i.e., their trading behavior is motivated by potential capital gains.

We provide structural estimates of the parameters of the model using new experimental data on five closed-book call-market sessions. New features of the data that are important for our study include individual behavior’s records, and the elicitation of cognitive reflection abilities and accuracy of price forecasts. Individual behavior’s records allow us to identify different traders’ types in the data (via the model), while cognitive reflection abilities and price forecast accuracy provide additional support for our identification strategy. The estimation is conducted by fitting aggregate simulated variables – prices and volume – to the corresponding aggregate experimental variables. According to the estimation, 11% of subjects are speculators, 44% of subjects are fundamental traders and the remaining subjects are noise traders. We show that simulated fundamental traders accumulate assets early and sell their units gradually to speculators and noise traders. Speculators accumulate a substantial number of assets during the boom and initiate the crash. Simulated fundamental traders and speculators end up with much lower asset holdings (close to zero) than noise traders. Speculators end up with the highest simulated terminal wealth levels, followed by fundamental traders. Noise traders end up with significantly lower wealth levels.

Next we use the simulated trading strategies and individual asset holdings data to identify trader types at a micro-level. The validity of our classification is also supported by individual characteristics of subjects (forecasts accuracy and cognitive reflection test scores) which were not used in the estimation. In particular, fundamental traders are much better in predicting the first-period price than other types, and noise traders are worse in price forecasting during the crash compared to fundamental traders. For instance, Lei et al. (2001) show that even if capital gains are not possible, the standard bubble-crash pattern persists. Lei and Vesely (2009) show that the pre-crash bubble period is much shorter than the actual asset market experiment started, designed to reduce subject confusion about the stochastic dividend process, entirely eliminates the bubble-crash pattern. Smith et al. (2000) show that if dividends are paid at the end of the trading horizon only (the least confusing design) the formation of bubbles is least likely. Kirchler et al. (2012) and Huber and Kirchler (2012) show that the main source for subject-confusion is the decreasing fundamental value process. However, Noussair et al. (2001), Noussair and Tucker (2014) and Baghestanian and Walker (2014) show that bubbles emerge also in environments with flat fundamental value. Other studies suggest that while confusion plays a role in the formation of bubbles and mispricing, strategic uncertainty and the lack of common expectations also play an important role, e.g., Akiyama et al. (2013) and Cheung et al. (2014).

3 For more details on explanatory and predictive theories, see Schotter (2009).

4 Note that noise traders in our model differ from Duffy and Unver near-zero-intelligence traders in that we do not assume that noise traders have weak foresight. The assumption of weak foresight is important for Duffy and Unver (2006) to generate the observed crash-patterns in the lab since they only have one type of traders.

5 We elaborate below that speculators are similar to Level-1 trader types, characterized by one step of iterated reasoning in their expectation formation (Stahl and Wilson, 1994, 1995; Costa-Gomes and Crawford, 2006; Crawford et al., 2013). In accordance with the Level-k literature we impose that their anchoring types (commonly referred to as L-k-type) are noise traders.

6 For instance, Lei et al. (2001) show that even if capital gains are not possible, the standard bubble-crash pattern persists.
to fundamental traders and speculators. Also, noise traders have much lower cognitive reflection skills (measured by the cognitive reflection test in Frederick (2005); CRT) than other types.

Our model and estimation results improve the understanding of the dynamics of bubbles and crashes. In the first period, noise traders under-predict the price compared to the fundamental value. As a result, they sell assets to the fundamental traders. After that, speculators buy aggressively (from fundamental and noise traders) since they predict an upward price trend and expect capital gains. At the peak, which is well above the fundamental value, speculators anticipate the upcoming crash and start selling to noise traders, who do not foresee the downward price trend. To summarize, noise traders are first taken advantage of by fundamental traders and then, during the crash, by speculators. Not surprisingly, they end up with the lowest terminal wealth compared to other classes of traders. These observations are consistent with the findings of Griffin et al. (2011), who show that both sophisticated and noise traders actively bought technology shares during the Tech bubble, while at the peak, sophisticated investors pulled capital out and noise traders continued to buy.

In order to test the predictive power of our model, we use our estimated model to predict behavior in another experiment where cash and asset endowments are increased by a factor of 5, thus keeping the cash-to-asset ratio constant. Our model predicts smaller bubbles with higher cash and asset endowments, and these predictions are confirmed with the out-of-sample experimental data. This indicates that the actual sizes of the endowments are important for price dynamics. This new finding complements the literature, which currently emphasizes the importance of the cash-to-asset ratio (e.g., Caginalp et al., 1998; Kirchler et al., 2012).

Our work complements the existing literature in several ways. We provide a framework that helps understand how bubbles and crashes are generated in experimental asset markets. Specifically, we identify heterogeneous trading strategies that generate bubble-crash patterns. Caginalp and Ilieva (2005) and Caginalp and Merdan (2007) provide a mathematical model with two types of traders (fundamental and momentum), which can be used to generate price dynamics similar to the ones observed in the lab. However, in their model crashes are generated because momentum traders become cash constrained. On the other hand, our model explains bubbles in the absence of cash constraints, but it requires three types of traders. This is important, since in our data, as well as in other experimental studies, traders are endowed with large amounts of cash and thus do not become cash constrained easily.

Duffy and Unver (2006) are the first to propose a computational model with noise traders to generate bubble-crash patterns. We depart from Duffy and Unver (2006) by introducing heterogeneous agents and by finding significant differences in behavior between traders’ types. The mix of heterogeneous agents also allows us to dispense with the assumption of weak foresight (i.e., the probability of being a buyer decreases with time), which generates a sharper crash in their environment.7

Haruvy and Nossair (2006) also propose heterogeneous traders and provide a valuable adaptation of the model of DeLong et al. (1990a) into the framework of experimental asset markets. Haruvy and Nossair (2006) focus on fitting price dynamics at the aggregate level, and are not interested in fitting trading volume paths. The main contribution of our approach relative to Haruvy and Nossair (2006) is that it allows us not only to fit trading prices, but also to fit trading volumes, which, in turn, allows us to describe how different types of traders participate in bubbles’ formation. To this end, we modify the types of traders considered in DeLong et al. (1990a), as in their model the trade volume is zero during a crash of a positive bubble, which is usually not observed in the data.8 We discuss further differences in the main body of the paper.

Overall, our approach contributes to the understanding of the mechanics of bubbles and crashes and can be potentially used to examine policies.9 To the best of our knowledge, our model is the first to focus on trading volumes at both the aggregate and individual levels, which in turn has important implications for the understanding of bubble formation. Indeed, our work sheds light on the questions of when and why bubbles start and crash in experimental asset markets and how different types of traders contribute to bubbles’ formation.

Section 2 presents the experimental data. Section 3 presents our model and its main building blocks. In Section 4 we estimate the parameters of the model based on aggregate variables. We then proceed and use the estimates to identify different types of traders in the experimental data. Section 5 shows that bubbles and crashes are generated by the interplay of speculators, fundamental and noise traders. Section 6 investigates the out-of-sample predictive power of our model with additional new data. Section 7 concludes the paper.

2. Experimental design and data

2.1. Experimental design and procedures

The experimental design builds on the seminal study of Smith et al. (1988). In the laboratory market subjects had the opportunity to trade assets with a stochastic dividend process. The market had a finite time horizon of 15 periods. At the

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6 Duffy and Unver (2006) use the notion of “near-zero-intelligence” traders.
7 Another difference is that we focus on a call-market trading institution rather than double auction.
8 In DeLong et al. (1990a) during a crash of a positive bubble, fundamental traders want to sell since the price is above the fundamental value, momentum traders want to sell since the price trend is negative and speculators want to sell too.
9 For example, the model can be used to evaluate the impact of asset-holdings caps (Lugovskyy et al., 2014) or short-selling constraints on bubble formation (Haruvy and Nossair, 2006). The model can also be used to study the effect of changes in the trading institution on price dynamics (Baghestanian et al., 2015).
end of each period, each unit of the asset in a trader’s inventory paid an uncertain dividend of 0, 8, 28, or 60 francs (the experimental currency) with equal probability (e.g., Smith et al., 1988; Boening et al., 1993; Caginalp et al., 2000, 2001; Haruvy et al., 2007; Hussam et al., 2008). Therefore, the expected value of the dividend payment in each period was 24 francs. It was publicly known that the dividend was independently drawn each period and the actual dividend paid in each period was the same for all traders.

