Optimal Trade Policy with Trade Imbalances∗

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Abstract

We characterize optimal trade policy in a dynamic trade model in which trade imbalances are generated endogenously. In absence of capital controls, import taxes and export subsidies exhibit a counter-cyclical behavior. Active use of capital control, however, dampens the time-variation of optimal trade taxes. Moreover, although in general the level of optimal trade policy in a period does not depend on the size of trade deficits in that period, optimal trade policy and equilibrium trade deficits move together under certain growth paths. Finally, we find that the optimal trade policy will discourage (encourage) the accumulation of foreign debt when the country is expected to grow faster (slower) than the rest of the world. Fitting our model to the United States economy from 1995 to 2016, we find that the welfare gains from optimal policy are largely due to static terms-of-trade effects, implying that gains from changing tariffs over time or using capital controls are small.

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1 Introduction

A salient feature of international trade is the presence of trade imbalances. The level and direction of these imbalances may be affected by various policies including trade policy and capital controls. How should governments conduct their trade policy under trade imbalances? The literature on trade policy has largely avoided this question by focusing on static models in which trade is balanced by assumption and, thus, various dynamic aspects of trade policies are overlooked. Our goal in this paper is to explore the relationship between inter-temporal trade and optimal trade policy.

The presence of inter-temporal dynamics poses several important questions regarding the optimal conduct of trade policy. First, in the presence of international capital flows, which make trade imbalances possible, governments could supplement trade policy with capital controls to manipulate the flow of goods and services across borders. The potential interdependence between capital controls and trade policies may have important implications about the design and benefits of trade agreements. For example, following negotiated trade liberalizations, governments may have an incentive to use capital controls more actively to manipulate their terms of trade, thereby frustrating the intent of trade agreements to some degree.\footnote{\hspace{1em}It is notable that shortly after its accession to the World Trade Organization, China was frequently accused of manipulating its exchange rate to affect the flow of goods and services.}

To what extent are such arguments valid? As a step toward addressing these questions, in this paper we characterize the interdependence of capital controls and trade policy.\footnote{\hspace{1em}\hspace{1em}A key difference between tariffs and capital controls is that tariffs could be imposed on trade flows at the sectoral level, while capital flow taxes or exchange rate manipulations cannot replicate a sector-specific tax on trade flows. In other words, compared to sector-specific tariffs, these policies are more blunt instruments that affect aggregate trade flows across all sectors.}

A second question is related to the pattern of optimal tariff over business cycles. Are optimal tariffs related to the size of the economy or to its growth rate? A country that is expecting a high growth rate in the future may find it optimal to run a deficit at the current period to smooth its consumption over time. Does the growth rate of the economy have any bearing on optimal conduct of trade policy?

Our first objective is to characterize the pattern of unilaterally optimal import and export taxes/subsidies across products and time.\footnote{\hspace{1em}We assume that the home government can impose arbitrary taxes on imports, exports, and accumulation of assets, while the foreign government is passive.} To this end, we use a two-country and multiple-product Ricardian model with time-varying labor productivity, in which the variation in productivity over time creates a role for international lending and borrowing for consumption-smoothing purposes. In this model, we show that within a period, optimal export tax/subsidies are differential but optimal import tariffs are uniform across products. This finding is similar to that of Beshkar and Lashkaripour (2019) and Costinot, Donaldson, Vogel, and Werning (2015), who study optimal trade policy under static Ricardian models. We, however, show that under our dynamic Ricardian model, optimality requires the variation of both import and export taxes/subsidies over time.

The key parameter that explains the variation in optimal trade taxes across time is the productivity of the home country relative to the rest of the world. In particular, we find that import taxes and export subsidies are counter-cyclical. Intuitively, this result is obtained because the government is interested in manipulating its inter-temporal terms of trade by reducing exports in booms and limiting imports in low-productivity periods. In other words, from the government’s point
of view, the households save too much in booms or, equivalently, they consume too much of the foreign good when productivity is low. The government’s policy response, therefore, would be to increase (decrease) the price of consumption in low-productivity periods (booms). This outcome may be achieved by increasing import tariffs and export subsidies in low-productivity periods.

Our second objective is to characterize the optimal capital control taxes and their interdependence with trade policies. We first make the observation that if the government has access to time-varying import and export taxes, capital control taxes are not necessary for the implementation of optimal policy. However, capital control taxes become useful if any of the trade tax instruments are unavailable, or if the government cannot vary them over time.

Our final objective is to provide a quantitative assessment of optimal trade and capital control policies in an environment with endogenous trade imbalances. We are particularly interested in (i) generating insights about the magnitude of variation in optimal policies over time under different constraints on policy instruments, and (ii) decomposing the gains from optimal policy into static and dynamic terms-of-trade gains.

We quantify our model using data on trade flows and production for the United States from 1995 to 2016. To do so, we extend the “exact hat algebra” methodology (Dekle et al. 2008) to the primal version of our dynamic Ramsey problem. This technique enables us to quantify the model and perform our optimal policy exercise using only estimates of elasticity of substitution and data on observed trade shares and production.

To measure the magnitude of variation in optimal policies, we consider two scenarios in which the set of available policy instruments is rich-enough to implement the optimal allocation. In our first exercise, we assume that the government of the United States chooses time-varying trade taxes (i.e., import tariffs and export subsidies) to implement its policy. We then consider a case in which export tax/subsidy is no longer available but capital control taxes are introduced to the set of available policy instruments. As discussed earlier, the government could achieve its optimal allocation under either of these scenarios. However, the pattern of optimal import tariffs depend critically on what other policy instruments are used by the government.

With trade taxes as policy instruments, optimal import tariffs and export subsidies show a significant variation over time. For the United States, optimal tariffs vary between 27% and 33%, and export subsidies vary between zero and 6%. When capital controls are used in lieu of export subsidies, the time-variation of import tariffs is virtually vanished—with tariffs hovering around 25% for the entire period.

These intertemporal patterns of import tariffs may be understood by noting that the government’s objective from modifying its policy over time is to achieve the desired intertemporal relative prices. Since trade taxes are applied to current consumption, they must vary over time to affect the relative price of aggregate consumption in different periods. In contrast, capital control is a direct tax on intertemporal trade and, hence, could generate the desired intertemporal relative prices without changes in tariffs.

As mentioned above, our second goal in the quantitative section is to measure the degree to which gains from optimal policy in our model emanates from static vs. dynamic terms-of-trade effects. To this end, we introduce constraints on available policy instruments that help isolate each of these channels for gains from optimal policy. First, we calculate gains from protection assuming that the only available policy instrument is a constant import tariff, which precludes manipulation.
of dynamic terms of trade. We find that this restriction has a negligible effect on the welfare gains from optimal policy, which implies that dynamic terms of trade gains are very small for the United States. Similarly, we find that the use of capital control taxes under free trade has very little welfare implications, which further reinforces the point that almost all gains from optimal policy come from static terms of trade gains. This finding also shows that under our model and observed trade flows, capital control taxes in the United States cannot effectively undermine free trade agreements.

Literature

The distinguishing feature of our paper is to characterize the optimal trade policy under a dynamic model in which trade imbalances are endogenously determined. This is in contrast with much of literature that either has used static models to analyze optimal trade policy or has ignored trade policy and focused on capital control. In particular, we build on two previous papers on optimal trade policy (Beshkar and Lashkaripour 2019, henceforth BL) and optimal capital control (Costinot, Lorenzoni, and Werning 2014, henceforth CLW). BL find the structure of optimal import and export taxes under a static general-equilibrium Ricardian model in which trade is balanced by assumption and, hence, inter-temporal considerations are absent. To focus on capital control, CLW assume free trade and adopt a dynamic endowment model in which the endowments are subject to exogenous changes over time.

The previous literature on quantitative analysis of trade policy (e.g., Ossa 2014, 2016) works under the assumption that trade is balanced. In fact, to fit their static models to data, which involves substantial trade imbalances, previous studies consider optimal policy in a counterfactual scenario in which trade is balanced. The salience of large trade imbalances in observed data cast doubt on the relevance of these static quantitative results for policy analysis. To our best knowledge, the current literature lacks a quantitative analysis of dynamic trade and capital control policies.

Under the assumption of free trade, Schmitt-Grohé and Uribe (2017) study optimal capital control policy for a small open economy. Similar to CLW, Schmitt-Grohé and Uribe (2017) also find that optimal capital control would reduce borrowing during recessions, causing a counter-cyclical protection pattern. Unlike our model, the main reason to use capital control in their model is to correct an inefficiency arising from the interaction of the collateral constraint with the price of the non-tradable goods. Heathcote and Perri (2016) consider a tax on interest income from international bonds that is proportional to the aggregate net foreign asset position, and show that the optimal level of such a tax is positive. Our analysis is more general as we do not impose any requirement on capital control taxes to be conditional on foreign asset positions.

Empirical evidence on the intertemporal pattern of trade policy is rare and the existing studies deliver diverging results. As discussed by Lake and Linask (2016), the conventional wisdom is that import protection is counter cyclical, which is supported by various observations on the use of temporary trade barriers within the framework of the WTO (e.g., Bown and Crowley 2013). However, more recent works such as Lake and Linask (2016) suggest that import protection, particularly in developing countries, are pro-cyclical. Our theoretical results, which are based on a terms-of-trade framework, are more in line with the conventional wisdom mentioned above.

Staiger and Sykes (2010) take on the issue of interdependence between trade policy and currency manipulation and ask if governments could frustrate the intent of trade agreements by manipulating the value of their currency. They conclude that the trade effects of such policies could
not be identified well-enough to make a judgement about whether these policies frustrate the intent of trade agreements. Our quantitative analysis lends support to this view by showing negligible effect of capital control taxes on welfare.

Bagwell and Staiger (1990) consider trade wars and self-enforcing trade agreement in a dynamic environment in which the countries’ endowments are subject to shock, but no inter-temporal trade takes place. The dynamics in their model come from the fact that governments could exchange trade policy concessions over time. Keeping the assumption of balanced trade in each period, Bagwell and Staiger (2003) extend their previous work to study trade policy over persistent business-cycle shocks.

While we find that unilaterally optimal trade and capital control policies do not generate significant fluctuations in trade imbalances, several papers in the literature find that the observed trade policies do in fact cause substantial changes in net trade flows.\(^5\) In particular, Alessandria and Choi (2019) find that the faster pace of tariff cuts in the United States compared to its trading partners was an important cause of the large expansion in the US trade deficit as a share of its total trade.\(^6\) Similarly, Alessandria, Choi, and Lu (2017) show that about 70% of the post-WTO surge in China’s trade surplus may be attributed to the decline in trade barriers over this time period. In these studies, the large effect of trade policy changes on trade imbalances are due to their transitory nature.\(^7\) Similarly, our optimal trade policies reduce the level of trade flows. However, they do not significantly change the variation in trade as a fraction of GDP.

Capital controls could interact with exchange rate policies and other government interventions—the so-called macro-prudential policies—that are aimed at enhancing financial stability. Using a model of small open economy, in which the government is unable to manipulate its intertemporal terms of trade using capital control, Davis and Presno (2017) show that capital controls allow optimal monetary policy to focus less on the foreign interest rate and more on domestic variables. Brunnermeier and Sannikov (2015) study the tradeoff between financial stability and optimal allocation of capital under a stochastic growth model with shocks to capital endowments and frictions in international capital markets that prevents risk sharing. In this paper we abstract from the potential use of capital controls as macro-prudential policy to focus on their terms of trade implications.

In Section 2, we present the basics of the model. In Section 3 we present the planner’s problem and establish our first result about the pattern of optimal trade policy under a dynamic trade model. In Section 4, we derive optimal trade policy and characterize its variation over time. In Section 4, we also consider capital control taxes as an additional policy instrument at the government’s disposal and discuss its interdependence with trade taxes. In Section 5, we consider a simple growth path and compute the optimal trade policy and the equilibrium levels of deficit and surplus. We introduce our quantitative analysis in Section 6. Finally, in Section 7, we conclude by discussing some of the potential implications of our analysis as well as further questions for future research.

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\(^5\)To be sure, note that we find that optimal policy reduces the size of trade imbalances, but it does not have a clear effect on the variation of imbalances over time.

\(^6\)Reyes-Heroles (2016) shows that the decline in the trade costs has been a major driver of trade imbalances. He estimates that about 69% of the increase in trade imbalances from 1970 to 2007 can be attributed to lower trade costs in goods markets.

\(^7\)More precisely, the large effect of trade policy changes in these studies is due to asymmetric pace of policy changes in different countries, which is a transitory phenomenon.
2 The Basic Model

We use a multi-period, two-country, $K$-industry, model in which consumption and production takes place in each period of time, $t$. Time, $t$, is assumed to be in $\{0, \cdots , T\}$ where $T$ is a finite natural number or $\infty$. Each country, $i, j \in \{h, f\}$, produces a distinct variety in each industry. We let $p_{t,i,k}^j$ and $x_{t,i,k}^j$ denote, respectively, period-$t$ price and quantity of consumption in country $j$ of country $i$’s variety of product $k$. With appropriate interpretation of subscripts and superscripts, bold-faced variables denote vectors and capitalized variables denote aggregate values.

Consumers can trade a one-period bond on the world capital market. In period $t$, the consumer in $j$ country can buy a claim for consumption in period $t+1$, denoted by $b_{t+1}^j$, at a price of $q_t^j$ per unit. Throughout our analysis, we assume that agents in both countries have perfect foresight about evolution of the fundamentals.\(^8\)

**Producers’ Problem** We adopt a Ricardian framework in which each goods is produced using labor as the only input to production. Labor productivity in industry $k$ of country $i$ at time $t$, denoted by $a_{t,i,k}$, is independent of the quantity of production. Labor is perfectly mobile across industries within the same country. The population of labor in each country is assumed to be constant over time, and we normalize the population in each country to 1. Producers are perfectly competitive and, thus, their price is equal to marginal costs of production:\(^9\)

$$p_{t,i,k} = \frac{w_{t,i}}{a_{t,i,k}},$$

where $w_{t,i}$ is the wage in country $i$ in period $t$.

**Consumers’ Problem** The lifetime utility of the representative consumer in country $j$ is given by

$$\sum_{t=0}^{T} \beta^t u \left(g \left(x_t^j\right)\right), \quad (1)$$

where $x_t^j$ is country $j$’s vector of consumption in period $t$, $g \left(x_t^j\right)$ is the aggregate consumption (i.e., utility) in period $t$ and $u \left(\cdot\right)$ is concave function. The consumer’s per-period budget constraint in country $j$ is given by

$$p_t^j \cdot x_t^j + q_t^j b_{t+1}^j = w_{t,j} + b_t^j - T_t^j, \quad (2)$$

where $T_t^j$ is a lump-sum tax paid by the consumers in country $j$ – note that $T_t^j$ can be negative in which case the government is making a transfer to the representative consumer in country $j$ in period $t$. Note that prices $p_t^j$ and $q_t^j$ are after tax prices where these taxes are imposed by the home government and are described below. The optimization problem of the consumer in country $j$ is

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\(^8\)As we will show later, policies are independent of history of the fundamentals and only depend on the contemporaneous value of fundamentals, i.e., productivity, trade costs and discount factors, etc. One can easily show that—very much the same as in Lucas Jr and Stokey (1983)—introducing uncertainty together with the assumption of complete markets does not change this result. Moreover, the relationship between optimal policy and fundamentals does not change with uncertainty. In other words, the only difference between the optimal allocations under perfect foresight and with shocks is that the Lagrange multiplier on the implementability constraint, (13), is different between the two specifications.

\(^9\)Absent domestic taxes, producer and consumer prices are equal $p_{t,i,k} = p_{t,i,k}^j$. 

6
to choose a vector of consumption and bonds for each period \( \{x^j_t\} \) to maximize its lifetime utility (1) subject to its per-period budget constraints (2). Letting \( \lambda^j_t \) denote the Lagrange multiplier on the budget constraint of country \( j \) consumers in period \( t \), the optimal consumer choice implies

\[
\beta^t \frac{du(\mathbf{g}(x^j_t))}{dx^j_{t,i,k}} = \lambda^j_t p^j_{t,i,k}, \, \forall i, k \tag{3}
\]

and

\[
q^j_t = \frac{\lambda^j_{t+1}}{\lambda^j_t}. \tag{4}
\]

Combining these conditions and noting that price index in the home country is given by \( P^h_t = \frac{p^h_{t,j,k}}{d\mathbf{g}(x^j_t)/dx^j_{t,i,k}} \) for any \( j, k \), the Euler equation for the home consumer may be written as

\[
q^h_t = \beta u'(\mathbf{g}(x^h_{t+1})) \frac{P^h_t}{P^h_{t+1}}. \tag{5}
\]

**Policy Instruments** We assume that the home government is policy active, while the foreign government takes a laissez faire approach. The policy instruments at the disposal of the home government include import and export tax/subsidy as well as a tax on international borrowing and lending of domestic households. Together with iceberg transport costs, \( d^j_{t,i,k} \), trade taxes create a wedge between home and foreign prices. To be specific, the ad valorem export tax on good \( k \) in the home country, \( \tau^h_{t,k} \), is implicitly given by:

\[
p^f_{t,h,k} \equiv (1 + \tau^h_{t,k}) d^f_{t,h,k} p^h_{t,h,k} x^f_{t,h,k}. \tag{6}
\]

Similarly, the import tax, \( \tau^h_{t,f,k} \), is implicitly given by:

\[
p^h_{t,f,k} \equiv (1 + \tau^h_{t,f,k}) d^h_{t,f,k} p^f_{t,f,k} \tag{7}
\]

Finally, the home government imposes a capital control tax, \( \tau^h_{t,b} \), which is a tax on the home consumer’s holding of foreign assets, namely,

\[
q^h_{t,b} \equiv (1 + \tau^h_{t,b}) q^f_t. \tag{8}
\]

Given home country’s policies, \( \tau^h_{t,h,k}, \tau^h_{t,f,k}, \tau^h_{t,b}, \) and \( T^h_t \), and the fact that the foreign country is passive (for which all these values are set to zero), an equilibrium in this economies is defined as follows:

1. Consumers maximize their utility in 1 subject to the budget constraint 2 while taking as given prices and policies.
2. Firms maximize profits, taking as given prices and policies.
3. Home government’s budget constraint is satisfied

\[
T^h_t = \sum_k \tau^h_{t,h,k} d^h_{t,h,k} p^h_{t,h,k} x^h_{t,h,k} + \sum_k \tau^h_{t,f,k} d^h_{t,f,k} p^f_{t,f,k} x^h_{t,f,k} + \tau^h_{t,b} q^h_{t,b} b^h_t. \tag{9}
\]

4. All markets, labor, goods and international bonds, clear. Note that bond market clearing is given by \( b^f_t + b^h_t = 0 \).
Marginal Utility Wedges  It is useful to define the wedge between the foreign and home marginal utilities. Note that absent any policies, the ratio of marginal utilities between the foreign and home country is equal to the relative shadow value of income in the two countries. Trade and capital control taxes create additional wedges between marginal utilities. In particular, letting
\[ \theta_{t,f,k} = \frac{du\left(g\left(x_{t}^{h}\right)\right)/dx_{t}^{h}}{du\left(g\left(x_{t}^{f}\right)\right)/dx_{t}^{f}} \frac{1}{\bar{d}_{t}^{f,h,k}} \]
denote the wedge between home and foreign marginal utility from the consumption of the foreign goods, the import tax satisfies the following condition:
\[ 1 + \tau_{t,f,k} = \frac{p_{t}^{h}}{p_{t}^{f}} = \frac{\lambda_{t}^{f}}{\lambda_{t}^{h}} \theta_{t,f,k}. \]  (9)

Similarly, intra-temporal wedge for the export good, \( \theta_{t,h,k} = \frac{du\left(g\left(x_{t}^{h}\right)\right)/dx_{t}^{h}}{du\left(g\left(x_{t}^{f}\right)\right)/dx_{t}^{f}} \frac{d_{t}^{f}}{d_{t}^{h}} \), is related to export tax in the following way:
\[ 1 + \tau_{t,h,k} = \frac{p_{t}^{f}}{d_{t}^{f}} = \frac{\lambda_{t}^{h}}{\lambda_{t}^{f}} \theta_{t,h,k}. \]  (10)

Finally, the inter-temporal wedge, \( \theta_{t,b} = \frac{u\left(g\left(x_{t+1}^{h}\right)\right)/u\left(g\left(x_{t}^{h}\right)\right)}{u\left(g\left(x_{t+1}^{f}\right)\right)/u\left(g\left(x_{t}^{f}\right)\right)} \) is related to capital control tax as follows:
\[ 1 + \tau_{t,b} = \frac{q_{t}^{h}}{q_{t}^{f}} = \theta_{t,b} \frac{p_{t}^{h}}{p_{t+1}^{h}} = \frac{\lambda_{t+1}^{h}}{\lambda_{t}^{h}} \frac{\lambda_{t+1}^{f}}{\lambda_{t}^{f}} \frac{P_{t}^{h}}{P_{t+1}^{h}}. \]  (11)

It is worth noting that the levels of the wedges on their own do not represent distortions. The variation of wedges over time or across goods, however, reflects distortions generated by taxes.

