Instructor
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Office hours
Mondays and Wednesdays, 10:30 am - 11:30 am or by appointment (Rawles Hall 432)

Meeting time and location
Day/Time: Mondays, Wednesdays and Fridays 9:05 am-9:55 am.
Location: Rawles Hall 104.
Attending the lectures is strongly encouraged.

Prerequisites

M511-512 (Real Variables I-II)

- Familiarity with the following topics will be helpful in understanding the materials in this class: Lebesgue measure theory, Convergence theorems, Differentiation and Integration, Normed linear spaces, Function spaces, Linear functionals, Banach spaces, Hilbert spaces, Schwartz class.
- You can also talk to the instructor for further reading of some prerequisite materials if needed.

Textbook

About the course
This course is an introduction to the energy methods and maximum principle methods in studying second-order partial differential equations. This course will cover the following topics:

- Distribution theory, Sobolev spaces
- Weak solutions to elliptic PDE
- Elliptic regularity theory
- Maximum principles
- Spectral theory of elliptic operators
- Linear parabolic and hyperbolic equations.

Most of the materials are taken from Chapters 5-7 and Appendix D in Evans’ book. Below is a tentative detailed plan.

Week 1: An overview of the course, Hölder spaces, Weak derivatives, Sobolev spaces
Week 2: Approximating a function in a Sobolev space by smooth functions, Extension Theorem
Week 3: Traces, Sobolev inequalities: Isoperimetric inequality, Gagliardo-Nirenberg-Sobolev
inequality

**Week 4:** Sobolev inequalities continued: Morrey’s inequality, Poincaré’s inequality, BMO; Rellich-Kondrachov compactness Theorem

**Week 5:** Weak solutions to second-order elliptic equations, Lax-Milgram Theorem, Energy estimates

**Week 6:** Existence theorems for weak solutions, Fredholm alternative

**Week 7:** Elliptic regularity theory: difference quotient and interior regularity

**Week 8:** Elliptic regularity: difference quotient and boundary regularity

**Week 9:** Maximum principles: weak maximum principle, strong maximum principle, Hopf’s lemma, Harnack’s inequality

**Week 10:** Eigenvalues and Eigenfunctions

**Week 11:** Second-order parabolic equations and existence of weak solutions via Galerkin approximations (approximate solutions, energy estimates and compactness)

**Week 12:** Regularity, Maximum principles, Harnack’s inequality for second-order parabolic equations

**Week 13:** Second-order hyperbolic equations and existence of weak solutions via Galerkin approximations

**Week 14:** Regularity and finite propagation speed for second-order hyperbolic equations

**Week 15:** Monotonicity formulas (time permitting)

**Grading policy**

Homework: 40%, Midterm: 20%, Final exam: 40%

**Homework**

There will be 5 written homework assignments to be handed in.

**Exams**

There will be one take-home midterm exam and one take-home final exam given at the end of the term.

**Recommendation for further reading**

The following books will be useful for a deeper treatment of some materials covered in the class: