CLEARING UP THE FISCAL MULTIPLIER MORASS: A BAYESIAN PERSPECTIVE

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with Eric M. Leeper (IU) and Nora Traum (NC State)

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SED
1. Present Value Multiplier:

\[
\text{Present Value Multiplier}(Q) = \frac{\sum_{t=0}^{Q} E_t \left( \prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta Y_{t+Q}}{\sum_{t=0}^{Q} E_t \left( \prod_{i=0}^{Q} R_{t+i}^{-1} \right) \Delta G_{t+Q}}
\]

2. Impact Multiplier: \( Q = 0 \)
HOW BIG/SMALL ARE FISCAL MULTIPLIERS?

IMF Working Paper 10/73 March 2010

1. 17 coauthors: model builders for policy institutions

2. Seven Structural Models: QUEST, GIMF, FRB-US, SIGMA BoC-GEM, OECD Fiscal, NAWM.

3. Conclude: “Robust finding across all models that fiscal policy can have sizeable output multipliers.”
FIGURE 1: Estimated Impact on GDP of Increase in Government Purchases of 1 Percent of GDP
Robust Finding?

- Cogan, Cwik, Taylor and Wieland (2010), Cwik and Wieland (2010)
  - Multipliers less than 1

- Uhlig (2010)
  - Long-run multipliers negative
Figure 5. Output and Government Spending: 40 years.
Motivation

Why do policy models yield very different conclusions for multipliers even when conditioning on same data set?

Answer: Multipliers are *conditional* statistics, so different specifications $\rightarrow$ different multipliers
Motivation

Why do policy models yield very different conclusions for multipliers even when conditioning on same data set?

Answer: Multipliers are *conditional* statistics, so different specifications $\rightarrow$ different multipliers

IMF WP10/73’s Response to Uhlig (2010) and Cogan et al. (2010):

- include hand-to-mouth agents
- focus on short-run & temporary stimulus
- model different types of fiscal-monetary interactions (Davig-Leeper (2009))
Open Question: To what extent does a DSGE model force a particular multiplier on the data?

- “black box” problem of DSGE models
- use Bayesian methodology to address issue
Our Contribution

• Build suite of nested models to determine important elements for multipliers.

• Use modified prior predictive analysis (PPA) to understand \textit{a priori} what restrictions are generated by DSGE model

• More general message: What does it mean for a prior to be “flat”?

• Distribution of object of interest should be “flat” relative to economic question at hand
**Findings**

- Model restrictions impose tight ranges on multipliers
- Rigidities and hand-to-mouth agents key for long run multipliers $> 0$
- Most important features for multiplier variation:
  - gov. spending process
  - hand-to-mouth agents
  - monetary-fiscal interactions
**Review of PPA**

- Standard Exercise [Lancaster (2004), Geweke (2010)]: used to evaluate model’s adequacy for given feature of data *before* estimation stage (model evaluation)

- $\theta$ parameters, $y$ data, $\omega$ vector of interest

\[
\begin{align*}
\theta^{(m)} & \sim p(\theta) \\
y^{(m)} & \sim p(y|\theta^{(m)}) \\
\omega^{(m)} & \sim p(\omega|y^{(m)}, \theta^{(m)})
\end{align*}
\]

- Compare distribution of $\omega$ to data
REVIEW OF PPA

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• \( \theta \) parameters, \( y \) data, \( \omega \) vector of interest

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\]

• Compare distribution of \( \omega \) to data

• computationally inexpensive
MODIFIED PPA

• Issue: What is multiplier in data? Requires model and identification

• $A_J$ DSGE model, $\theta$ parameters of DSGE, $\omega = $ multipliers

  Draw $\theta^{(m)} \sim p(\theta)$
  Solve DSGE Model
  Calculate $\omega^m|\theta^{(m)}$
  Form $p(\omega|A_j)$
MODIFIED PPA