3 Optimal Trade Policy

To find the optimal policy of the home government, we use the primal approach as in Lucas Jr and Stokey (1983), by solving a planning problem in which the equilibrium quantities are directly chosen by the government. We then find the set of trade and capital control taxes that implement the optimal allocation. Later, we will also consider scenarios in which the government’s policy space is subject to varying degrees of constraints imposed by trade agreements and domestic rules.\footnote{CLW and Costinot, Donaldson, Vogel, and Werning (2015) have demonstrated the benefits of applying this approach to international trade policy problems.}

Under the primal approach, the planner’s problem is to choose the vector of allocations for all periods, \( x_{t}^{h} \), to maximize the welfare of the representative consumer in the home country, i.e.,
\[ \max_{\{x_{t}^{h}\}_{t=0}^{T}} \beta^{T} u \left( g \left( x_{t}^{h} \right) \right), \quad (P) \]
subject to
1. Per-period labor-market clearing conditions:\(^{11}\)
\[
\left( x_{t,h}^h + d_{t,h}^f \odot x_{t,h}^f \right) \cdot \frac{1}{a_{t,h}} = 1,
\]
\[
\left( d_{t,h}^f \odot x_{t,f}^h + x_{t,f}^f \right) \cdot \frac{1}{a_{t,f}} = 1,
\] (12)

2. Implementability condition:
\[
\sum_{t=0}^{T} \beta^t \nabla u \left( g \left( x_t^f \right) \right) \cdot x_t^f = \sum_{t=0}^{T} \beta^t \left[ \nabla u \left( g \left( x_t^f \right) \right) \right]_f \cdot a_{t,f}. \] (13)

where \( \left[ \nabla u \left( g \left( x_t^f \right) \right) \right]_f \) is the vector marginal utilities for the good that the foreign country produces. This condition requires that the total expenditure allocated to foreign consumers by the home planner is equal to the value of their income.

3. No domestic distortion in either country (given that the available tax instruments are levied only on international exchanges), which implies that marginal utilities from consumption of the domestically-produced goods in each country are proportional to their input requirement:
\[
\frac{\partial g \left( x_t^f \right)}{\partial x_t^f} = \frac{\lambda_t^f}{a_{t,f,k}}, \] (14)
\[
\frac{\partial g \left( x_t^h \right)}{\partial x_t^h} = \frac{\lambda_t^h}{a_{t,h,k}}. \] (15)

A few features of this optimal policy problem are worth mentioning. First, since the government distributes its tax revenues in a lump-sum fashion, the budget constraint of the domestic consumer imposes no extra constraint on the planning problem. Second, since taxes are only imposed on international flows, production is domestically efficient. Equation (14) specifies that for any two goods produced in each country, marginal rate of substitution must be equal to marginal rate of transformation or, equivalently, the relative productivities.

The first order condition of the planner’s problem (P) with respect to the allocation of the import good, \( x_{t,f,k}^h \), may be written as:
\[
\beta^t \frac{\partial u \left( g \left( x_t^h \right) \right)}{\partial x_t^h} = \frac{\eta_t^f}{a_{t,f,k}},
\]
where \( \eta_t^f \) is the Lagrange multiplier on the resource constraint of the foreign country \((14)\). Moreover, substituting the equilibrium price, \( p_{t,f,k} = \frac{u_{t,f}}{a_{t,f,k}} \), in the foreign consumer’s optimality condition we obtain
\[
\beta^t \frac{\partial u \left( g \left( x_t^f \right) \right)}{\partial x_t^f} = \frac{\lambda_t^f u_{t,f}}{a_{t,f,k}}.
\]

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\(^{11}\)Recall that \( d \) denotes trade costs. Moreover, we use \( \odot \) to denote element-wise multiplication of vectors.
The above two equations imply that

$$\theta_{t,f,k} \equiv \frac{\beta^t \partial_u (g(x^t_h)) / \partial x_{t,f,k}}{\beta^t \partial_u (g(x^t_f)) / \partial x_{t,f,k}} = \frac{\eta^f_t}{\lambda^f_t w_{t,f}}.$$ 

The left-hand side of this equation is the optimal wedge between the home and foreign’s marginal utility of consuming the \(t, f, k\) variety, while the term on the right-hand side includes only economy-wide variables. Therefore, we confirm that the optimality of uniform tariffs under a static Ricardian model with balanced trade (BL, Costinot, Donaldson, Vogel, and Werning 2015) carries over to a dynamic Ricardian model with trade imbalances.

**Proposition 1.** Under a dynamic Ricardian model: (i) The optimal import tariffs are uniform across goods but generally differential across periods. (ii) If intra-temporal preferences, \(g\), takes a CES form, then the optimal export taxes are also uniform across goods, in a given period.

This Proposition states that, generally, optimality requires uniform import tariffs and differential export taxes, while optimal export taxes will be also uniform in the case of CES preferences. The intuition behind the optimality of uniform tariff within each period can be understood by considering the motive of the home country to use tax policies. In particular, the home country wishes to impose taxes in order to reduce the income, i.e., the present value of wages, in the foreign country. Since labor is perfectly mobile across sectors and its movements cannot be controlled by the home government, relative prices of foreign exports (home imports) are determined solely by relative productivities. Thus, the home government is unable to affect relative income from production of these goods by taxing them differentially. This implies that imposing differential taxes simply create distortions for the domestic consumer without any revenue benefits for the government. This implies that tariffs must be uniform within a period.

Optimal tariffs, however, are not uniform across periods. This result follows because wages are not equalized across periods and, thus, time-varying tariffs can affect the relative inter-temporal prices of imports and exports. Finally, as stated in part (ii) of Proposition 1, under CES preferences, optimal export taxes are also uniform across products. However, for more general preferences, export taxes are differential across products.\(^{12}\)

4 **Intertemporal Structure of Optimal Trade Policies**

In comparison to static trade policy analyses, the problem of optimal policy under a dynamic setting has at least two novel features. First, trade policy could fluctuate over time, which creates the possibility for tariffs to affect saving and investment decisions in expectation of future changes in trade policy. For instance, if governments announce a commitment to gradually reduce import tariffs over time, households are induced to decrease their current consumption in order to save for consumption at more favorable prices in the future.\(^{13}\) A second feature of trade policy in dy-

\(^{12}\)This result reflects the terms-of-trade motive of the government, which is to induce a monopoly markup over the competitive exporters’ price. The optimal markup for a product may be achieved by levying an export tax that reflects the import demand elasticity for that product in the foreign country.

\(^{13}\)Similarly, a surge in the United States’ trade deficit in 2018 was attributed to a looming trade war with China and other countries. Other examples include increased investment in the export sector in expectation of free trade agreements (McLaren, 1994), increased imports in anticipation of tariff hikes (Alessandria et al., 2019), or trade flow dynamics that are induced by asymmetric phase-in of tariff cut commitments across countries (Alessandria and Choi, 2019; Alessandria et al., 2017).
namic settings, which will be discussed in the next section, is the emergence of an additional tax instrument, namely, an inter-temporal trade or capital control tax, which may be a complement or substitute for trade policy.

In this section, we provide a few theoretical insights into determinants of optimal trade policy under some simplifying assumptions. In particular, we assume there are no trade costs, i.e., \( d_{t,j,k} = 1 \), and preferences are CES within each period.

As we know from part (ii) of Proposition 1, under CES preferences, optimal import and export taxes are uniform within a period. This result implies that under CES preferences the home government’s optimal policy problem reduces to choosing taxes on aggregate import and export volumes, rather than on individual traded varieties. Therefore, adopting CES preferences suppresses the variation of optimal trade taxes across products.

Formally, we assume that

\[
g(x_{jt}) \equiv X_{jt} \equiv \left[ \sum_{k} \alpha_k \left( x_{jt,h,k} \right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},
\]

and we let \( X_{jt,i} \) denote the aggregate volume of goods exported from \( i \) to \( j \) in period \( t \). We can similarly define aggregate productivity as

\[
A_{t,i} = \left[ \sum_{k} (\alpha_k)^{\sigma} \left( a_{t,i,k} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.
\]

Moreover, we assume that intertemporal preferences are also characterized by a CES function, namely,

\[
u(X_{jt}) = \frac{\eta}{\eta - 1} \left( X_{jt} \right)^{\frac{\eta - 1}{\eta}},
\]

where, \( \eta \) is the intertemporal elasticity of substitution.

Under CES preferences, therefore, we can rewrite the planner’s problem (P) using aggregate values only, namely,

\[
\max \{ X_{h,j} \}_{t=0}^{T} \sum_{t=0}^{T} \beta^t u \left( X_{h,t} \right) \quad (P')
\]

subject to

\[
X_{t,h}^j + X_{t,h}^f = A_{t,h},
\]

\[
X_{t,f}^j + X_{t,f}^f = A_{t,f},
\]

\[
\sum_{t=0}^{T} \beta^t \left( \frac{du(X_{t,f}^f)}{dX_{t,f}^f} X_{t,f}^f + \frac{du(X_{t,h}^f)}{dX_{t,h}^f} X_{t,h}^f \right) = \sum_{t=0}^{T} \beta^t \frac{du(X_{t,f}^f)}{dX_{t,f}^f} A_{t,f}.
\]

Before introducing the necessary conditions for optimality, note that the necessary conditions would be also sufficient if our programming problem is convex. This is guaranteed when the constraint set is strictly convex and objective of the maximization problem is strictly concave. Standard concavity properties of the CES function together with concavity of \( u \) guarantee that the objective is strictly concave. To ensure the convexity of the constraint set, we make the following assumption:
**Assumption 1.** \( \sigma \geq \eta \geq 1 \).