Given the dividend process, the fundamental value of the asset could be calculated at any time within the experiment. More specifically, the fundamental value could be calculated as the expected value of the dividend in each period (24 francs) times the number of periods remaining (including the current period). The fundamental value of the asset was, therefore, declining from 360 francs in period 1 to 24 francs in period 15, and assets became worthless at the end of period 15. At the beginning of the experiment, each trader was endowed with two units of the asset and a cash balance of 2000 francs.

Traders had the opportunity to buy and sell assets in each period via a closed-book call market. Conditional on asset and cash constraints, subjects submitted buy and/or sell limit orders. Buy orders were ordered from highest to lowest and sell orders from lowest to highest. The price was determined by the intersection of these schedules. If they were overlapping, the lowest market clearing price possible was then determined to be the trading price in period t, at which transactions were executed. If no such price existed, i.e., if the entire demand schedule happened to be below the aggregate supply schedule, the highest bid-price was reported to the subjects. No transactions were executed at this price.

Subjects could not purchase more units than they could afford nor sell more units than they had in their inventories, i.e., negative cash balances and short selling were not allowed. Inventories of assets and cash balances were carried over from period to period. No interest was paid on cash holdings and there were no transaction costs. At the beginning of each period traders also made forecasts of the transaction price for that period. They were paid for the accuracy of their forecasts. All earnings from forecasting accumulated in a separate account from the traders’ cash on hand, and thus these payments did not affect the market capital asset ratio.

At the beginning of each session, subjects were provided the instructions of the first task of the experiment. The instructions for all tasks were projected on an overhead. The first stage in all sessions consisted of a cognitive reflection test (Frederick, 2005) to measure the cognitive reflection ability of all subjects (see Appendix A for more details). This stage was hand-run with the subjects providing their answers to the three questions on a decision sheet. Subjects were given as much time as they needed to complete the three questions. Subjects received two dollars for each correct answer at the end of the session. Once everyone finished, the decisions sheets were collected and the instructions for the second stage were handed out. The market instructions were read aloud in front of the subjects. Afterwards, the subjects were given 5 min to complete a short quiz. The experimenter went over the answers on an overhead and then started the market. The subjects were privately paid their earnings for all stages of the experiment. Throughout the experiment, subjects were encouraged to ask questions at any time. The questions were asked and addressed privately to avoid the possibility of biasing the entire group.

The experiment consisted of five markets conducted at Indiana University. In four out of the five sessions (Sessions 1, 3, 4 and 5) nine subjects participated in the experiment and eight subjects participated in Session 2. Subjects were recruited from undergraduate courses via the IELab Recruiting System. Many of the subjects had taken part in previous experiments in economics and other disciplines, but no subjects had participated in markets of comparable designs and each subject participated in only one market of this study. The markets were computerized and programmed with the z-Tree software package. At the end of a session, each subject’s final holdings of francs were converted to dollars at the predetermined and publicly known conversion rate of 148 francs to 1 US dollar. Each session lasted approximately 80 min including instructional period and payment of subjects. Subjects earned on average $24.

2.2. Data description

In this section we focus on the main features of the experimental data. Fig. 1 shows the experimental trading prices for the five sessions and the average price over sessions as well as the average traded quantities.

The dashed straight line in Fig. 1(a) depicts the fundamental value of the asset, the other dashed curves depict the transaction prices of each session, and the solid curve depicts the average transaction prices. The price dynamics show the standard bubble-crash pattern (e.g., Smith et al., 1988; Boening et al., 1993; Caginalp et al., 2000, 2001; Haruvy et al., 2007; Hussam et al., 2008; Williams, 2008). Furthermore, Fig. 1(b) indicates that trading volumes are nonnegligible. Some bubble measures, capturing different bubble aspects, are presented in Table 2. Next, we provide a computational model that generates price patterns, volume patterns, and bubble measures that are similar to the ones observed in the data.

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10 They were paid 50 francs for the forecast within 10%, 20 francs for within 25%, and 10 francs for within 50% of actual price. We followed Haruvy et al. (2007) for the forecast rewards structure.
11 The instructions for all stages of the experiment are available upon request.
12 As stated by Frederick (2005), the CRT measures “cognitive reflection – the ability or disposition to resist reporting the response that first comes to mind.”
13 See Fischbacher (2007) for a discussion of the z-Tree software package.
3. The model

We propose a model similar in spirit to the models of Duffy and Ünver (2006) and Haruvy and Noussair (2006). We construct the model to resemble the laboratory economy described in Section 2. The model captures important features of the experimental data. In what follows we provide a general description of the market environment, define and characterize individual trader types, and conclude with a summary of the simulation steps.

3.1. Simulated asset market environment

In our market environment $N$ agents interact in $T$ periods and trade a single financial asset. Initially each agent $i$ is endowed with $x_0^i$ units of cash and $y_0^i$ units of the financial asset. At the end of every period the asset pays random dividends drawn with equal probability from a commonly known support $\{d_1, d_2, d_3, d_4\}$, with $d_1 = 0$ and $d_1 < d_2 < d_3 < d_4$. The expected dividend is denoted as $d = \frac{1}{4} \sum_{i=1}^{4} d_i$. Since our model is supposed to fit the laboratory environment, the dividend support is $\{0, 8, 28, 60\}$. In general, the support does not necessarily have to be restricted to four values or to an i.i.d. dividend process. The fundamental value of the asset in every period is common knowledge and given by

$$FV_t = \tilde{d}(T - t + 1) \quad \text{for} \quad t = 1, \ldots, T.$$

Under rational expectations and risk neutrality, prices should equal the fundamental value.

In every trading period $t = 1, \ldots, T$ traders may either buy or sell units of the financial asset (or remain inactive). During the experiments traders were allowed to submit both bids and asks simultaneously, potentially for multiple units. To capture this feature in the simulations, we subdivide each trading period into $S$ submission rounds.

In each of the $s = 1, \ldots, S$ submission rounds a trader is either a seller or a buyer (the decision or probability to be buyer or seller is described below). Trader $i$ in submission round $s$ and trading period $t$ can submit an ask price, $a_{i,s,t}$, for one unit of the asset if she is a seller and may submit a bid, $b_{i,s,t}$, if she is a buyer. These features of the model allow us to capture the fact that the experiments traders can submit both bids and asks in a given trading period, and to track the volume of the traded asset during the experiments. Since we do not allow for short selling or borrowing, in every trading period, traders cannot sell more units than in their holdings, and buyers cannot exceed their cash holdings.

In every submission round in trading period $t$ all the bids and ask prices are aggregated to obtain the price and quantity traded. That is, as in the experiments, the trading institution used to determine market clearing prices is a closed-book call market.\footnote{See also Gode and Sunder (1993) for commodity markets.}

\footnote{Bids are ordered from highest to lowest to obtain the inverse demand schedule, while asks are ordered from lowest to highest to obtain the inverse supply schedule. The trading price is determined as the intersection of the inverse demand and supply schedules.}

\footnote{In closed-book call markets traders observe only the market clearing price (on their screens) but are not informed about the identities of the traders who sell or buy units of the asset. Typically they also do not observe the trading volume in this market environment.}

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In each round, any buyer who submits a bid above the market-clearing price $p^*_t$ buys one unit of the asset at $p^*_t$. Thus, the updated cash and unit holdings of a buyer are
\[
x_{i,t}^d = x_{i,t-1}^d - p^*_t \mathbb{1}_{\{b^i_{s,t} > p^*_t\}} \quad \text{and} \quad y_{i,t}^d = y_{i,t-1}^d + \mathbb{1}_{\{b^i_{s,t} > p^*_t\}}
\]
for each submission round $s$ of trading period $t$. The symbol $\mathbb{1}_C$ is an indicator function, taking the value 1 if condition $C$ is satisfied and 0 otherwise.

