Assignment 1

Due at the start of class on Wednesday, September 4.

1. For the vectors
\[ \vec{u} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} \text{ and } \vec{v} = \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix} \]
compute
(a) \(2\vec{u} + 3\vec{v}\)  (b) \(\vec{u} \cdot \vec{v}\)  (c) \(|\vec{u}| + |\vec{v}|\)  (d) \(\vec{u} \times \vec{v}\)

2. Let \(\vec{w} = (1, 3, -1)\) and \(\vec{z} = (2, 0, \alpha)\). For which value of \(\alpha\) is \(\vec{w}\) orthogonal to \(\vec{v}\)? Prove that there are no real numbers \(\alpha\) such that the angle between \(\vec{v}\) and \(\vec{w}\) is \(\pi/3\).

3. Verify that the vector \(\vec{v} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})\) is a unit vector (that is, has length 1). Then determine the magnitude of the projection of the vector \(\vec{w} = (6, 2, 2)\) onto the direction of \(\vec{v}\).

4. Find the parametric equation for the line in \(\mathbb{R}^3\) that passes through the point \((1, -2, 3)\) and that is parallel to the vector \((-1, 0, 5)\). Sketch this line.

5. Find the parametric equation of the line passing in \(\mathbb{R}^3\) passing through the points \((3, 1, 0)\) and \((2, 3, 2)\).

6. For the vectors \(\vec{u}\) and \(\vec{v}\) given in problem 1, verify using the dot product that your answer for part 1d is indeed orthogonal to both \(\vec{u}\) and \(\vec{v}\). Then find the equation of the plane having \(\vec{u} \times \vec{v}\) as its normal vector that passes through the point \((3, 1, 2)\).

7. Find the equation of the plane passing through the points \((1, 1, 2)\), \((3, -1, 6)\) and \((5, 2, 0)\).
8. The line given parametrically by \( \mathbf{r}(t) = (1 + 2t, 2 + 5t, 1 - 3t) \) for \( t \in \mathbb{R} \) passes through a plane at the point \((1, 2, 1)\) and this line is perpendicular to the plane. Find the equation of the plane. Write your final answer in the form \( ax + by + cz = d \).
Assignment 2

Due at the start of class on Friday, September 13.

Note: All pages numbers in Kreyszig refer to the online edition, not our special class edition.

1. The span of a set of vectors is the collection of all linear combinations of those vectors. Describe the set of all vectors \((x, y, z)\) in the span of the set

\[ S = \{(1, 2, 3), (2, 4, -1)\} \]

in the form of an equation relating \(x, y\) and \(z\).

*Hint: Geometrically, what kind of object is the span of \(S\)?*

2. Suppose you are trying to solve a system of 2 linear equations in 3 variables (say \(x, y\) and \(z\)). Explain *geometrically* how you know there are either no solutions or infinitely many solutions.

3. Determine all solutions to the linear system

\[
\begin{align*}
x + 3z &= 4 \\
y - 4z &= 1
\end{align*}
\]

Out of Kreyszig

Section 7.3 pp 280-281: 8, 9, 10, 12, 13, 14, 21.

*For 7.3, problems 8,9,10,12 work with the augmented matrix and use elementary row operations to find an equivalent matrix in row echelon form. Show your steps and then obtain the answer from this reduced matrix. For problems 13, 14 and 21 you can do the same or just write down the augmented matrix and use Matlab or whatever program you want to find (and write down) an equivalent matrix in row echelon form and solve from there.*
Assignment 3

Due at the start of class on Friday, September 20.

Note: All pages numbers in Kreyszig refer to the online edition, not our special class edition.

1. Determine the rank and determine the set of all vectors in the nullspace of the matrices given in problem 6 and problem 7 of page 287 of Kreyszig. (Ignore the instructions in the book for these problems.)

2. For parts (a) and (b) determine whether or not $\vec{w}$ is in the span of $\{\vec{u}, \vec{v}\}$ by first writing the corresponding augmented matrix and then using Gaussian elimination.

   \[
   (a) \quad \vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}, \quad \text{(b)} \quad \vec{u} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} -3 \\ 1 \\ 9 \end{pmatrix}
   \]

3. Let $\vec{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \vec{v} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \vec{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Determine whether or not the set $S = \{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent by rephrasing the question as a linear system of three equations in three unknowns and then solving the linear system via Gaussian elimination.

4. Give an example of a set of 3 vectors $\{\vec{u}, \vec{v}, \vec{w}\}$ in $\mathbb{R}^3$ is linearly dependent even though the 3 sets $\{\vec{u}, \vec{v}\}$, $\{\vec{u}, \vec{w}\}$ and $\{\vec{v}, \vec{w}\}$ are all linearly independent.
5. Determine whether or not the following collection of vectors in $\mathbb{R}^4$ is or is not a subspace of $\mathbb{R}^4$. Explain.
   
   (a) All vectors $\vec{v} = (v_1, v_2, v_3, v_4)$ such that $v_1 + v_2 + v_3 + v_4 = 0$.
   
   (b) All vectors $\vec{v} = (v_1, v_2, v_3, v_4)$ such that $v_1 + v_2 + v_3 + v_4 = 1$.
   
   (c) All vectors $\vec{v} = (v_1, v_2, v_3, v_4)$ such that $v_1 \geq v_2$.

6. Suppose that the $k$ vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ are mutually orthogonal. That means that $\vec{v}_i \cdot \vec{v}_j = 0$ whenever $i \neq j$. Assume that none of these vectors is the vector $\vec{0}$. Try to prove that this collection of vectors is linearly independent.
   
   *Hint: Try an argument “by contradiction” in which you assume otherwise; namely, assume to the contrary that the collection is linearly dependent and try to reach a contradiction.*