In other words, we assume that goods are more substitutable within a period than across periods. As we show in Appendix A, under this assumption, the optimization \( (P') \) is equivalent to one in which the constraint set is convex. This implies that the first-order conditions are necessary and sufficient for optimality.

Using the properties of the CES preferences, and letting \( \mu \) to denote the Lagrange multiplier of the implementability constraint 17, the FOC with respect to \( X_{t,h}^h \) may be written as

\[
\left( \frac{X_t^h}{X_t^f} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \left( \frac{X_{t,h}^h}{X_{t,h}^f} \right)^{-\frac{1}{\sigma}} = \mu \left[ 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X_t^f}{X_t^f} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \right],
\]

\[ \text{(18)} \]

The LHS of this equation, which is the marginal-utility wedge for the consumption of the home good, namely, \( \theta_{t,h} \equiv \frac{d}{dX_t^h} \), is decreasing in \( X_{t,h}^h \). The RHS of this equation, which is the marginal cost of consuming the home good for the home consumer, is increasing in \( X_{t,h}^h \). Similarly, the FOC with respect to \( X_{t,f}^h \) may be written as

\[
\left( \frac{X_t^h}{X_t^f} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \left( \frac{X_{t,f}^h}{X_{t,f}^f} \right)^{-\frac{1}{\sigma}} = \mu \left[ 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X_t^f}{X_t^f} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \right],
\]

\[ \text{(19)} \]

where, the LHS represent the marginal-utility wedge for the consumption of the foreign good, \( \theta_{t,f} \equiv \frac{d}{dX_t^f} \).

It is useful to write the above optimality conditions in terms of trade shares (\( \lambda_{t,f} \equiv \frac{X_t^f}{X_t^f} \)), local consumption shares (\( \pi_{t,f} \equiv \frac{X_t^f}{X_t^f} \)).

\[
\frac{1}{\mu \theta_{t,h}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \frac{\lambda_{t,f}^f}{\pi_t}, \tag{20}
\]

\[
\frac{\theta_{t,f}}{\mu} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \frac{\lambda_{t,f}^f}{\pi_t} + \frac{1}{\sigma} \pi_t, \tag{21}
\]

Formulas (20) and (21) illustrate the main determinants of optimal wedges—or equivalently, optimal taxes as we describe below. First, as expected, it highlights the role of trade (im)balances. In particular, if trade was balanced, expenditure must equal income in each period and, thus, \( \lambda_{t,f}^f = \pi_t^f \). As an example, in periods when \( \pi_t^f \) is high relative to \( \lambda_{t,f}^f \), i.e., when the foreign country’s expenditure is high relative to its output, the wedge on exports must be high. This is resulting from the increased monopoly power of the home government when the foreign country is buying a

\[ \text{14} \text{See Appendix D for the derivation of these formula.} \]

\[ \text{15} \text{We refer to the share of country } j \text{ output that is consumed in country } j \text{ as local consumption shares.} \]
higher share of home output. Second, both intra- and inter-temporal elasticities matter for optimal trade policies. Finally, the home government has extra motive to create a wedge on imports – captured by the term \( \sigma \frac{1}{\pi_t^f} \) in (21). Mathematically, this is coming from the fact that taxing imports, has the extra effect of reducing the level of wages in the foreign country, which is given by the right hand side of (17).

4.1 Optimal Import Tariffs and Export Subsidies

So far we have characterized optimal marginal utility wedges using a primal approach to optimization. We now use the relationship between wedges, prices, and policy instruments to determine the level of policies that could implement these optimal wedges.

Trade taxes and subsidies in a given period determine the ratio of relative prices in the home and foreign country, \( \frac{\rho_{t,f}}{\rho_{t,h}} = \frac{\theta_{t,f}}{\theta_{t,h}} \). Therefore, using the definition of trade taxes (9-10), the optimal level of total trade restrictions in period \( t \) may be written as

\[
(1 + \tau_{t,f}) (1 + \tau_{t,h}) = \frac{\theta_{t,f}}{\theta_{t,h}},
\]

where, \( \theta_{t,f} \) and \( \theta_{t,h} \) are the optimal wedges given by equations (20-21).

In a static model, in which trade is balanced in each period, the above formula is the only restriction on policies that is needed for the implementation of optimal allocation. That is, in a static model, what matters for optimality in a given period is the level of total protection, i.e., \( (1 + \tau_{t,f}) (1 + \tau_{t,h}) \), rather than the specific levels of import and export tax/subsidy.\(^{16}\) However, in our dynamic framework, policies should also induce optimal intertemporal wedges. To this end, the individual levels of optimal import and export taxes should also satisfy the following intertemporal conditions:

\[
\frac{1 + \tau_{t,f}}{1 + \tau_{t-1,f}} = \frac{\theta_{t,f}}{\theta_{t-1,f}},
\]

and

\[
\frac{1 + \tau_{t,h}}{1 + \tau_{t-1,h}} = \frac{\theta_{t,h}}{\theta_{t-1,h}}.
\]

To characterize the time-variation of optimal trade taxes, it is useful to rewrite the optimality conditions using fraction of outputs consumed in each country. Letting \( z_t \equiv \frac{A_{t,h}}{A_{t,f}} \) denote the relative productivity of home to foreign country in period \( t \), the FOCs may be written exclusively in terms of \( z_t, \pi_{t}^h, \pi_{t}^f \) and the parameters of the model (See Appendix B). Therefore, in each period, the fraction of home and foreign production that is consumed at home is pinned down by the relative productivity, \( z_t \), in that period. The effect of future and past productivities on the current allocation operates only through the time-invariant Lagrange multiplier, \( \mu \). This observation also implies that

**Proposition 2.** Up to a normalization, the optimal import and export taxes/subsidies in period \( t \) are uniquely determined by the relative productivities in period \( t \), i.e., \( z_t \).

\(^{16}\)This indeterminacy reflects Bond's (1990) argument that the solution to optimal trade policy determines all tariff levels relative to a numeraire.
This proposition implies that the size and direction of current trade balance has no bearing on the current optimal trade policy. That is because two periods with equal relative productivities—and hence identical optimal policy—could have very different levels of trade imbalances. Therefore, in general, there is no relationship between optimal trade policy and trade balance in a given period.

To describe the behavior of taxes over time, we start from a simple case of an economy where the elasticity of substitution between and across periods are equal, i.e., \( \sigma = \eta \). In this case, equations (9) and (10) reduce, respectively, to

\[
\left( \frac{1 - \pi_{f}^{t}}{1 - \pi_{f}^{h}} \right)^{-\frac{1}{\sigma}} = 1 - \frac{1}{\sigma} + \frac{1}{\sigma \pi_{f}^{t}}.
\]

and

\[
\left( \frac{1 - \pi_{h}^{t}}{1 - \pi_{h}^{h}} \right)^{-\frac{1}{\sigma}} = 1 - \frac{1}{\sigma}.
\]

As it can be observed, in this case, \( \pi_{t,i}'s \) are independent of \( z_{t} \). This implies that when \( \sigma = \eta \), under the optimal policy of home, the fraction of output that is consumed in each country remains constant over time.

**Corollary 1.** *If \( \sigma = \eta \), the optimal trade policy is invariant to variations in \( z_{t} \).*

To provide intuition for this result, note that in this setup, the home government’s main goal is to diminish the foreign country’s factor income by manipulating the demand for foreign output. When \( \sigma = \eta \), the relative prices of the foreign country’s production is independent of their imports. This would imply that home exports should not be taxed, i.e., their relative prices should stay the same over time. Furthermore, homogeneity of the problem implies that import taxes are independent of \( z_{t} \) and thus constant over time.

The intuition provided above for determinants of wedges suggest that when \( z_{t} \) is low and income in the foreign country is relatively high, it should be optimal to have high export taxes and low import tariffs. Unfortunately, we cannot show this analytically. We can, however, show that

**Proposition 3.** *Under optimal trade policy, the share of foreign production that is consumed abroad, \( \pi_{f}^{t} \), is increasing in \( z_{t} \).*

This Proposition implies that there are forces towards making import taxes counter-cyclical and export taxes are pro-cyclical. In other words, optimal trade policy encourages a pro-cyclical consumption pattern. This finding is similar to the key finding of CLW for the case of capital control taxes under free trade.

To obtain an intuition about this result, note that the government is interested in reducing the national saving rate during booms, in order to reduce the country’s demand for imports during low-productivity periods, thereby achieving a better inter-temporal term-of-trade.\(^{17}\) This goal may be achieved by a higher import tariff during recessions or a higher export tax in booms.

\(^{17}\)In other words, from the perspective of the government, under free trade, consumers consume too much of the foreign good in low-productivity periods and too little of the domestic good in booms. Relatedly, the saving rate of the consumers in booms (i.e., periods with high relative productivities) are too high from the government’s point of view.
4.2 Capital Control Policy

Our analysis so far shows that trade policy instruments alone are sufficient to implement the optimal allocation and, thus, capital control taxes are redundant when the government has unconstrained access to trade policy instruments. Moreover, we showed that in contrast to the static version of the model in which the import and export taxes are perfectly substitutable (and, hence, one of the policy instruments is redundant), under a dynamic model, both import and export taxes are generally necessary for the implementation of the optimal policy. As we now show, however, the capital control tax could substitute one of the trade policy instruments.