In each round, any seller who submits an ask below the market-clearing price $p^*_t$ sells one unit at $p^*_t$. The updated cash and unit holdings of a seller are
\[
x_{i,t}^s = x_{i,t-1}^s + p^*_t \mathbb{1}_{\{a^i_{s,t} < p^*_t\}} \quad \text{and} \quad y_{i,t}^s = y_{i,t-1}^s - \mathbb{1}_{\{a^i_{s,t} < p^*_t\}}
\]
for each submission round $s$ of trading period $t$. Next, we explain how market-clearing prices are determined in each submission round. A market-clearing trading price is any price which satisfies
\[
|B| = |\{i = 1, \ldots, N : b^i_{s,t} > p_{s,t}\}| = |A| = |\{i = 1, \ldots, N : a^i_{s,t} < p_{s,t}\}|
\]
where $|U|$ denotes the cardinality of the set $U$. That is, the market-clearing price is a price at which the number of buyers equals the number of sellers. Let $b^*_t$ be the lowest bid price in $B$ and $b_{s+1}$ be the highest bid price outside of $B$. Similarly, let $a^*_t$ be the highest ask in $A$ and $a_{s+1}$ be the lowest ask outside of $A$. Then, for any $\lambda \in (0, 1)$ the following price clears the market:
\[
p_{s,t} = \lambda \min\{b^*_t, a_{s+1}\} + (1 - \lambda) \max\{b_{s+1}, a^*_t\}.
\]

In the experiments, the market-clearing price was defined as the lowest price at which there was an equal number of sellers and buyers. We use $\lambda = 10^{-2}$ to determine the corresponding simulation market-clearing price. Whenever the entire demand schedule happened to be below the supply schedule, the highest bid price was taken as a proxy for the market-clearing price. No trades are executed at this price during the simulations.\footnote{In the model, the case in which identical bids or asks are submitted happens with probability zero. In this hypothetical case, buyers and/or sellers are determined randomly among agents with identical bids or asks.}

After the 5th (last) submission round in trading period $t$, the random dividend $D_t$ is realized and traders update their cash holdings according to:
\[
x_{i,t+1}^d = x_{i,t}^d + D_t y_{i,t}^d.
\]

The period market-clearing price is defined as the average of market-clearing prices $\{p_{s,t}^*\}_{s=1}^S$ over the submission rounds:
\[
p_t = \frac{1}{S} \sum_{s=1}^S p_{s,t}^*.
\]

The simulated market-clearing prices will then be compared with the observed market-clearing prices in Section 4.

3.2. Trader types

There are three classes of agents in the model: noise traders, fundamental traders and speculators. Noise traders are equally likely to be either buyers or sellers in each period, and their bid/ask price is determined by the previous period clearing price and a noise term. Fundamental traders tend to buy when the price is below and tend to sell when the price is above the fundamental value. Speculators form their price expectations taking into account the presence of noise traders in the spirit of Level-1 traders. They buy when the price is expected to increase and sell otherwise, i.e., their trading behavior is motivated by potential capital gains. In what follows, we provide a more precise description of these trader types.

3.2.1. Noise traders

Let $k_1$ denote the number of fundamental traders, $k_2$ denote the number of speculators, where $k_1, k_2 \in \{0, 1, \ldots, N\}$ and $k_1 + k_2 \leq N$. Then, the number of noise traders is $(N - k_1 - k_2)$. As in Duffy and Unver (2006), we impose some assumptions on the behavior of noise traders and model their bidding behavior. At the beginning of every submission round $s$ in every period $t$, each noise trader is a buyer with probability $\pi_t$ and a seller with probability $1 - \pi_t$. We assume that $\pi_t = \pi = 0.5$. We depart from Duffy and Unver (2006) who assume that the probability of being a buyer is decreasing over time and initially equal to 0.5. Specifically they assume that $\pi_t = \max\{0.5 - \phi t, 0\}$ with $\phi \in [0, 0.5]$. Duffy and Unver (2006) refer to this as 'weak foresight' assumption. This assumption implies that as $t$ increases, the excess supply increases under weak foresight and generates a sharper downturn of prices. Duffy and Unver (2006) need this assumption to generate the crash pattern, given that their model consists of only noise traders. We do not impose the 'weak foresight' assumption since in our model we have heterogeneous types, and the crash is generated by the interplay of different types of traders.
If a noise trader is selected to be a buyer in trading period $t$ and submission round $s$, she submits a bid subject to cash availability. Her bid is of the form

$$b_{s,t}^{i} = \min \{ (1 - \alpha) \epsilon_t + \alpha p_{t-1}, x_{s,t} \} ,$$

where $\alpha \in [0, 1]$ is the price anchoring parameter, $\epsilon_t \sim U[0, \kappa FV_t]$, $\kappa \geq 0$, $x_{s,t}$ denotes the current cash holdings of agent $i$, and $p_{t-1}$ is the market-clearing price in period $t - 1$. A higher $\alpha$ indicates that noise traders place more weight on the previous period price and less weight on the noise term when submitting bids or asks. A higher $\kappa$ indicates that noise traders are more likely to pay above the fundamental value. No bid is submitted by a noise buyer whenever her cash holdings are zero, i.e., if $x_{s,t} = 0$.

Similarly, if a noise trader is selected to be a seller in trading period $t$ and submission round $s$, she submits an ask subject to her unit holdings. Her ask is of the form

$$a_{s,t}^{i} = (1 - \alpha) \epsilon_t + \alpha p_{t-1} ,$$

where $\alpha \in [0, 1]$ and $\epsilon_t \sim U[0, \kappa FV_t]$. No ask is submitted by the noise seller whenever her unit holdings are zero, i.e., if $y_{s,t} = 0$.

That is, noise traders submit bids and asks based on the previous period price and a noise term. The parameter $\alpha$ captures the behavioral notion that anchoring effects may be important — in our context the relevant anchor is the market clearing unit price in the previous period.

3.2.2. Fundamental traders

Fundamental traders are modeled as a combination of classes of agents suggested by Cason (1992) and Haruvy and Noussair (2006). Cason (1992) provides a simulation model for goods markets where market-clearing prices are determined via a closed-book call-market institution. Haruvy and Noussair (2006) present a simulation model for double auctions asset markets. The authors define passive investors as traders who buy assets at the current standing ask price if that price is below the fundamental value. Similarly, they sell assets whenever the current standing bid price is above the fundamental value of the stock.

We modify the adaptive agents considered by Cason (1992) and the passive investors considered by Haruvy and Noussair (2006) to model adaptive fundamental or simply fundamental traders. A fundamental trader computes in every period the bound

$$l_t = \omega^f l_{t-1} + (1 - \omega^f) p_{t-1} - \tilde{d} \quad \omega^f \in (0,1) ,$$

where $\omega^f$ is the adaptive expectation parameter and $l_0 = FV_1 + \tilde{d}$. A smaller $\omega^f$ indicates that fundamental traders put more weight on past market clearing prices, when forming expectations about the next period’s price.

The intuition is straightforward: $l_t$ serves as a proxy for the expected market-clearing price in period $t$, which is unknown by the agent at the time of the bid/ask submission. When fundamental traders form expectations about the price, they take into consideration both previous prices and fundamental values. We subtract $\tilde{d}$ to control for the decreasing fundamental value and thus our updating rule for $l_t$ differs from the one suggested by Cason (1992). Notice that $l_t$ does not depend on $s$, i.e., we do not allow fundamental traders to learn within the period, which is in line with the experimental design. The functional form of $l_t$ suggests that price expectations are formed adaptively. Empirical evidence for adaptive expectation formation, used by subjects during asset market experiments, is provided by Haruvy et al. (2007), Smith et al. (1988) and Williams (1987).

While noise traders are randomly determined to be buyers or sellers, a fundamental trader decides whether to be a buyer or a seller. If the bound is below the fundamental value, $l_t \leq FV_t$, and her cash holdings are positive, $x_{s,t} > 0$, then she chooses to be a buyer and submits a bid as follows:

$$b_{s,t}^{i} = \min \{ B_{s,t}^{i}, x_{s,t} \} \quad B_{s,t}^{i} \sim U[l_t, FV_t] .$$

That is, if the trader believes that the market-clearing price is below the fundamental value and she has enough cash, she submits a bid $B_{s,t}^{i} \sim U[l_t, FV_t]$. If, on the other hand, the bound is above the fundamental value, $l_t > FV_t$, and her asset holdings are positive, $y_{s,t} > 0$, then she chooses to be a seller and submits an ask from the interval between the fundamental value and the bound:

$$a_{s,t}^{i} \sim U[FV_t, l_t] .$$

Bids and asks are random to allow for some decision errors. Naturally, if agents cash holdings are zero, she cannot submit a bid, and if her asset holdings are zero, she cannot submit an ask.

3.2.3. Speculative traders

There are $k_s$ speculators, whose trading behavior is motivated by expected capital gains. Speculator $i$ decides whether to buy or sell assets based on her expectations in the beginning of period $t$ (i.e., before submitting a bid or an ask in $t$) about market clearing prices in periods $t$ and $t+1$. If

$$E'(p_{t+1}) > E'(p_t) ,$$
speculator $i$ decides to submit a bid order expecting to make capital gains by selling in the following period. According to speculators, bids in the interval $[E(p_t), E(p_{t+1})]$ are profitable, and, allowing for decision errors, speculators’ bids are given by:

$$b_{i,t}^{b,i} = \min(b_{i,t}^s, x_{s,t}), \quad B_{i,t}^b \sim U[E(p_t), E(p_{t+1})].$$  \hspace{1cm} (6)

If on the other hand

$$E(p_{t+1}) \leq E(p_t)$$

speculator $i$ decides to sell and submits an ask order of the form:

$$a_{i,t}^{s,i} \sim U[E_i(p_{t+1}), E(p_t)].$$  \hspace{1cm} (7)

If a speculator’s cash holdings are zero, she cannot submit a bid, and if her asset holdings are zero, she cannot submit an ask. We further impose that $a_{i,t}^{s,i} = b_{i,t}^{p,i} = FV_t$ in the last two periods of the simulation, since the potential for speculation is limited (see also Haruvy and Noussair (2006) for a similar approach).

We use a Level-$k$ modeling approach to compute speculators’ expectations. Specifically we assume that speculators are Level-1 traders, best responding against a benchmark population of Level-0 noise traders. The average equilibrium price process in a population consisting solely of noise traders takes the form:

$$E(p_t) = \alpha p_{t-1} + (1 - \alpha) \frac{k}{2} FV_t.$$

The assumption that speculators know the actual values of $\alpha$ and $k$ is too strong, especially in a closed-book call market. We therefore assume that speculator $i$ forms expectations in the following way:

$$E(p_t) = \gamma_1 p_{t-1} + \gamma_2 FV_t \quad \gamma_1 \in [0, 1], \gamma_2 \geq 0,$$

where $\gamma_1$ and $\gamma_2$ are the expectation parameters for speculators. When forming expectations about the current period price, a higher $\gamma_1$ indicates that speculators place more weight on the previous period price, while a higher $\gamma_2$ indicates that speculators place more weight on the current fundamental value.

Iterating (8) one period forward yields $E(p_{t+1})$ as a function of publicly observed variables $FV_t, FV_{t+1}, p_{t-1}$, and $p_{t-2}$. It can be shown that it is generally possible to find parameter values under which speculators initially buy the asset and sell it toward the end of the trading horizon under our specification of bids and asks, thereby inducing a price crash.

3.3. Simulation steps

Next, we summarize the main simulation steps within a given period $t$:

1. At the beginning of submission round $s \in \{1, \ldots, S\}$:
   (a) Each noise trader becomes either a buyer or a seller with equal probability. Buyers submit bids and sellers submit asks according to rules (2) and (3), respectively.
   (b) Each fundamental trader computes bound $l_t$ and decides to become a buyer if the bound is below fundamental value and a seller if it is above fundamental value. Buyers and sellers submit bids and asks according to rules (4) and (5), respectively.
   (c) Each speculator computes expectations of future prices based on (8) and decides to become a buyer if he expects a price increase, $E(p_{t+1}) > E(p_t)$, and a seller if he expects a price decrease, $E(p_{t+1}) \leq E(p_t)$. Each buyer submits a bid according to rule (6) and each seller submits an ask according to rule (7).

2. After all the bids and asks are submitted, the market-clearing price is computed. The market-clearing price is then reported (no additional information is revealed to the traders) and trades are executed. Traders’ cash and unit holdings are updated accordingly.

3. The process above is repeated for the $S$ submission rounds. After trades in the $S$th submission round are executed, the asset pays its random dividends and cash holdings are updated accordingly.

4. Model and data: estimation

In this section, we estimate the parameters of the structural model using aggregate prices and quantities. Based on this optimal fit with respect to aggregate variables, we characterize the strategies used by speculators, fundamental and noise traders. The results are then used to identify the different types of traders in the data. We will show that there is a tight relationship between the terminal wealth levels of agents and their trading strategies in the simulations, which is also

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Table 1
Parameter estimates for the model of noise traders, fundamental traders, and speculators for the experimental data.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>αf</th>
<th>κ</th>
<th>δ</th>
<th>γ1</th>
<th>γ2</th>
<th>k1</th>
<th>k2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7648</td>
<td>0.2738</td>
<td>7.301</td>
<td>3</td>
<td>0.1351</td>
<td>3.7926</td>
<td>44%</td>
<td>11%</td>
</tr>
</tbody>
</table>

Notes: α is the price anchoring parameter for noise traders. A higher α indicates that noise traders place more weight on the previous period price and less weight on the noise term when submitting bids or asks. αf is the adaptive expectation parameter for fundamental traders. A smaller αf indicates that fundamental traders put more weight on past market clearing prices, when forming expectations about the next period’s price. κ is the parameter for the support of the distribution of the noise term for noise traders. A higher κ indicates that noise traders are more likely to pay above the fundamental value. γ1, γ2 are the expectation parameters for speculators. When forming expectations about the current period price, a higher γ1 indicates that speculators place more weight on the previous period price. A higher γ2 indicates that speculators place more weight on the current fundamental value. S is the number of submission rounds within a period. In each submission round, a trader can submit a bid or an ask for one unit. k1, k2 are the numbers of fundamental traders and speculators, respectively.

confirmed by the experimental data. The latter finding indicates that the simulations capture important features of the data also at the individual level.

We start by obtaining estimates for the model parameters. Given the initial cash and asset-holdings endowment from the experimental design (2000 francs and 2 units), the number of traders, N, and the number of trading periods, T, the model presented in Section 3 consists of eight free parameters: the price anchoring parameter of the noise traders α, the noise-support parameter κ of the noise traders, the learning parameter of the fundamental traders αf, the number of submission rounds S, the expectations parameters for speculators γ1 and γ2, the number of fundamental traders k1 and the number of speculators k2.

For a given set of parameters (α, αf, κ, S, γ1, γ2, k1, k2), number of traders N, number of periods T, and individual endowments \(x_{0t}^{N}\) and \(y_{0t}^{N}\), each simulation run \(m = 1, 2, \ldots, M\) will generate a price sequence \(p_{m}^{T}, p_{m}^{2}, \ldots, p_{m}^{N}\), an \(N \times T\) matrix of cash holdings and an \(N \times T\) matrix of asset holdings. The results can be interpreted as the simulation-equivalent of one experimental session. Let \(\hat{P}_{t}(\alpha, \alpha_{f}, \kappa, S, \gamma_{1}, \gamma_{2}, k_{1}, k_{2})\) and \(\hat{Q}_{t}(\alpha, \alpha_{f}, \kappa, S, \gamma_{1}, \gamma_{2}, k_{1}, k_{2})\) denote the average simulated asset price and quantity in period t. Let \(\hat{P}_{t}\) and \(\hat{Q}_{t}\) denote the average observed trading price and volume in period t from our experiments. In order to obtain parameter estimates we used a grid-point search algorithm,\(^{19}\) minimizing the following objective function:

\[
SSE(\alpha, \alpha_{f}, \kappa, S, \gamma_{1}, \gamma_{2}, k_{1}, k_{2}) = \sum_{t=1}^{15} \left( \frac{\hat{P}_{t}(\alpha, \alpha_{f}, \kappa, S, \gamma_{1}, \gamma_{2}, k_{1}, k_{2}) - \hat{P}_{t}}{FV_{t}} \right)^{2}
+ \sum_{t=1}^{15} \left( \frac{\hat{Q}_{t}(\alpha, \alpha_{f}, \kappa, S, \gamma_{1}, \gamma_{2}, k_{1}, k_{2}) - \hat{Q}_{t}}{TSU_{t}} \right)^{2}.
\]

Note that we normalized the price difference by the fundamental value of period 1 (FV1) and the quantity difference by the total stock of units (TSU). The equation above computes the sum of the squared differences of the simulated average variables to the corresponding observed variables.

The parameters provided in Table 1 are estimated by minimizing the distance between actual and simulated prices and quantities according to Eq. (9). Our results indicate that, in addition to noise traders, there are nonnegligible fractions of fundamental traders (44%) and speculators (11%).

Table 2 provides several bubble measures computed using the data and the model. We provide several measures since they capture different aspects of bubbles and thus together give a better picture of bubbles than each measure separately. A quick look at the simulated bubble measures reveals that they are close to the actual data measures, which provides additional support for the model. Fig. 2 also depicts the simulated and actual prices and quantities. Importantly, the model provides a good fit not only for prices (as in Duffy and Unver (2006) and Haruvy and Noussair (2006)), but also for trading volumes.

To the best of our knowledge, there are only two models that provide a theoretical framework for bubbles in experimental asset markets when cash constraints are not necessarily binding (see Caginalp and Ilieva 2005) for an explanation of bubbles where cash constraints are important). In what follows we discuss how we contribute to this literature. In particular, we compare our approach with Duffy and Unver (2006) and Haruvy and Noussair (2006), who propose simulation models which provide a good fit to the aggregate price data.