• Issue: What is multiplier in data? Requires model and identification

• $A_j$ DSGE model, $\theta$ parameters of DSGE, $\omega = $ multipliers

  Draw $\theta^{(m)} \sim p(\theta)$
  Solve DSGE Model
  Calculate $\omega^m|\theta^{(m)}$
  Form $p(\omega|A_j)$

• PPA gives entire range of possible multipliers
Our Model

1. forward-looking, optimizing agents
2. utility from consumption and leisure
3. capital and labor inputs in production
4. monopolistic competition
5. nominal & real frictions
6. fiscal and monetary policy
7. open economy features
Nested Specifications

- Model 1: Basic RBC
Nested Specifications

- Model 1: Basic RBC
- Model 2: RBC with real frictions
Nested Specifications

• Model 1: Basic RBC

• Model 2: RBC with real frictions

• Model 3: NK model with sticky prices and wages
NESTED SPECIFICATIONS

- Model 1: Basic RBC
- Model 2: RBC with real frictions
- Model 3: NK model with sticky prices and wages
- Model 4: NK model with hand-to-mouth agents
Nested Specifications

- Model 1: Basic RBC
- Model 2: RBC with real frictions
- Model 3: NK model with sticky prices and wages
- Model 4: NK model with hand-to-mouth agents
- Model 5: NK model with open economy features
**Model 1: Basic RBC**

- CRRA, time-separable utility

\[
E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\xi}}{1+\xi} \right]
\]

- Cobb-Douglas production

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha}
\]

- Law of motion for capital:

\[
K_t = I_t + (1 - \delta)K_{t-1}
\]
MODEL 1: BASIC RBC

- GBC:

\[ B_t + \tau^K_t R^K_t K_{t-1} + \tau^L_t W_t L_t + \tau^C_t C_t = R_{t-1} B_{t-1} + G_t + Z_t \]

- capital tax, labor tax, government consumption, transfers follow

\[ \hat{X}_t = \rho_x \hat{X}_{t-1} + (1 - \rho_x) \gamma_x \hat{s}^b_{t-1} + \epsilon^x_t \]

where \( \hat{s}^b_{t-1} = B_{t-1}/Y_{t-1} \)
Model 1: Basic RBC

- 5,000 draws from priors: $\gamma \sim N^+(2, 0.6)$, $\xi \sim N^+(2, 0.6)$, $\rho_x \sim B(0.5, 0.2)$, $\gamma_x \sim N^+(0.2, 0.05)$

- Priors similar to Smets and Wouters (2003) and others

- Other parameters fixed at well known values (e.g., $\beta = 0.99$)
## Model 1: Basic RBC

<table>
<thead>
<tr>
<th>Variable</th>
<th>Impact</th>
<th>4 quart.</th>
<th>10 quart.</th>
<th>25 quart.</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Prob}(PV \frac{\Delta Y}{\Delta G} &gt; 1)$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta C}{\Delta G} &gt; 0)$</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta I}{\Delta G} &gt; 0)$</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
<td>&lt;0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>
**Model 1: Basic RBC**

- **Total Output PV**
  - Graph showing the relationship between total output and its present value (PV).
  - X-axis: Time period (0-200).
  - Y-axis: Present value.

- **Total Consumption PV**
  - Graph showing the relationship between total consumption and its present value.
  - X-axis: Time period (0-200).
  - Y-axis: Present value.

- **Wealth Consumption PV**
  - Graph depicting the wealth consumption and its present value.
  - X-axis: Time period (0-200).
  - Y-axis: Present value.