Recalling that $\tau_{t,b}$ denotes the capital control tax in period $t$, the relationship between all policy instruments and wedges are given by

$$\frac{1 + \tau_{t,f}}{1 + \tau_{t-1,f}} = \frac{1}{1 + \tau_{t-1,b}} \frac{\theta_{t,f}}{\theta_{t-1,f}},$$

(25)

and

$$\frac{1 + \tau_{t,h}}{1 + \tau_{t-1,h}} = (1 + \tau_{t-1,b}) \frac{\theta_{t,h}}{\theta_{t-1,h}}.$$

(26)

Now suppose that export taxes/subsidies are exogenously set to zero for all periods, but the home government has access to import tariffs and capital control taxes. Equations (25) and (26) make it clear that the government could still achieve the optimal allocation using only import tariffs and capital controls.

To understand this result, note that optimal policy implies a set of relative prices within and across periods. To set intra-temporal relative prices, one trade policy instrument (i.e., import or export tax) is sufficient. To set inter-temporal relative prices, we can either use a capital control tax which creates a wedge between current and future consumptions, or a combination of import and export taxes. In particular, after eliminating export tax/subsidy, the government can respond with an adjustment in import tariffs that preserves the terms of trade within each period. Moreover, the government uses capital control taxes/subsidies to preserve the inter-period terms of trade.

Cyclical behavior of optimal tariffs and capital controls

As we discussed in the Introduction, an important trade policy question concerns the cyclical behavior of optimal tariffs. In the discussion following Proposition 3, we have already discussed that under optimal policy, consumption shows a pro-cyclical pattern and that, in the absence of capital control taxes, optimal import tax and export subsidies are counter-cyclical. We now ask if the introduction of capital control taxes have any impact on the time-variability of import tariffs.

Setting export tax/subsidies equal to zero for all periods, optimal tariffs should satisfy

$$\tau_{t,f} = \frac{\frac{1}{\sigma} \frac{1}{\pi_t} f}{1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \frac{\lambda_t f}{\pi_t}}.$$

To understand the variation of optimal tariffs in presence of capital control, we can compare the above formula for optimal tariffs—absent export taxes - with equation (21). In particular, if $\pi_t f$ and $\pi_t / \lambda_t f$ comove over time, the fluctuations in $\tau_{t,f}$ as defined above is dampened relative to those of the optimal import and export taxes. Note that $\pi_t f$ is the ratio of consumption of domestic goods to
output while $\pi_i^f / \lambda_i^f$ is the ratio of total consumption to total output. Since these two statistics are often positively correlated, the above suggests that absent export taxes and with capital controls optimally chosen, variations in import tariffs are dampened. Our quantitative exercise suggests that this is in fact the case: with optimal capital control taxes, optimal tariffs do not vary as much over time.

We can also discuss the cyclical behavior of optimal capital controls. While we cannot show this analytically, it is often the case that import taxes are procyclical while export taxes are countercyclical. When the government shuts down export taxes and uses capital controls instead, these cyclical properties translate to a relationship between growth and capital control. In particular, if the home country is expected to have a high growth relative to the foreign country in the next period, it is optimal to impose a tax on accumulation of foreign debt. This is reminiscent of the finding in CLW who find a similar result assuming free trade.

5 Economic Growth, Trade Imbalances and Optimal Trade Policy

In this section, we discuss the implications of economic growth on trade imbalances and optimal trade policy. To this end, we study some likely exogenous growth scenarios within our model and compute optimal trade taxes and equilibrium trade imbalances over time.

In light of proposition 2, under the unilaterally optimal allocation, the households in the home country consume a greater fraction of domestic production in periods in which home output is relatively larger. Moreover, during these periods, the fraction of foreign production that is consumed at home decreases. As we illustrate in Section 3, this trade-off leads to an import tariff that decreases with relative endowment and an export tax that increases with relative endowment.

The above result implies that over a period of high growth, one in which the productivity of the home country increases relative to the foreign country, we must observe an increase in export taxes and a decrease in import tariffs. This is illustrated in Figure 1. In this figure, we consider two countries that start at the same level of endowment, while the home country grows at an annual constant rate of 4% and the foreign country grows annually at rate 2%. We assume that this growth lasts for 10 years and subsequently endowments in the two countries stay constant. We normalize export tax to 0 at time 0. As we see, over this period the home country increases its export tax while at the same time it reduces its import tariffs. Moreover, the combined change in export and import taxes implies greater restriction in trade in periods with greater relative productivity.
As we have shown, the difference between inter-temporal and intra-temporal elasticity of substitution is one of the determinants of the variation in tax policies. In our calculations, we have assumed that $\eta = 1.1$ while we allow $\sigma$ to vary. The values of $\sigma$ we consider are 5–15. Note that $\sigma - 1$ is equivalent to the trade elasticity in an Armington model - see Caliendo and Parro (2014) - and its estimated values are in this range. As $\sigma$ increases, we see that the level of export subsidies declines while its variation increases. This illustrates a trade-off between intra- and inter-temporal terms of trade manipulation. As $\sigma$ increases, imports become very elastic relative to import and thus within-period terms of trade manipulation is not very beneficial to the home government. Furthermore, the benefits of inter-temporal terms of trade manipulation increases and thus both export subsidies and import taxes change more.

Figure 1: Export taxes (left) and import tariffs (right) over time

Figure 2: Trade deficit in the home country over time
In Figure 2, we plot trade deficit in the home country over time and for various levels of $\sigma$. While in our model, there is no particular relationship between deficit and trade policies, since home country finds it optimal to borrow – due to the fact that its income is growing relative to its trading partner - as deficit decreases, export subsidies increase and import taxes decrease.

6 Quantitative Analysis

In this section, we provide a quantitative version of the model described above and fit it to the economy of the United States. To do so, we adjust our model by allowing for time-varying trade costs and discount factors in order to perfectly fit the data on GDP, exports and imports in the United States and the rest of the world from 1995 to 2016. During this time period, the U.S. adopted historically-low tariffs after the inception of the WTO in 1995. Therefore, we fit the data under the assumption that the trade policy of the US in this period is free trade.

We rewrite the optimal policy problem in terms of moments observable in the data and thereby extending the so-called exact hat algebra to optimal policy problems. Using this approach, we calculate the optimal trade and capital control taxes under various scenarios. We mainly consider three scenarios: first, unilaterally optimal trade policy without any restrictions; second, optimal capital controls assuming free trade obligations under WTO; and third, unilaterally optimal tariffs when capital control and export taxes are unavailable.

6.1 The Hat-Algebra for the Primal Approach

Gravity models of international trade, such as the one we use in this paper, lend themselves very well to quantitative analyses. In general, quantifying the effect of trade costs (including transport costs and trade taxes) on trade flows requires the estimates of various structural parameters of the model such as productivities and preference shocks. Under gravity models, however, the only parameters that need to be calibrated are trade elasticities. In particular, the effects of changes in trade taxes may be quantified without any knowledge on other parameters of the model that are invariant to changes in trade taxes.

This feature of the gravity models can be seen by rewriting the equilibrium conditions in terms of changes in the variables as a response to changes in trade policy. This technique, originally known as calibrated share form in the Computational General Equilibrium literature, was reintroduced to the quantitative trade literature by Dekle, Eaton, and Kortum (2007). Costinot and Rodríguez-Clare (2014) provide a detailed discussion of this method, which they refer to as the “exact hat algebra”. Finally, Ossa (2016) provides a survey of this method as applied to the problem of optimal trade policy.

To evaluate the quantitative effects of trade and capital control policy, we first adapt the hat-algebra technique to our setting, which involves the primal approach to a dynamic Ramsey problem. Before doing so, we first extend the model to allow for time-varying discount rates. In

\[ This is different from the previous papers in this literature that use a static trade model and the dual approach in computing the optimal policy (Beshkar and Lashkaripour, 2019; Ossa, 2014). \]
particular, we assume that preferences are given by
\[
\sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta^s \left( \sum_{k=h,f} \left( X_{t,k}^j \right)^{1-\frac{1}{\sigma}} \right)^{1-\frac{1}{\eta}} \frac{1}{1-1/\eta}.
\]

This change generates enough flexibility in the model in order to match any pattern of trade deficit over time.\footnote{One might think that we also need to introduce home bias in order to match trade flows. However, under CES preferences, home-bias act identical to trade costs. As a result, we do not need to introduce home-bias.}

Consider the Planner’s problem \((P')\) modified to allow for time-varying discount rates defined above. Referring to variables with a bar as the free-trade variables, i.e., those observed in the data, and using the hat-algebra notation, \(\hat{y} \equiv \frac{y}{\bar{y}}\) for any variable \(y\), we can write the resource constraints (16) as
\[
X_{t,j}^j \hat{X}_{t,j}^j + d_{t,h}^f \hat{X}_{t,j}^f \hat{X}_{t,j}^{-j} = A_{t,j},
\]
for \(j = h, f\) and all \(t\). Letting \(\pi_{t,j} \equiv \frac{X_{t,j}^j}{A_{t,j}}\) denote the fraction of country \(j\) output that is consumed domestically, these conditions may be written as
\[
\pi_{t,j} \hat{X}_{t,j}^j + (1 - \pi_{t,j}) \hat{X}_{t,j}^{-j} = 1.
\]
Similarly, we can rewrite the implementability constraint (17) and the utility of the representative consumer in the home country in hat-algebra form. This results in the following optimization problem:
\[
\max_{\{\hat{X}_{t,j}^i\}_{i,j}} \sum_{t=0}^{\infty} \alpha_t^h \left( \sum_j \lambda_{t,j}^h \left( \hat{X}_{t,j}^h \right)^{1-\frac{1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}} \left( \frac{1}{1-\frac{1}{\eta}} \right)^{1-\frac{1}{\eta}}, \tag{P1}
\]
subject to
\[
1. \text{ Implementability condition}
\]
\[
\sum_{t=0}^{\infty} \alpha_t^f \left( \hat{X}_{t}^f \right)^{1-\frac{1}{\sigma}} = \sum_{t=0}^{\infty} \frac{\alpha_t^f \lambda_{t,f}^f}{\pi_t^f} \left( \hat{X}_{t}^f \right)^{\frac{1}{2}-\frac{1}{\eta}} \left( \hat{X}_{t,f}^h \right)^{-\frac{1}{\sigma}},
\]
\[
2. \text{ Resource constraints}
\]
\[
\pi_t^h \hat{X}_{t,h}^h + \left(1 - \pi_t^h\right) \hat{X}_{t,h}^f = 1,
\]
\[
\pi_t^f \hat{X}_{t,f}^f + \left(1 - \pi_t^f\right) \hat{X}_{t,f}^h = 1,
\]
where, \( \sum_j \lambda_{t,j}^j \left( \hat{X}_{t,j}^f \right)^{1 - \frac{1}{\beta}} = \left( \hat{X}_t^f \right)^{1 - \frac{1}{\beta}} \).

In the above optimization problem, \( \alpha_j^j \) is the time-0 value of time-\( t \) expenditure as a fraction of the present value of income at time 0. Additionally, \( \lambda_{t,k}^j \) is the fraction of the expenditure in period \( t \) in country \( j \) which is spent on the output of country \( k \).

The above formulation of the optimal policy problem highlights the sufficient statistics that are required to be measured in the data in order to solve for optimal policy. These sufficient statistics include \( \alpha_j^j \): the share of total expenditure in country \( j \) that is spent in period \( t \), \( \lambda_{t,k}^j \): the share of \( j \)'s expenditure that is spent on \( k \)'s output in period \( t \), and \( \pi_j^j \): the share of output of \( j \) that is consumed domestically. It, thus, implies that both the level and changes in productivity, trade costs, and preference shocks affect optimal policies only indirectly through the sufficient statistics. Therefore, in order to understand optimal policy and its determinants, it suffices to measure these sufficient statistics. As we explain below, we construct \( \alpha_j^j \), \( \lambda_{t,k}^j \), and \( \pi_j^j \) based on the data on total output (GDP) of United States and the rest of the world and import and export volumes.

In addition to the problem above, we also consider two alternative policy exercises: one in which the home government (U.S.) is constrained by trade agreements to keep intra-temporal trade undistorted and one in which the home government can only change import tariffs. These assumption imply that their associated wedges, intertemporal wedge and intratemporal wedge on home goods, must be zero. In the associated optimal policy problems, we impose these constraints.

### 6.2 Data and Calibration

As we have shown, the optimal policy problem is fully determined by the parameters \( \alpha_j^j \), \( \pi_j^j \), and \( \lambda_{t,k}^j \). Here, we describe how we map our model to the data in order to measure these statistics.

We use World Bank’s data on real imports and exports and GDP for the United States and the rest of the world for the time period of 1995–2016. As we need time-0 measurement of expenditures, we calculate real interest rate for the U.S. and a collection of other countries. In particular, for the U.S., we use data on 10-year treasury notes and U.S. inflation measured using the CPI. We use the difference between the two to compute our measure of annual real interest rate.\(^{20}\) Finally, in order to calculate real interest rates in the rest of the world, we use the data provided by Jordà et al. (2019) on 17 advanced economies. In each country, we calculate the (ex-post) real interest rate as the difference between long-term interest rates and inflation according to CPI. We then calculate a weighted average of gross interest rate across these countries weighted by their corresponding GDP (measured according to PPP).

Two observations are important to note. First, in 1995, U.S.’s international investment position is negative and around 3.6% of GDP. This justifies to some extent the choice of the zero initial assets for both countries. Second, during this time period, the United States has run an increasing trade deficit – as a fraction of GDP. In order to have our model match the second observation, i.e., accumulation of debt over the time period, we assume that time starts at 1995 and runs until infinity. We assume that after 2017, both home and foreign country grow at rate 1.5% annually while annual real interest rates are 2%.\(^{21}\) Moreover, we assume that trade costs and discount factors are constant after 2017.

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\(^{20}\)While the data on 10-year Treasury Inflation Protected Securities does not date back to 1995, for the period in which yields exist, our constructed measure is very close to the 10-year TIPS yield. We show both series in Figure 10 in the Appendix.

\(^{21}\)While these are somewhat optimistic estimates for growth and interest rates in the future, we also consider other
In order to map our model to the data, we simplify by assuming that total consumption in the country is equal to GDP plus imports less exports. Given this assumption, for the period of 1995 to 2017, we can calculate \( \pi^j_t \) and \( \lambda^j_{t,k} \) as follows:

\[
\pi^j_t = \frac{GDP^j_t - EX^j_t}{GDP^j_t},
\]
\[
\lambda^j_{t,j} = \frac{GDP^j_t - EX^j_t}{GDP^j_t - EX^j_t + IM^j_t}, \lambda^{j-1}_{t,j} = 1 - \lambda^j_{t,j}
\]

Moreover, as \( \alpha^j_t \) is the share of the time-0 value of expenditure at \( t \) in total time-0 expenditure, it is given by

\[
\alpha^j_t = \frac{GDP^j_t - EX^j_t + IM^j_t}{P} \cdot \frac{1}{(1 + r^j_1) \cdots (1 + r^j_{2017})}
\]

where \( r^j_t \) is real interest rate and \( I^j \) is time-0 wealth defined by

\[
I^j = GDP^j_{1995} + \frac{GDP^j_{1996}}{1 + r^j_{1996}} + \cdots + \frac{GDP^j_{2017}}{(1 + r^j_{1996}) \cdots (1 + r^j_{2017})} \cdot \frac{1}{1 + 0.015}
\]

In the above, we have used the assumption that after 2017, GDP grows at rate 1.5% while real interest rates are 2%. While the above calculations give us the required sufficient statistics in the periods for which we observe data, we can use properties of the model and the assumption of stationarity after 2017 to calculate the corresponding values after 2017. We describe these calculations in Appendix F.

Finally, we choose parameter values for \( \sigma \) and \( \eta \) in line with the macro and trade literatures. We assume that \( \sigma = 5 \) and \( \eta = 0.5 \) in our baseline calculations. We also consider variations in these parameters (including, for example, \( \sigma = 10 \) and \( \eta = 1/3 \)) to understand their effect on optimal taxes.  

In Figure 3, we show the measured statistics. Notably, \( \alpha_t \) rises for the United States in the period leading up to the trade collapse of 2008 and then collapses in 2008. Moreover, as our optimal wedge formulas (20 and 21) identify, the main statistics that affects optimal taxes are \( \lambda^j_{t,f}/\pi^j_t \) and \( \pi^j_t \). Recall that \( \lambda^j_{t,f}/\pi^j_t \) is the ratio of GDP to expenditure in the rest of the world. As Figure 3 shows, this ratio exhibits a hump-shape behavior. As U.S. accumulates trade deficit in the period leading to the financial crisis and trade collapse of 2008, this ratio rises until 2005. It then falls until 2008 and then remains constant. As we show below, optimal trade taxes exhibit a very similar behavior.

\footnote{values for growth and real interest rates after 2017; namely, more pessimistic values for growth of 0.5% and real interest rate of 1%. Our results remain roughly unchanged.}

\footnote{Note that for these values of \( \sigma \) and \( \eta \), Assumption 1 is violated and the constraint set in the optimal policy problem is not concave. As a result, optimality conditions might not describe the optima. To address this issue, we use a global method for optimization and our computations suggests that the solution is unique.}
6.3 Results

6.3.1 Optimal Unrestricted Taxes

We start by describing the results of our optimal policy exercise when the U.S. unilaterally chooses its optimal trade policy in an unrestricted fashion. Figure 4 depicts optimal trade taxes over time.\(^{23}\) As it can be seen the main feature of optimal trade taxes are their U-shaped behavior over time; U-shaped behavior in case of export taxes and hump-shape in case of import taxes. This is mainly coming from the behavior of trade deficit and how it relates to production in the rest of the world. As mentioned earlier, \(\lambda_{t,f}^f / \pi_t^f\), or the ratio of total output to total expenditure, rises in the first half of the time period considered—see Figure 3. In other words, total expenditures are declining relative output in the foreign country. This implies that marginal utilities in the the rest of the world are higher in these periods. Since the main goal of trade taxes are to reduce (time-0) income in the rest of the world, when such taxes have a higher impact on income in rest of the world, they should be used more. This is in fact the case for the first half of the time period since expenditures are falling relative to income and as a result marginal utilities are rising. Therefore both import

\(^{23}\)By Lerner symmetry, we can normalize one of the taxes – and shift all other taxes accordingly. We thus choose export taxes to be 0 in 2017.
tariffs (export taxes) should increase (decrease) in order to achieve this goal. Note that for the period after 2017, since both countries are assumed to be growing at the same rate, the export and import taxes are constant.

Figure 4: Export (top panel) and Import (bottom panel) Taxes in the baseline calibration with $\sigma = 5, \eta = 1/2$.

Formulas (20) and (21) shed light on the behavior of optimal taxes. In particular, as they show wedges (or equivalently, taxes) are determined by the ratio of GDP to total expenditure in the rest of the world$^{24}$, $\frac{\lambda_{t,f}}{\pi_t} = \frac{p_{t,f} X_{t,f}}{p_{t,f} A_{t,f}}$, the difference between the inverse of intratemporal and intertemporal elasticity of substitution, $\frac{1}{\eta} - \frac{1}{\sigma}$, and the inverse of the share of production consumed domestically

$^{24}$We know that $\lambda_{t,f} = \frac{p_{t,f} X_{t,f}}{p_{t,f} X_{t,f}}$ and $\pi_t = \frac{p_{t,f} X_{t,f}}{p_{t,f} A_{t,f}}$. Therefore, $\frac{\lambda_{t,f}}{\pi_t} = \frac{p_{t,f} A_{t,f}}{p_{t,f} X_{t,f}}$ is the ratio of total output to total expenditure in country $f$. 

23
in the foreign country. Note that since $\sigma = 5$, the term $\frac{1}{\sigma} \pi^f_t$ does not generate a lot of variations in import taxes. In contrast, $\frac{1}{\eta} - \frac{1}{\sigma}$ = 2 − 0.2 = 1.8. Therefore, variations in $\lambda^f_t/\pi^f_t$ are more important relative to those of $\pi^f_t$ in determining optimal taxes.

As we have mentioned earlier, the same allocations can be implemented using capital controls, i.e., taxes on international asset holdings (or debt) together with an intratemporal tariff on imports. Figure 5 depicts import tariffs and capital control taxes. Note that in the absence of export taxes, import tariffs play the role of the intratemporal wedge, i.e., the difference between the relative price of the two goods faced by consumers at home and abroad. This corresponds to $(1 + \tau^f_t)(1 + \tau^h_t) - 1$. Since import and export taxes mainly move in opposite directions, its values remains mainly unchanged.