\(^{19}\) We first specified a coarse grid for all our parameters to obtain estimates for \(k_{1}, k_{2} \in \{1, 2, 3, \ldots, 9\}\). We then fixed the obtained type distribution and refined the grids for the remaining parameters. We re-initiated this procedure for \(S \in \{1, 2, 3, 4, 5\}\) to obtain an estimate for \(S\). As in Duffy and Unver (2006), we set \(p_{0} = 0\).
Thus, traders’ volume are data.

Table 2
Bubble measures: call markets, five sessions.

<table>
<thead>
<tr>
<th>Bubble measure</th>
<th>Formula</th>
<th>Experimental data Mean (Std. Err.)</th>
<th>Simulation Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turnover</td>
<td>( \sum_{t=1}^{15} q_t / TSU )</td>
<td>2.00 (0.37)</td>
<td>2.37</td>
</tr>
<tr>
<td>Amplitude</td>
<td>( \max_t \left{ \left( \frac{P_{t-\phi t}}{P_{t-1}} \right) - \min_t \left{ \left( \frac{P_{t-\phi t}}{P_{t-1}} \right) \right} \right} )</td>
<td>5.11 (3.9)</td>
<td>4.43</td>
</tr>
<tr>
<td>APD</td>
<td>( \frac{1}{n} \sum_{t=1}^{15}</td>
<td>P_t - FV_t</td>
<td>)</td>
</tr>
<tr>
<td>PD</td>
<td>( \frac{1}{n} \sum_{t=1}^{15} (P_t - FV_t) )</td>
<td>154.64 (57.31)</td>
<td>155.49</td>
</tr>
<tr>
<td>RAD</td>
<td>( \frac{1}{n} \sum_{t=1}^{15} \max(P_t - FV_t) )</td>
<td>0.97 (0.35)</td>
<td>0.98</td>
</tr>
<tr>
<td>RD</td>
<td>( \frac{1}{n} \sum_{t=1}^{15} \min(P_t - FV_t) )</td>
<td>0.94 (0.34)</td>
<td>0.97</td>
</tr>
<tr>
<td>RPAD</td>
<td>( \frac{1}{n} \sum_{t=1}^{15} \frac{</td>
<td>P_t - FV_t</td>
<td>}{</td>
</tr>
<tr>
<td>Haessel</td>
<td>( R^2 ) of OLS regression: ( P_t = \alpha + \beta FV_t + \epsilon_t )</td>
<td>0.29 (0.25)</td>
<td>0.36</td>
</tr>
<tr>
<td>Boom</td>
<td>( \max { \tau : (P_{\tau+1}, FV_{\tau+1}) &gt; (FV_{\tau+1}, FV_{\tau}) } )</td>
<td>13.4 (0.9)</td>
<td>14</td>
</tr>
<tr>
<td>Trend Dur.</td>
<td>( \max { \tau : P_t - F_t &lt; P_{t+1} - FV_{t+1} &lt; \cdots &lt; P_{t-(\tau-1)} - FV_{t-(\tau-1)} } )</td>
<td>5.8 (2.6)</td>
<td>4</td>
</tr>
</tbody>
</table>

Duffy and Ünver (2006) allowed for weak foresight assuming that the probability of becoming a buyer for noise traders decreases over time, i.e., \( \pi_t = \max \{ 0.5 - \phi t, 0 \} \). In contrast to Duffy and Ünver (2006) we depart from the weak foresight assumption and introduce heterogeneous traders.

The model in Haruvy and Noussair (2006) is based on the standard setup introduced by DeLong et al. (1990a). The authors are mainly interested in fitting prices, while our model also fits trading volumes. Note that in DeLong et al. (1990a) the trading volume is zero during a crash of a positive bubble, while positive trading volumes are observed in experimental asset markets data.20 Thus, it is not surprising that Haruvy and Noussair (2006) underestimate the observed trading volume in 20 out of their 22 sessions. We show below that an accurate estimation of the turnover and dynamic patterns of the trading volume is important for the understanding of bubbles and crashes. We discuss further differences between the two approaches in Appendix C.

Fig. 2. Data and simulated aggregate variables. Dashed lines: simulations. Solid lines: data. Straight dashed line: FV. (a) Prices. (b) Quantities.

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20 In DeLong et al. (1990a) during a crash of a positive bubble, fundamental traders want to sell since the price is above the fundamental value, momentum traders want to sell since the price trend is negative and speculators want to sell too. Zero trade volumes is not a problem for fitting price dynamics.
Furthermore, 

4.1. Simulated and experimental strategy comparison

One of the key features of our data is that we track not only aggregate but also individual behavior of traders. Thus we can use our estimates (see Table 1) to compare the simulated individual behavior with the actual behavior of traders. The main message of this section is that the model does a good job at describing individual data, even though the parameters are estimated using aggregate data. The model also predicts patterns for asset holdings and terminal wealth levels for each type of trader which are consistent with the data.

Fig. 3(a) illustrates the simulated asset holdings of the three classes of traders. We observe that fundamental traders initially marginally accumulate assets and then, when prices are above the fundamental value, gradually sell their units. Speculators accumulate assets while they predict an upward move in trading prices. When speculators predict a decrease in prices, they start selling their assets to noise traders. Fig. 3(b) depicts the individual asset holding paths in an environment consisting only of NZI traders in the call-market analog of Duffy and Ünver (2006). Clearly, Fig. 3(a) and (b) shows that the two models provide different predictions for traders’ asset holdings.

Next we use the simulated trading strategies and individual asset holdings data to identify trader types at a micro-level. To this end, for each trader \( i \), we regressed (controlling for heteroscedasticity) his individual asset holdings in period \( t \), \( u_{i,t} \), separately on the average period \( t \) asset holdings of simulated speculators \( u_{SP}^{i} \) and simulated fundamental traders \( u_{F}^{i} \):

\[
    u_{i,t} = \beta_{SP}^{i} + \beta_{F}^{i} u_{F}^{i} + \epsilon_{i,t} \quad \text{and} \quad u_{i,t} = \beta_{SP}^{i} + \beta_{F}^{i} u_{F}^{i} + \epsilon_{i,t}.
\]  

(10)

Afterwards we tested for every trader whether \( \beta_{F}^{i} > 0 \) or \( \beta_{SP}^{i} > 0 \) at a 5% significance level. Thus, whenever \( \beta_{F}^{i} \) was strictly greater than zero at a 5% level for subject \( i \), we classified that trader as a fundamental trader. We used a similar procedure to identify speculators. Whenever both coefficients were significantly greater than zero, we classified the subject as the type for which the coefficient was more significant. We classified a subject as a noise trader if neither \( \beta_{F}^{i} \) nor \( \beta_{SP}^{i} \) were strictly greater than zero. Table 3 shows the number of identified speculative, fundamental and noise traders for every session. We observe that with the exception of Session 2, the fractions of fundamental traders and speculators are relatively stable across sessions. Overall we identify 12 fundamental traders (27.3%) and 11 speculators (25%) in the data.

Fig. 4(a) shows the simulated asset holding dynamics and Fig. 4(b) depicts the asset holding dynamics of the representative speculative, fundamental, and noise traders from the data. Fig. 4(a) and (b) indicates similar overall trading patterns between representative traders and simulated traders.

Next, we show that our model predicts accurately terminal wealth levels for each trader type, which provides further support for our identification strategy. The simulated speculative, fundamental, and noise traders earn on average 4121, 2876, and 2207 francs respectively. Table 3 reports the median and average terminal wealth levels that each identified trader type earned in the experiment. The data’s and model’s terminal wealth levels for each type are not statistically different from each other for noise and fundamental traders, while for speculators they are statistically different. Furthermore, the ranking

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21 There was one instance in which a subject sold all units in the first period and remained inactive for the remainder of the session. Hence his vector of asset holdings only contains zeros. Clearly, this generates an identification challenge for our regression based approach. We classified this subject as fundamental trader, since the dynamics are better reflected by fundamental trading behavior in our model.

22 We generated “representative” speculative, fundamental, and noise traders by averaging each period \( t \)’s asset holdings for each trader-type group.

23 The one-sample Wilcoxon signed-rank \( p \)-values for noise traders, fundamental traders, and speculators are 0.89, 0.23, and 0.08, respectively. The unit of observation used in the tests is the session average of the terminal wealth for each type of traders. That is the number of observations is five for each one-sample test in this section.
of terminal wealth across trader types predicted by the model is consistent with the one observed in the data: speculators, identified in our data, earn more than fundamental traders (although not significantly more; two-sample Wilcoxon signed-rank p-value = 0.50) and speculative and fundamental traders earn more than noise traders (the two-sample Wilcoxon signed-rank p-values are 0.04 and 0.08, respectively).24

Our trader type identification is also supported by exogenous characteristics of subjects, such as the cognitive reflection test and the accuracy of price forecasts.

Consistently with the terminal wealth ranking, fundamental and speculative traders perform better than noise traders in the CRT test (Mann–Whitney p-values are 0.02 and 0.03, respectively). On the other hand, performances in the CRT tests are not statistically different between speculative and fundamental traders (Mann–Whitney p-value is 0.77). The median and average CRT scores for each trader group are reported in Table 3.

### 5. When and why bubbles start and crash?

The decomposition of traders performed in Section 4 allows us to discuss when and why bubbles start and crash. In this section, we will use the data from our experiment to address the following questions. Who is responsible for the emergence and amplification of bubbles? Who generates the crash?

For each trader group, we compute average asset holdings in each session for every period t and denote them by \( u_t^S \), \( u_t^F \), and \( u_t^N \) for fundamental, speculative, and noise traders respectively. Next we identify several periods of interest in the experiment’s trading horizon: the first trading period, the boom periods, the price-peak period,25 and the crash periods. Finally, for each trader group, we analyze the trading patterns within and between these periods of interest. We present our findings in a series of observations summarized in Fig. 5.26

**Observation 1 (Initial Accumulation).** In period 1, only fundamental traders do not under-predict the trading price, which turns out to be below the fundamental value. As a result, in this period fundamental traders substantially increase their asset holdings.

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24 The unit of observation used in the tests is the session average of the terminal wealth for each type of traders. That is the number of observations is 10 for each two-sample test in this section.

25 Session-specific peak periods are 3, 12, 8, 10, and 7.

26 Fig. 5 illustrates a representative session but clearly our analysis employs all the sessions.
Fig. 5. The mechanics of bubbles and crashes.

Fig. 6. Deviations of price forecasts from actual prices.

Fig. 6 indicates that in the first period fundamental traders submit higher and more accurate price forecasts than speculators and noise traders (Mann–Whitney $p$-values are 0.009 for both tests). Fundamental traders accumulate shares of the undervalued asset by buying mainly from noise traders. The average asset-holdings increase of fundamental traders is from 2 to 2.8 shares. Noise traders decrease their asset holdings from 2 to 1.5 shares.

**Observation 2 (Boom).** During the Boom, speculators increase their asset holdings on average from 2.1 to 3.7. Fundamental traders decrease their asset holdings on average from 2.8 to 0.8.

**Observation 2** indicates that speculators (and to a minor extent noise traders) fuel the bubble by buying shares from fundamental traders. Noise traders increase their asset holdings on average from 1.5 to 1.8.

**Observation 3 (Peak).** At price peaks, speculators decrease their asset holdings on average from 3.7 to 3.3, fundamental traders from 0.8 to 0.7 units, while noise traders increase their asset holdings on average from 1.8 to 2.1.

---

27 We measure the accuracy of price predictions by the deviation of the individual price forecasts from the actual trading prices. For the Mann–Whitney test, we took the median individual price forecasts deviation for each group of traders and each session. As a result we have five observations for each type of traders for each period. Fig. 6 plots the medians of these five observations for each group and period.
At the price peak, speculators sell their assets to noise traders. Fundamental traders decrease their asset holdings only marginally at the price peak, as most fundamental traders have already sold the majority of their shares.

**Observation 4 (Crash).** Noise traders expect a higher price than both fundamental and speculative traders during the crash. Thus they tend to buy during the crash.

As illustrated by Fig. 6, during the crash periods, noise traders over-predict the prices compared to fundamental traders and speculators. This difference is statistically significant, as illustrated by the regression in Appendix D with individual absolute deviations of the price forecast from price as dependent variable. Namely, the interaction terms of the Crash-Dummy with both speculator and fundamental trader dummies are negative and statistically significant, indicating that during the crash both speculators and fundamental traders have lower price expectations than noise traders. This is consistent with noise traders buying during the crash from speculators and fundamental traders, whose asset holdings decrease on average from 3.3 to 0.3 units and from 0.7 to 0.3 units, respectively. The average asset holdings of speculators and fundamental traders in period 15 are 0.3, whereas the average asset holdings of noise traders in period 15 are 3.9.

These observations are consistent with the findings of Griffin et al. (2011), who show that both sophisticated and noise traders actively bought technology shares during the Tech bubble, while at the peak, sophisticated investors pulled capital out and noise traders continued to buy.

To summarize, bubbles and crashes are generated by the interplay between speculators, noise and fundamental traders. First, as prices start below the fundamental value, fundamental traders buy from noise traders initiating a positive price trend. Next, speculators buy from fundamental traders during the boom and sell their shares to noise traders during the crash. Speculation is profitable in this environment due to the presence of noise traders who are willing to buy the overvalued asset in later periods.

### 6. Comparative statics and out-of-sample predictions

In this section we test the predictive power of our model using a different data set. We use our estimated model to predict behavior in another experiment and then compare it with the actual data. In our initial experimental design we endowed subjects with 2 shares and 2000 francs. We ran five additional closed-book call-market sessions with different subjects, in which we only changed the initial endowments to 10 shares and 10,000 francs, keeping the cash/asset ratio constant. Note that we do not re-estimate the model parameters to best fit the price and trading volume paths of the new data. Instead, we simulated the market environment with the new endowments using the previously estimated parameters (see Table 1). This provides a more rigorous test of our model.

Fig. 7 shows the simulated and actual (average) trading prices under the original endowments (2, 2000) as well as the simulated and actual (average) trading prices under the modified endowments (10, 10,000). Table 4 provides a comparison
of simulated and actual bubble measures under the new endowments (see Table 2 for the original endowments bubble measures).

We observe that our model provides accurate comparative statistics: a proportional increase in asset and cash-endowments, which keeps the asset/cash ratio constant, reduces the size of bubbles in experimental closed-book call markets significantly. In the model bubbles are smaller with higher endowments because the probability that noise traders run out of shares, and thus cannot sell when they happen to be sellers, is lower when they are endowed with more shares. Also fundamental traders, whose arbitraging behavior tends to stabilize prices, are longer active in the market if they are initially endowed with a larger amount of shares, given that the parameter S is fixed. Note that in both environments, traders rarely run out of cash. As a result our model predicts smaller bubbles in the treatment with high endowments than in the treatment with the low endowments.

Our model underestimates the new trading volume (not surprisingly, since we did not re-estimate the model and thus kept the number of submission rounds at S = 3), but it predicts correctly that the turnover decreases under higher endowments.

We used the same method to identify the different types of traders in the data. That is, we used the average simulated trading strategies from the original experimental design (2, 2000) to ensure a proper out-of-sample analysis. Table 5 presents the distributions of trader types for each session. Overall, we obtain a similar distribution to the one associated with the original design, namely 26.7% of subjects are identified as speculators and 28.9% as fundamental traders.

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Speculators</th>
<th>Fundamental</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45%</td>
<td>22%</td>
<td>33%</td>
</tr>
<tr>
<td>2</td>
<td>11%</td>
<td>45%</td>
<td>45%</td>
</tr>
<tr>
<td>3</td>
<td>22%</td>
<td>33%</td>
<td>45%</td>
</tr>
<tr>
<td>4</td>
<td>33%</td>
<td>22%</td>
<td>56%</td>
</tr>
<tr>
<td>5</td>
<td>22%</td>
<td>22%</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Trader types summary statistics: out of sample.

The unit of observation used in the Mann–Whitney and Wilcoxon signed-rank tests used in this section is the session average of the terminal wealth for each type of traders. That is the number of observations is 10 for each two-sample test and 5 for each one-sample test in this section.
investigate how changes in the environment affect the dynamics of bubble formation. It can also be used to guide the design of policies that may help to dampen or eliminate bubbles. We leave a more detailed exploration of these topics for further research.

7. Conclusion

This paper provides a behavioral model for experimental closed-book call markets with three types of agents: fundamental, speculative and noise traders. We estimate the parameters of the model using experimental data. The model allows us to identify different types of traders in the data. Moreover, the validity of our trader type identification is confirmed by exogenous characteristics of subjects, such as the cognitive reflection test and the accuracy of price forecasts.

We find that our three types of traders are important to explain the mechanics of bubbles and crashes. Specifically, fundamental traders buy from noise traders in initial periods initiating an upward trend in prices. Next, speculators buy from fundamental traders during the boom. Finally, noise traders buy from speculators during the crash. Importantly, speculation is profitable in this environment due to the presence of noise traders who are willing to buy the overvalued asset in later periods.

We used our model to obtain out-of-sample predictions. The model predicts smaller bubbles if the cash and asset endowments are higher, keeping the cash-to-asset ratio constant. We confirm this prediction with additional out-of-sample data. In addition, it can be used to analyze how changes in the environment (e.g., changes in the dividend processes, trading institutions, etc.) affect the dynamics of bubble formation. For example, Baghestanian et al. (2015) apply this model to different trading institutions including double auction and tâtonnement. Furthermore, the model can also be used to guide the design of policies that may help dampen bubble formation.

Appendix A. Frederick’s cognitive reflection test (CRT)

The Frederick’s CRT consists of the following three questions (answers in brackets):

• A bat and a ball cost $1.10 in total. The bat costs $1 more than the ball. How much does the ball cost? (5)
• If it takes five machines five minutes to make five widgets, how long would it take 100 machines to make 100 widgets? (5)
• In a lake, there is a patch of lilypads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half the lake? (47)

Frederick (2005) shows that the number of correct answers on the previous three questions is positively correlated with subject-specific results on other cognitive reflection ability tests such as the Wonderlic Personnel Test (WPT) and the “need for cognition scale” (NFC). Frederick also shows that there is a tight correlation between CRT scores and scores of subjects on the Scholastic Achievement Test (SAT) and the American College Test (ACT). Table A.6 shows the CRT score distributions over the five sessions. The table reports the fractions of subjects answering one, two, three or none of the questions correctly, the average number of correct answers as well as the median number of correct answers. We observe that typically about 2/3 of the subjects answer 0–1 questions correctly and around 1/3 of the subjects answer 2–3 questions correctly. The last row in Table A.6 shows the CRT score distribution reported in Frederick (2005). Frederick’s sample consisted of 3428 students. We observe a very similar distribution indicating that our sample is fairly representative.

Appendix B. CRT, price forecasts, and terminal wealth

In this section we show that there is a positive correlation between exogenous characteristics of subjects, such as CRT and initial price forecast, and their terminal wealth.

The first two columns of Table B.7 show that there is a statistically significant positive correlation between the CRT performance (i.e., the number of correct answers) and terminal wealth.  

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30 A 12-min, 50-item test used by the National Football League and other employers to assess the intellectual abilities of their prospective hires.
31 An 18-item test, which measures the endorsement of statements like “the notion of thinking abstractly is appealing to me” (Cacioppo et al., 1984).
32 This test was developed by Frederick (2005), See Appendix A for more details.
33 Naturally, terminal wealth does not include earnings from forecast accuracy and CRT.
34 Since the specific dividend paths could affect terminal wealth levels we followed also a secondary approach: for each subject we divided their individual terminal wealth levels, $x_i$, by the session-specific aggregate terminal wealth levels, $\sum x_i$. The correlation between the resulting fractions, $\frac{x_i}{\sum x_i}$, and the CRT results remain unchanged and significant (p-values are 0.03 and 0.04 for Kendal and Spearman correlations, focusing on deviations from the FV respectively; p-values are 0.04 and 0.041 for Kendal and Spearman correlations, focusing on deviations from the actual trading price, respectively). CRT results remain unchanged and significant if we use one-sided tests (p-values are 0.059 and 0.06 for Kendal and Spearman correlations, respectively). This procedure controls for session-specific dividend realization paths: a high dividend realization at an early trading period may affect the price dynamics differently than a high dividend realization at a later point in time.
Table A.6
Cognitive reflection test: scores by session.

<table>
<thead>
<tr>
<th>Session</th>
<th>N correct answers</th>
<th>Zero</th>
<th>One</th>
<th>Two</th>
<th>Three</th>
<th>Average</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td></td>
<td>0.318</td>
<td>0.341</td>
<td>0.136</td>
<td>0.205</td>
<td>1.23</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td></td>
<td>0.56</td>
<td>0.22</td>
<td>0.11</td>
<td>0.11</td>
<td>0.78</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td></td>
<td>0.125</td>
<td>0.5</td>
<td>0.125</td>
<td>0.25</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>S3</td>
<td></td>
<td>0.44</td>
<td>0.22</td>
<td>0.11</td>
<td>0.22</td>
<td>1.11</td>
<td>1</td>
</tr>
<tr>
<td>S4</td>
<td></td>
<td>0.11</td>
<td>0.55</td>
<td>0.11</td>
<td>0.22</td>
<td>1.44</td>
<td>1</td>
</tr>
<tr>
<td>S5</td>
<td></td>
<td>0.34</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>1.33</td>
<td>1</td>
</tr>
<tr>
<td>Frederick (2005)</td>
<td></td>
<td>0.33</td>
<td>0.28</td>
<td>0.23</td>
<td>0.17</td>
<td>1.24</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A.7
Correlation between terminal wealth and exogenous subjects’ characteristics.

<table>
<thead>
<tr>
<th>CRT correct answers</th>
<th>Kendall</th>
<th>Spearman</th>
<th></th>
<th>Prediction₁ − FV₁</th>
<th>Kendall</th>
<th>Spearman</th>
<th></th>
<th>Prediction₁ − P₁</th>
<th>Kendall</th>
<th>Spearman</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20 (0.09)</td>
<td>0.26 (0.09)</td>
<td>-0.22 (0.04)</td>
<td>-0.33 (0.03)</td>
<td>-0.21 (0.05)</td>
<td>-0.30 (0.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: p-values are reported in parentheses.

Columns 3 and 4 report a statistically significant negative correlation between the terminal wealth and forecast deviation from the fundamental value for the 1st period price. That is, the closer is the subject’s price forecast in period 1 to the fundamental value of period 1, the higher is his terminal wealth.

Alternatively, the price forecast in period 1 can be evaluated against the realized price in that period. Columns 5 and 6 report statistically significant negative correlation between this measure and terminal wealth. The results are robust to using the alternative benchmark: better initial price predictions result in higher terminal wealth.35

Appendix C. Individual behavior: DeLong et al. (1990a)

In what follows, we analyze our data through the lenses of the well-established model of DeLong et al. (1990a) (DSSW) as applied to experimental markets by Haruvy and Noussair (2006) (HN). The main point is to show that our model provides a better framework to understand our data. DSSW present a heterogeneous agent model consisting of trend-chasers or feedback traders, fundamental or passive traders, and rational speculators.36

C.1. Identification of trader types using Haruvy and Noussair (2006)

Following DSSW and Haruvy and Noussair (2006) (HN), the functional forms of the demand functions, D(·), of trend chasers (feedback traders), fundamental or passive traders (FV traders) and speculators are given in Eqs. (11)–(13), respectively:

\[ D(p_{t-1}, p_{t-2}) = -\delta + \beta(p_{t-1} - p_{t-2}) \]  (11)

\[ D(p_t) = -\alpha(p_t - FV_t) \]  (12)

\[ D(p_{t+1}, p_t) = \gamma(E(p_{t+1}) - p_t), \]  (13)

where \( \delta, \beta, \alpha \) and \( \gamma \) are all non-negative parameters, \( E(·) \) is the expectations operator, \( FV_t \) is the fundamental value of the asset in period \( t \), and \( p_t \) is the uniform transaction price in period \( t \).

We use the same procedure as HN to classify every subject (for every session separately) into one of the three types:

(a) if the change in a subjects’ asset holdings does not have the opposite sign as the difference of the transaction prices in periods \( t-1 \) and \( t-2 \), she is classified as a trend-chaser for that particular period;

(b) if the difference between the fundamental value of the asset in period \( t \) and the transaction price in period \( t \) does not have the opposite sign as the change in a subjects’ asset holdings in period \( t \), she is classified as fundamental trader in period \( t \);

---

35 Similarly to the CRT–wealth correlations, the results are robust to re-calculating the correlations using the session-wide normalized terminal wealth levels as an experimental performance measure.

36 Their model provides a theoretical framework to investigate the potential consequences of destabilizing rational speculation in financial markets. For similar and additional follow-up contributions see also Hart and Kreps (1986), DeLong et al. (1990b), Cutler et al. (1990), Hirshleifer (2001), Abreu and Brunnermeier (2003) and Brunnermeier and Pedersen (2005).
(c) if the difference between the average transaction price in period $t + 1$ and the average transaction price in period $t$ does not have the opposite sign as the change in a subjects’ asset holdings for that period, she is classified as speculator in period $t$.

The number of periods in which each subject belongs to every single type are counted, yielding a vector of three scores for every participant. A subject is classified as agent of the type for which he has the highest score, provided that the score is greater than or equal to $\tau = 8$. If a subject has a maximum score lower than 8, that subject does not belong to any of the three types and is classified as “other”. If a subject’s maximum score is the same for two (three) types and is at least 8, the participant is assigned a weight of $1/2$ ($1/3$) to each type for which he has the same maximum score.

We applied this approach to our data and we obtained very similar trader types distributions to Haruvy and Noussair (2006) (see Table VI in their paper). Namely 33.7% of our subjects are passive traders, 39.4% are feedback traders, 26.9% are speculators, and 0% are others.

C.2. Comparison between Haruvy and Noussair (2006) and our model

Haruvy and Noussair (2006) estimate the type distribution and the associated demand parameters by minimizing the root mean square error (RMSE) between average session prices and simulated prices. Before applying their methodology to our data, we perform a simple statistical test to check whether our data are indeed generated by a model consistent with the assumptions of DSSW and HN. In particular, we follow the approach of Bossaerts et al. (2007), we rewrite the demand functions (11)–(13) as:

$$D_{i,t}(\cdot) = D_{i,t}(\cdot) + \epsilon_{i,t},$$

where $i$ represents one of the three types from above. We add a mean-zero-noise-term, $\epsilon_{i,t}$, to the demand functions of every type, capturing random factors which may affect individual demands. Along the lines of Bossaerts et al. (2007), we assume that these noise terms are identically and independently distributed (iid) over time but not necessarily across types.

Next, letting $N_1, N_2$ and $N_3$ be the fractions of feedback traders, passive traders and speculators, respectively (which sum to one), we compute the equilibrium prices as:

$$p_t = -N_1\delta + N_2\alpha FV_t + N_3\gamma \bar{p}_{t+1} + N_1\beta p_{t-1} - N_1\beta p_{t-2} + f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}),$$

where $f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})$ is a mean zero, iid random noise term. Using the belief formation structure of HN, the first step equilibrium prices are

$$p_t = -N_1\delta + N_2\alpha FV_t + N_3\gamma FV_{t+1} + N_1\beta p_{t-1} - N_1\beta p_{t-2} + f(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}).$$

By using the fact that in experimental asset markets the fundamental value decreases in every period by the average dividend-payment, $\bar{d}$, i.e., $FV_{t+k} = FV_t - \bar{d}k$, iterating (15) one period forward and taking expectations, we obtain the following Level-1 expectations for speculators:

$$E[p_{t+1}] = FV_t - N_1\delta + N_3\gamma \bar{d} + N_1\beta - \bar{d} + N_2\alpha FV_t + N_3\gamma \bar{d} (p_t - p_{t-1}).$$

Plugging (16) into (14) and collecting terms yields a price process of the form:

$$p_t = m_0 + m_1 FV_t + m_2 p_{t-1} + m_3 p_{t-2} + \eta_t,$$

where $m_0 = -\frac{N_1\beta N_2\alpha}{N_2\alpha + N_3\gamma}$, $m_1 = \frac{1}{N_2\alpha + N_3\gamma}$, $m_2 = \frac{N_1\delta + N_3\gamma \bar{d}}{N_2\alpha + N_3\gamma}$, $m_3 = -\frac{N_1\beta}{N_2\alpha + N_3\gamma}$, and $\eta_t$ is an iid noise term with mean zero.

Based on the non-negativity constraints on the parameters in the models of DSSW and HN, the coefficients $m_2$ and $m_3$ should have opposite signs and $m_3$ should not be equal to zero. Thus the validity of the non-negativity constraints imposed by DSSW and HN can be tested via the following null hypotheses:

$$H_0 : (\text{sign}(m_2) \neq \text{sign}(m_3)) \text{ and } m_3 \neq 0.$$  

---

37 See Eqs. (1)–(3) in DSSW for $\alpha$, $\beta$ and $\gamma$. See HN, p. 1142, for the non-negativity constraint on $\delta$.

38 The coefficients are well defined if $1 - \frac{N_1\gamma \bar{d}}{N_2\alpha + N_3\gamma} \neq 0$. If that is the case, note that $1 - \frac{N_1\gamma \bar{d}}{N_2\alpha + N_3\gamma}$ can be either greater than or less than zero. If it is greater than zero, $m_2$ is non-negative and $m_3$ is strictly positive (if all types of agents are present in the model). A similar argument holds if $1 - \frac{N_1\gamma \bar{d}}{N_2\alpha + N_3\gamma}$ is less than zero.
Table C.8
Testing DeLong et al. (1996a)∗.  

<table>
<thead>
<tr>
<th></th>
<th>(1) Random effects</th>
<th>(2) Fixed effects</th>
<th>(3) Between</th>
<th>(4) Arellano, Bond</th>
<th>(5) Blundell, Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>m2</td>
<td>0.510** (0.032)</td>
<td>0.510* (0.100)</td>
<td>2.21 (0.466)</td>
<td>0.510** (0.016)</td>
<td>0.543** (&lt;0.001)</td>
</tr>
<tr>
<td>m3</td>
<td>−0.059 (0.713)</td>
<td>−0.070 (0.678)</td>
<td>−0.370 (0.783)</td>
<td>−0.071 (0.604)</td>
<td>−0.027 (0.809)</td>
</tr>
<tr>
<td>N</td>
<td>60</td>
<td>60</td>
<td>68</td>
<td>55</td>
<td>60</td>
</tr>
</tbody>
</table>

p-values in parentheses.
* p < 0.10
** p < 0.05

Table D.9
Comparison of forecast deviations from price across trader types.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FV-Dummy</td>
<td>−5.433 (0.37)</td>
<td>2.462 (0.40)</td>
</tr>
<tr>
<td>Spec-Dummy</td>
<td>0.952 (0.06)</td>
<td>26.35 (2.89)</td>
</tr>
<tr>
<td>Period1-Dummy</td>
<td>−105.0** (&lt;−7.65)</td>
<td>127.1† (5.93)</td>
</tr>
<tr>
<td>Crash-Dummy</td>
<td>106.1† (17.13)</td>
<td>−65.2** (−4.61)</td>
</tr>
<tr>
<td>FV-Dummy × Period1-Dummy</td>
<td>−78.79** (−4.44)</td>
<td>−78.79** (−4.44)</td>
</tr>
<tr>
<td>Spec-Dummy × Period1-Dummy</td>
<td>−56.14** (−4.61)</td>
<td>−56.14** (−4.61)</td>
</tr>
<tr>
<td>Spec-Dummy × Crash-Dummy</td>
<td>−78.79** (−4.44)</td>
<td>−78.79** (−4.44)</td>
</tr>
<tr>
<td>Constant</td>
<td>−6.400 (−0.22)</td>
<td>−33.30 (−2.43)</td>
</tr>
<tr>
<td>N</td>
<td>660</td>
<td>660</td>
</tr>
<tr>
<td>R²</td>
<td>0.0004</td>
<td>0.207</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the absolute deviation of the price forecast from the actual price for a given subject in a given period. Driscoll and Kraay (1998) standard errors, estimated using the procedure implemented in STATA as xstcc, developed by Hoechle (2007). t-statistics in parentheses.
* p < 0.10.
** p < 0.05.
*** p < 0.01.

To test (18) we take first differences of the variables in (17) and estimate the coefficients m2 and m3 via standard dynamic panel methods.39 Columns (4) and (5) of Table C.8 show the Arellano and Bond (1991) (AB) and Blundell and Bond (1998) (BB) estimates for m2 and m3 respectively. Note that for all our estimates both coefficients either have the same sign or we cannot reject the null hypothesis that m3 is equal to zero.40 Since we can reject the joint hypothesis in (18), we conclude that the DSSW model does not generate price sequences consistent with our data.

Appendix D. Forecast accuracies and types

The dependent variable is the absolute deviation of the price forecast from the actual price for a given subject in a given period (Table D.9). Since the actual price is determined by actions of all subjects in a given session, in addition to serial correlation, we can expect spatial (cross-sectional) correlation within a session. The typical way of dealing with this problem would be clustering the standard errors. The number of sessions, however, is too small for clustering the standard errors at the session level – typically, it is recommended to have at least 30–40 clusters. Thus, instead of clustering, we estimate using Driscoll and Kraay (1998) standard errors, which are robust to arbitrary spatial and serial correlation. Importantly, Vogelsang (2012) shows that the Driscoll and Kraay (1998) standard errors are consistent even in specifications with time fixed effects. This allows us to include the Period1-Dummy which is 1 if period is 1 and 0 otherwise and Crash-Dummy which

39 Note that the first difference of FV is constant and therefore dropped from the estimation.
40 Columns (1)–(3) show the fixed effect, random effects and between estimates for m2 and m3 respectively.
is 1 for periods 12 through 15 and 0 otherwise. For estimation, the procedure is implemented in STATA as xstcc, developed by Hochtyle (2007).

References

Crawford, V.P., Iriberri, N., 2007. Level-k auctions: can a non-equilibrium model of strategic thinking explain the winner’s curse and overbidding in private-value auctions? Econometrica 75 (6), 1721–1770.