- **Subst. Consumption PV**
  - Graph illustrating the substitution consumption and its present value.
  - X-axis: Time period (0-200).
  - Y-axis: Present value.
Model 1: Basic RBC

Intuition Straightforward:


- $\uparrow G \rightarrow$ negative wealth and substitution effects, crowding out

- Consumption, Investment falls

- Increase in public demand cannot offset decrease in private demand
Model 2: RBC with Real Frictions

Add to Model 1

- Habit formation in utility

\[ E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t - \theta C_{t-1})^{1-\gamma}}{1 - \gamma} - \frac{L_t^{1+\xi}}{1 + \xi} \right] \]

\[ \theta \sim B(0.5, 0.2) \]

- Capacity utilization: \( \psi(v_t) \) cost per unit of \( K \)

\[ v = 1, \psi(1) = 0, \frac{\psi''(1)}{\psi'(1)} = \frac{\psi}{1-\psi}, \psi \sim B(0.6, 0.15) \]
Model 2: RBC with Real Frictions

• Investment adjustment costs

\[ K_t = (1 - \delta)K_{t-1} + \left[ 1 - s \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \]

where \( s(1) = s'(1) = 0 \), and \( s''(1) = s > 0 \), \( s \sim N(6, 1.5) \)
Model 2: RBC with Real Frictions

- Investment adjustment costs

\[ K_t = (1 - \delta)K_{t-1} + \left[ 1 - s \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \]

where \( s(1) = s'(1) = 0 \), and \( s''(1) = s > 0 \), \( s \sim N(6, 1.5) \)

- Aggregate resource constraint:

\[ Y_t = C_t + G_t + I_t + \psi(v_t)K_{t-1} \]
**Model 2: RBC with Real Frictions**

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<tbody>
<tr>
<td>$\text{Prob}(PV \frac{\Delta Y}{\Delta G} &gt; 1)$</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta C}{\Delta G} &gt; 0)$</td>
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Model 2: RBC with Real Frictions

Total Output PV

Total Consumption PV

Wealth Consumption PV

Subst. Consumption PV
Model 2: RBC with Real Frictions
Model 2: RBC with Real Frictions

- More dispersed range of multipliers

- Agents and firms want to smooth consumption and investment

- Smaller wealth effects (agents care about $c_t, c_{t-1}$), larger substitution effects (more sensitive to price changes)

- Same policy implications
Model 3: Sticky Price & Wage

Add to Model 2

- Monopolistically competitive intermediate goods & labor services

\[ Y_t = \left[ \int_0^1 y_t(i) \frac{1}{1+\eta_p} \, di \right]^{1+\eta_p} \]

- Price & wage stickiness via Calvo (1983)
Model 3: Sticky Price & Wage

Add to Model 2

- Monopolistically competitive intermediate goods & labor services

\[ Y_t = \left[ \int_0^1 y_t(i) \frac{1}{1+\eta p} \, di \right]^{1+\eta p} \]

- Price & wage stickiness via Calvo (1983)
  - prob. \( 1 - \omega_p \) re-optimize
  - prob. \( \omega_p \) partial indexation: \( p_t = \pi_{t-1}^{x_p} p_{t-1} \)
Model 3: Sticky Price & Wage

- Monetary policy via Taylor rule

\[ \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[ \phi_{\pi} \hat{\pi}_t + \phi_y \hat{Y}_t \right] + \epsilon_t \]
# Model 3: Sticky Price & Wage

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<tr>
<td>$\text{Prob}(PV \frac{\Delta Y}{\Delta G} &gt; 1)$</td>
<td>0.35</td>
<td>0.01</td>
<td>$&lt;0.01$</td>
<td>0.00</td>
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Model 3: Sticky Price & Wage

Total Output PV

Total Consumption PV

Wealth Consumption PV

Subst. Consumption PV
MODEL 3: STICKY PRICE & WAGE

Total Output PV

Total Consumption PV

Wealth Consumption PV

Subst. Consumption PV
Model 3: Sticky Price & Wage

- Much larger multipliers

- sticky prices → firms respond to a government spending increase by increasing production rather than their price

- Sub Effect: sticky wages → wage substitution effect is now often positive (increasing real wages)

- CB doesn’t raise nominal rate enough initially to keep real rate from falling

- Wealth Effect: initial real value of debt higher (than flex price case), requires larger fiscal adjustment
Model 4: Non-Savers