Capital control taxes mostly have values (positive or negative) around 0.5%. Note that these taxes are on stocks of debt and asset holdings. To give a perspective, for a value of 4% for the interest rate, a 1% tax on the stock of assets, is equivalent to a tax of $1.04/0.04 \times 1\% = 26\%$ on asset income. Note that the last values of capital control is somewhat large (-2.5%) because of the abrupt change in allocations before and after 2016. Moreover, optimal capital control taxes are 0 after 2017 since both countries grow at the same rate and real interest rates are stationary.

![Figure 5: Import Tariffs (left panel) and capital control taxes in the baseline calibration in percents.](image)

6.3.2 The Effect of Optimal Policy on Trade Flows and Relative Prices

We now describe the effect of optimal policy on trade flows and real exchange rates. To start, consider exports as a fraction of GDP. Since (before-tax) price of exports and GDP are the same, then under optimal policy we must have

$$\frac{EX^*_{t,h}}{GDP^*_{t,h}} = \left(1 - \pi^h_t\right)\tilde{X}^f_{t,h},$$

where, $\pi^h_t$ is the share of U.S. output that is consumed domestically and $\tilde{X}^f_{t,h}$ is the solution of the program (P1); asterisks denote optimal values. Similarly, we can show that before tax value of
Figure 6: Trade shares – import and export shares as a fraction of GDP; The left panel is for the economy with optimal trade taxes while the right panel is for the status quo economy.

Imports as a fraction of GDP is given by
\[ \frac{IM_{t,h}^*,h}{GDP_{t,h}^*} = \left( 1 - \lambda_{t,h}^* \right) \frac{\hat{X}_{t,f}^h}{X_{t,h}^h} \frac{\left( \hat{X}_{t,f}^h \right)^{1 - \frac{1}{\eta}}}{\left( \hat{X}_{t,h}^h \right)^{\frac{1}{\eta}}} \left( 1 + \tau_{f,t} \right) \frac{\left( \hat{X}_{t,f}^h \right)}{\left( \hat{X}_{t,h}^h \right)} \left( \hat{X}_{t,f}^h \right)^{-\frac{1}{\eta}} \left( \hat{X}_{t,h}^h \right)^{-\frac{1}{\eta}}. \]

Using the above formula, we can compare trade flows under optimal policy and the status quo economy. Figure 6 depicts trade flows under optimal policy as well as in the status quo economy. As expected, there is less trade occurring under optimal policy. Moreover, the variation in trade shares over time is not very different under the optimal policy and the status quo. In particular, the growth rate of export share between 1995 and 2017 is 44.5% under optimal policy while it is 41.8% for the status quo economy. Additionally, for import shares these values are 67.5% vs. 66.3%, respectively. Finally, net trade flows as measured by the ratio of exports to imports is lower under optimal policy. This is mainly reflecting the fact that under optimal policy, the U.S. is able to raise a significant revenue and thus imports are always above exports.

We can also compare the real exchange rates, the ratio of the price level in the U.S. and the rest of the world, under optimal policy and status quo allocations. Similar to trade flows, we can calculate the change in real exchange rate by realizing that
\[ \frac{RER_{t+1}^*}{RER_t^*} = \frac{RER_{t+1}}{RER_t} \times \frac{\left( \frac{\hat{X}_{t+1}^f}{X_{t+1}^f} \right)^{-\frac{1}{\eta}}}{\left( \frac{\hat{X}_t^h}{X_t^h} \right)^{\frac{1}{\eta}}} \times \frac{1 + r_{t+1}^f}{1 + r_{t+1}^h} \times \frac{\left( \frac{\hat{X}_{t+1}^f}{X_{t+1}^f} \right)^{-\frac{1}{\eta}}}{\left( \frac{\hat{X}_t^h}{X_t^h} \right)^{\frac{1}{\eta}}}, \]
where, the superscript * refers to allocations under optimal policies. Figure 7 depicts the evolution of real exchange rate over time under optimal policy in comparison to the status quo allocations.

\[25\text{we show these relations in the Appendix F}\]
\[26\text{For ease of comparison of the two graphs in Figure 6, we have omitted the trade shares for 2018 and afterwards. Import and export shares under optimal policy are 29.6% and 24.5%, respectively. Import and export shares in the status quo economy are 40.3% and 40.8%. Under status quo, import shares must be lower than export shares to pay for the accumulated trade deficits in 1995-2017.}\]
We see that real exchange rate under optimal policy is more volatile. This is mainly reflecting the fact that export taxes are U-shaped and import taxes are hump-shaped. This causes the home economy to grow slower than the foreign country initially and as a result the real exchange rate becomes more volatile under optimal policy. As a benchmark, it is useful to compare our optimal taxes to the measured wedges – those that justify the trade flows in a neoclassical model of trade (see Levchenko, Lewis, and Tesar 2010 and Alessandria, Kaboski, and Midrigan 2013). Note that our model allows for these wedges since they are often measured in a frictionless model, i.e., non-variable trade costs and absent taste shocks. Since our model is flexible enough to match any pattern of trade (we allow for variable trade costs and taste shocks), trade taxes act as distortions in addition to the wedges measured by these studies. In comparison to this literature, we see that optimal taxes move similar to measured wedges except that the magnitude of changes are much smaller. In particular, as shown by Levchenko et al. (2010), measured wedges seem to be positively correlated with trade flows (as percent of GDP). A similar pattern holds in our setup. However, the variation in our optimal taxes (changes of around 5 percentage points) are lower than that measured by Levchenko et al. (2010) (trend to peak change of around 10-20 percentage points).

Finally, we consider the effect of variations in the elasticity of substitution (intra- and inter-temporal) on optimal taxes. In order to make the results comparable, we normalize all taxes – on import and exports – so that the export taxes are 0 when the economy becomes stationary. As it can be seen, $\sigma$ is more important for the level of import taxes. This is mainly because $\sigma$ is the main determinant of intratemporal wedge – it is lower for higher $\sigma$’s. As a result, since export taxes do not vary a lot with $\sigma$, import taxes have to be much lower to accommodate a lower intratemporal wedge.

6.3.3 Optimal Restricted Taxes

Here, we consider restricting the policies used by the home country. In particular, we consider two exercises: one in which United States can only choose capital control taxes optimally and
Figure 8: Optimal export taxes (left panel) and import taxes (right panel) for various values of elasticities

one in which the U.S. only chooses import tariffs optimally. The first exercise sheds light on the
degree to which capital control taxes, which are not restricted under the WTO agreement, could
substitute for the lost policy space due to the constraints imposed by the WTO on trade taxes. The
second exercise is mainly motivated institutional constraints in the United States. In particular,
export taxes are banned by the US constitution. Moreover, since the Bretten Woods agreement,
the United States has strongly supported a policy regime that excludes capital controls. Together,
they shed light on the strength of the incentives to deviate from trade agreement and how trade
imbalances interplay with these incentives.

Figure 9 depicts the optimal tariffs and capital controls when these are the only tools available
to the government. Note that in case of tariffs only, tariffs are hump-shaped similar to import
taxes. Moreover, the magnitude of the changes are similar in from 1995 to 2004, they increase by
around 5 percentage points. Note that this increase come at a loss relative to the unrestricted policy
problem. This is because in that problem, the home government wishes to keep the intratemporal
wedge between domestic and foreign relative prices relatively stable. As for the capital control
case only, we observe that the variation in capital controls is reduced relative to optimal ones.

Figure 9: Optimal Tariffs (left panel) and capital control with restrictions on policies

In Table 1, we report the welfare gains under optimal restricted policies in consumption equiva-
ence units and compare them with the unrestricted policy case. As it can be seen, capital controls
can barely achieve any gains while tariffs can almost attain all the gains from optimal policy. In other words, capital controls cannot be a substitute for static terms of trade manipulation. This is mainly because of the way expenditure shares vary over time and across U.S. and the rest of the world. In particular, as shown in Figure 3 except for the years 1995 to 2000, $\alpha_j^t$’s move together which implies that the gains from dynamic terms of trade manipulation are not very large. In the Table, we also report the gains from imposing a constant tariff over time. We observe that constant tariffs are also very close to the optimum. Thus the gains from time varying trade taxes are not very large.

<table>
<thead>
<tr>
<th>Change in Welfare</th>
<th>Capital Control Only</th>
<th>Constant Tariff</th>
<th>Tariff Only</th>
<th>Unrestricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 5, \eta = 0.5$</td>
<td>0.001%</td>
<td>1.771%</td>
<td>1.772%</td>
<td>1.773%</td>
</tr>
<tr>
<td>$\sigma = 5, \eta = 0.33$</td>
<td>0.002%</td>
<td>1.772%</td>
<td>1.773%</td>
<td>1.775%</td>
</tr>
<tr>
<td>$\sigma = 10, \eta = 0.5$</td>
<td>0.001%</td>
<td>0.807%</td>
<td>0.808%</td>
<td>0.809%</td>
</tr>
<tr>
<td>$\sigma = 10, \eta = 0.33$</td>
<td>0.002%</td>
<td>0.808%</td>
<td>0.809%</td>
<td>0.811%</td>
</tr>
</tbody>
</table>

Table 1: Welfare calculations for restricted and unrestricted optimal policy; Numbers are in consumption equivalence units

7 Conclusion

In this paper, we analyzed unilaterally-optimal trade and capital control policies under a dynamic model with one factor of production. Our analysis sheds light on changes in optimal trade taxes over time and the viability of different tax instruments, i.e., tariffs, export subsidies, and capital control, as optimal policy instruments. In a quantitative version of our model, one which we are able to match the behavior of trade flows and production between the United States and the rest of the world, we show that import and export taxes must comove with trade flows. However, the gains from this variation is small. In particular, a constant tariff can achieve almost all of the gains from switching to optimal policy. This implies that capital controls have a very limited capacity in replicating optimal trade policies.

