Use mainly when $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$.

**Likelihood Ratio Test (LRT)**

$$\phi(x) = \begin{cases} 1, & \lambda(x) \leq c_0, \\ 0, & \lambda(x) > c_0 \end{cases} \quad 0 \leq c \leq 1$$

Reject for small values of the ratio.

$$\lambda(x) = \frac{\sup_{\theta \in \Theta_0} L(\theta | x)}{\sup_{\theta \in \bar{\Theta}} L(\theta | x)}$$

LRT statistic based on $x$. Restricted parameter space.

**Numerators:** Max probability of the observed sample over all parameters in $H_0$ (null hypothesis).

**Denominator:** Max probability of the observed sample over all possible parameters.

The supremum can be obtained by MLE under the restricted and unrestricted parameter spaces, then

$$\lambda(x) = \frac{L(\hat{\theta}_0, \text{MLE} | x)}{L(\hat{\theta}, \text{MLE} | x)} \max \left\{ L(\hat{\theta}_0, \text{MLE}), L(\hat{\theta}, \text{MLE}) \right\}$$

$$= \min \left\{ 1, \frac{L(\hat{\theta}_0, \text{MLE} | x)}{L(\hat{\theta}_0, \text{MLE} | x)} \right\}$$

In case you have $T(x)$ sufficient statistic for $\theta$, then

$$\lambda(x) = \lambda^*(T(x))$$

since by Factorization thm...

$$\lambda(x) = \sup_{\theta \in \Theta_0} \frac{L(\theta | x)}{L(\theta_0 | x)} = \sup_{\theta \in \Theta_0} \frac{f(x | \theta)}{f(x | \theta_0)}$$

depends on $x$ only through $T(x)$

$$= \sup_{\theta \in \Theta_0} \frac{g(T(x) | \theta) h(x)}{g(T(x) | \theta_0) h(x)} = \lambda^*(T(x))$$

Method: 1) derive MLE's $\hat{\theta}_0, \hat{\theta}$ / 2) compute $\lambda(x) / 3)$ find $W(x)$