Add to Model 3

- Non-savers consume entire per period disposable income

$$c_t^N = (1 - \tau_t^L)w_tL_t^N + Z_t^N$$
MODEL 4: NON-SAVERS

Add to Model 3

- Non-savers consume entire per period disposable income

\[ c_t^N = (1 - \tau_t^L)w_t L_t^N + Z_t^N \]

- Set wage to average of savers
Model 4: Non-Savers

Add to Model 3

- Non-savers consume entire per period disposable income

\[ c_t^N = (1 - \tau_t^L)w_tL_t^N + Z_t^N \]

- Set wage to average of savers

- Crucial parameter: percentage of non-savers

\[ \mu \sim B(0.3, 0.1) \]
## Model 4: Non-Savers

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<tr>
<td>$\text{Prob}(PV \frac{\Delta Y}{\Delta G} &gt; 1)$</td>
<td>0.88</td>
<td>0.32</td>
<td>0.07</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta C}{\Delta G} &gt; 0)$</td>
<td>0.84</td>
<td>0.46</td>
<td>0.18</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta L}{\Delta G} &gt; 0)$</td>
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Model 4: Non-Savers

- Total Output PV
- Total Consumption PV
- Wealth Consumption PV
- Subst. Consumption PV
MODEL 4: NON-SAVERS

Total Output PV

Total Consumption PV

Wealth Consumption PV

Subst. Consumption PV
Model 4: Non-Savers

- Much, much larger impact multipliers, similar long-run multipliers
- Intuition straightforward: nonsavers are nonsavers
- *The* most crucial parameter value
MODEL 5: OPEN ECONOMY

Add to Model 4

- Two large symmetric countries ($H$ & $F$)
- Complete financial markets
- $C$ and $I$ consist of domestic and imported goods

\[ Q_t^C = \left[ (1 - \nu_c) \frac{1}{\mu_c} (C_H^t)^{\frac{\mu_{C-1}}{\mu_C}} + \nu_c \frac{1}{\mu_C} (C_F^t)^{\frac{\mu_{C-1}}{\mu_C}} \right]^{\frac{\mu_C}{\mu_{C-1}}} \]

- $G$ non-traded
Model 5: Open Economy

- Home market domestic demand:
  \[ y_t^H(i) = Y_t^H \left( \frac{p_t^H(i)}{P_t^H} \right)^{-\frac{1+\eta_p^p}{\eta_p^p}} \]

- Home market foreign demand:
  \[ m_t(i) = M_t^* \left( \frac{p_t^{H*}(i)}{P_t^{H*}} \right)^{-\frac{1+\eta_p^p}{\eta_p^p}} \]

- Local currency pricing
**Model 5: Open Economy**

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<tr>
<td>$\text{Prob}(PV \frac{\Delta Y}{\Delta G} &gt; 1)$</td>
<td>0.81</td>
<td>0.27</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\text{Prob}(PV \frac{\Delta C}{\Delta G} &gt; 0)$</td>
<td>0.82</td>
<td>0.48</td>
<td>0.23</td>
<td>0.02</td>
<td>&lt;0.01</td>
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<td>$\text{Prob}(PV \frac{\Delta I}{\Delta G} &gt; 0)$</td>
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MODEL 5: OPEN ECONOMY

Total Output PV

Total Consumption PV

Wealth Consumption PV

Subst. Consumption PV
**Model 5: Open Economy**

- **Total Output PV**
- **Total Consumption PV**
- **Wealth Consumption PV**
- **Subst. Consumption PV**
Model 5: Open Economy

- smaller multipliers

- import-substitution effect: increases in government expenditures induce a substitution away from domestically produced goods towards imported goods.

- Multipliers are smaller still when government spending is a traded good as part of the increase in government spending is “leaked” to the foreign country
How much do multipliers vary on average due to particular parameter?

- Draw $\tilde{\theta} = [\tilde{\theta}_1 \ldots \tilde{\theta}_n]'$ from $p(\theta)$. Calculate $\tilde{\omega}|\tilde{\theta}_n$

- Let $\tilde{\theta}^i = [\tilde{\theta}_1 \ldots E[\theta_i] \ldots \tilde{\theta}_n]'$. Calculate $\tilde{\omega}^i|\tilde{\theta}^i$

- Calculate $\sqrt{\frac{\sum_{j=1}^{M} (\tilde{\omega}_j - \tilde{\omega}_j^i)^2}{M}}$
### RMSDs for NK Open Economy Model.

<table>
<thead>
<tr>
<th>Impact $\frac{\Delta C}{\Delta G}$</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\mu$, fraction of non-savers</td>
<td>0.28</td>
</tr>
<tr>
<td>$\rho_G$, lagged govt cons resp.</td>
<td>0.16</td>
</tr>
<tr>
<td>$\theta_c$, habit formation</td>
<td>0.12</td>
</tr>
<tr>
<td>$\rho_r$, lagged interest rate resp.</td>
<td>0.12</td>
</tr>
<tr>
<td>$\gamma$, risk aversion</td>
<td>0.09</td>
</tr>
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<tr>
<th>$PV\infty$ $\frac{\Delta C}{\Delta G}$</th>
<th>Value</th>
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<td>$\rho_G$, lagged govt cons resp.</td>
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</tr>
<tr>
<td>$\gamma$, risk aversion</td>
<td>0.08</td>
</tr>
<tr>
<td>$\rho_r$, lagged interest rate resp.</td>
<td>0.06</td>
</tr>
<tr>
<td>$\omega_w$, wage stickiness</td>
<td>0.06</td>
</tr>
<tr>
<td>$\xi$, inverse Frisch labor elast.</td>
<td>0.06</td>
</tr>
</tbody>
</table>
### RMSDs for NK Open Economy Model.

| Impact $\frac{\Delta Y}{\Delta G}$ |  
|----------------------------------|---|
| $\mu$, fraction of non-savers | 0.19 |
| $\rho_G$, lagged govt cons resp. | 0.12 |
| $\psi$, capital utilization | 0.15 |
| $\rho_r$, lagged interest rate resp. | 0.10 |
| $\theta_c$, habit formation | 0.08 |

| $PV_\infty \frac{\Delta Y}{\Delta G}$ |  
|-----------------|---|
| $\rho_G$, lagged govt cons resp. | 0.30 |
| $\rho_r$, lagged interest rate resp. | 0.07 |
| $\omega_w$, wage stickiness | 0.06 |
| $\xi$, inverse Frisch labor elast. | 0.06 |
| $\phi_\pi$, interest rate resp. to inflation | 0.05 |
**Alternative MP-FP Interaction**

- Multipliers depend on MP-FP interaction
  - Davig & Leeper (2009), Christiano, Eichenbaum, Rebelo (2009)

- Calculate multipliers for passive monetary and active fiscal policy regime
  - FP unconstrained: doesn’t control $B$ growth
  - MP satisfies equilibrium conditions: $R$ adjusts less than 1-1 with $\pi$
## Model 5: Open Economy PMAF

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<tr>
<td>Prob($PV \frac{\Delta Y}{\Delta G} &gt; 1$)</td>
<td>1.00</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td>Prob($PV \frac{\Delta C}{\Delta G} &gt; 0$)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Prob($PV \frac{\Delta I}{\Delta G} &gt; 0$)</td>
<td>0.73</td>
<td>0.53</td>
<td>0.45</td>
<td>0.44</td>
<td>0.47</td>
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CONCLUSION

• DSGE specification matters! If not careful, results can be imposed on data

• Most important features for multiplier variation:
  • gov. spending process
  • hand-to-mouth agents
  • monetary-fiscal interactions
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• Broader message: use PPA to shine light on DSGE black box