In this model, we have tried to tackle the question of optimal trade policy and trade imbalances in the simplest possible framework. In particular, in our model, intratemporal trade arises from preference for variety (i.e., Armington) while intertemporal trade occurs due to consumption smoothing reasons. We, thus, have made several potentially important simplifications that can be important and are left for future work. Namely, in our framework preferences are independent of past consumption. As a result, the model cannot generate the high degree of persistence observed in the data. Introducing such features (such as habit formation; see Alessandria et al. (2013) among others) can create a history dependence in optimal policy and might lead to a relationship between optimal trade policy and deficit (or country’s asset position). Moreover, we have abstracted from capital accumulation. In such an environment, optimal trade policy could potentially differ across capital and consumption goods. This in turn implies that capital control policies resemble trade policies to a greater extent. We leave the investigation of these questions for future work.
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Appendix

A Convexity of the optimization problem

Assumption 1 guarantees that the following conditions hold:

1. \( \sum_{t=0}^{T} \beta^t u'(X^f_t) X^f_t = \sum_{t=0}^{T} \beta^t \left( X^f_t \right)^{1-\frac{1}{\eta}} \) is concave in \( \left( X^f_1, X^f_2, \ldots \right) \).

2. \( \sum_{t=0}^{T} \beta^t u'(X^f_t) \frac{dX^f_t}{dX^f_{t,h}} A_{t,f} \equiv \sum_{t=0}^{T} \beta^t \left( X^f_t \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \left( X^f_{t,f} \right)^{-\frac{1}{\sigma}} A_{t,f} \) is convex in \( \left( X^f_1, X^f_2, \ldots \right) \) and \( \left( X^f_{1,f}, X^f_{2,f}, \ldots \right) \).

Under these conditions, in turn, the implementability condition (17) can be replaced by the inequality

\[
\sum_{t=0}^{T} \beta^t \left( \frac{d u(X^f_t)}{dX^f_{t,h}} X^f_{t,h} + \frac{d u(X^f_t)}{dX^f_{t,f}} X^f_{t,f} \right) - \sum_{t=0}^{T} \beta^t \frac{d u(X^f_t)}{dX^f_{t,f}} A_{t,f} \geq 0
\]

Moreover, if two allocations satisfy the above inequality, then their convex combination also satisfies it. Thus standard arguments – see Luenberger (1997) – imply that we have a convex optimization problem and that first-order conditions are necessary and sufficient.

B Relative Productivity and Optimal Wedges

Letting \( z_t \equiv \frac{A_{t,h}}{A_{t,f}} \), the equations (18) and (19) may be written as

\[
(\pi_{t,h})^{-\frac{1}{\sigma}} \left( \left[ (\pi_t^h z_t)^{1-\frac{1}{\sigma}} + (1 - \pi_t^f)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \\
\mu \left( 1 - \pi_t^h \right)^{-\frac{1}{\sigma}} \left( \left[ ((1 - \pi_t^h) z_t)^{1-\frac{1}{\sigma}} + (\pi_t^f)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right)^{\frac{1}{\sigma} - \frac{1}{\eta}} \\
= 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \frac{\left( \pi_t^f \right)^{-\frac{1}{\sigma}}}{((1 - \pi_t^h) z_t)^{1-\frac{1}{\sigma}} + (\pi_t^f)^{1-\frac{1}{\sigma}}}. 
\]
\[
(1 - \pi_t^f)^{-\frac{1}{\eta}} \left( \left[ \left( \pi_t^h z_t \right)^{1 - \frac{1}{\sigma}} + \left( 1 - \pi_t^f \right)^{1 - \frac{1}{\sigma}} \right] \sigma - 1 \right)^{\frac{1}{\sigma - 1}} \eta
\]

\[
\mu \left( \pi_t^f \right)^{-\frac{1}{\sigma}} \left( \left[ \left( 1 - \pi_t^h z_t \right)^{1 - \frac{1}{\sigma}} + \left( \pi_t^f \right)^{1 - \frac{1}{\sigma}} \right] \sigma - 1 \right)^{\frac{1}{\sigma - 1}} \eta
\]

\[
= 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \pi_t^f \right)^{-\frac{1}{\sigma}} \left( \left( 1 - \pi_t^h z_t \right)^{1 - \frac{1}{\sigma}} + \left( \pi_t^f \right)^{1 - \frac{1}{\sigma}} \right)
\]

The only variables that show up in these two conditions are \(\pi_t^f\) and \(\pi_t^h\). Moreover, \(z_t\) is the only time-varying parameter that shows up in these two equations. Therefore, in each period, the fraction of local output that is consumed locally in each country, \(\pi_t^f\) and \(\pi_t^h\), are pinned down by the relative productivity of the two countries in that period, \(z_t\).

### C Proof of Proposition 1

Consider the optimal taxation problem in primal form

\[
\max \sum_{t=0}^{T} \beta^t u \left( g \left( x_t^h \right) \right)
\]

subject to

\[
\sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k,j} x_{i,j,k} \frac{\partial}{\partial x_{i,j,k}} g \left( x_t^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k} \frac{\partial}{\partial x_{f,k}} g \left( x_t^f \right) y_{t,f,k}
\]

\[
\sum_{k} y_{t,j,k} = 1
\]

\[
x_{t,j,k}^h + x_{t,j,k}^f = y_{t,j,k}
\]

\[
\frac{\partial}{\partial x_{t,j,k}} g \left( x_t^f \right) = \frac{\lambda_t^j}{a_{t,j,k}}
\]

First, we can simplify the above by realizing that \(y_{t,j,k}\) can be substituted out. This is as follows: \(y_{t,h,k}\) does not affect neither the implementability constraint nor the marginal utility restriction. Therefore, we can replace the resource constraint with

\[
\sum_{k} x_{t,j,k}^h + x_{t,j,k}^f = 1
\]
For the foreign country, we can use the marginal utility constraint and replace it in the implementability constraint to get

\[
\sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k,j} x_{t,j,k}^f \frac{\partial}{\partial x_{j,k}} g \left( x_t^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^i \right) \right) \sum_{k} \lambda_t^f \frac{y_{t,f,k}}{a_t,f,k} \\
= \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^i \right) \right) \lambda_t^f \sum_{k} \frac{y_{t,f,k}}{a_t,f,k} \\
= \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^i \right) \right) \lambda_t^f
\]

Therefore, the above problem becomes

\[
\max_{t=0}^{T} \sum \beta^t u' \left( g \left( x_t^h \right) \right)
\]

subject to

\[
\sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^i \right) \right) \sum_{k,j} x_{t,j,k}^f \frac{\partial}{\partial x_{j,k}} g \left( x_t^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \lambda_t^f \\
\sum_{k} x_{t,j,k}^h + x_{t,j,k}^f = 1 \\
\frac{\partial}{\partial x_{t,j,k}} g \left( x_t^f \right) = \frac{\lambda_t^f}{a_t,j,k}
\]

Note that the last constraint is slack for the home country. This is because with and without the home government would like to set marginal utilities for domestic production proportional to the inverse of productivity. Formally, when we remove these constraints – for the home country, the solution must satisfy

\[
\beta^t u' \left( g \left( x_t^h \right) \right) \frac{\partial}{\partial x_{t,h,k}} g \left( x_t^h \right) = \frac{\gamma_t^h}{a_t,h,k}
\]

This implies that the constraint must be satisfied, since by setting \( \lambda_t^h = \frac{\gamma_t^h}{\beta u'(g(x_t^h))} \), the above becomes the constraint.

Next, consider the first order condition of the above problem with respect to \( x_{t,f,k}^h \):

\[
\beta^t u' \left( g \left( x_t^h \right) \right) \frac{\partial}{\partial x_{f,k}} g \left( x_t^h \right) = \frac{\gamma_t^f}{a_t,f,k}
\]

Comparing this to the marginal utility constraint associated with the foreign country, we see that taxes on imports should be uniform. To see that, note that we have

\[
\beta^t u' \left( g \left( x_t^h \right) \right) \frac{\partial}{\partial x_{f,k}} g \left( x_t^h \right) = \frac{\gamma_t^f}{a_t,f,k} \\
\frac{\partial}{\partial x_{f,k}} g \left( x_t^f \right) = \frac{\lambda_t^f}{a_t,f,k}
\]
The above implies that
\[
\frac{\beta' u' (g (x^h_k)) \frac{\partial}{\partial x_{j,k}} g (x^h_k)}{\beta' u' (g (x^h_j)) \frac{\partial}{\partial x_{j,k}} g (x^h_j)} = \frac{\beta' u' (g (x^h_k)) \frac{\partial}{\partial x_{j,k'}} g (x^h_k)}{\beta' u' (g (x^h_j)) \frac{\partial}{\partial x_{j,k'}} g (x^h_j)}
\]
for any two sectors \(k, k'\).

D  Derivation of Optimal Policy Formula (22-24)

The FOC with respect to \(X^h_{t,h}\) is given by:

\[
u'(X^h_t) \frac{dX^h_t}{dX^h_{t,h}} + \mu \left[ -u''(X^f_t) X^f_t \frac{dX^f_t}{dX^h_{t,h}} - u'(X^f_t) \frac{dX^f_t}{dX^h_{t,h}} \right] + \left( u''(X^f_t) \frac{dX^f_t}{dX^h_{t,h}} + u'(X^f_t) \frac{d^2X^f_t}{dX^h_{t,h}^2} \right) A_{t,f} = 0
\]

(27)

Noting that \(u'(X^h_t) = (X^h_t)^{-\frac{1}{\eta}}, u''(X^h_t) = -\frac{1}{\eta} (X^h_t)^{-1 - \frac{1}{\eta}}, \frac{dX^f_t}{dX^h_{t,h}} = \left( \frac{X^f_t}{X^h_t} \right)^{\frac{1}{\eta}}, \text{ and } \frac{d^2X^f_t}{dX^h_{t,h}^2} =
\]

\[
\left( \frac{x^f_t x^f_t}{x^h_{t,h} x^f_{t,h}} \right)^{\frac{1}{\eta}} \frac{1}{\sigma x^h_t},
\]

we obtain condition (18) in the text:

\[
\frac{(X^h_t)^{\frac{1}{\eta} - \frac{1}{\eta}} (X^h_{t,h})^{-\frac{1}{\eta}}}{\mu (X^f_t)^{\frac{1}{\eta} - \frac{1}{\eta}} (X^f_{t,h})^{-\frac{1}{\eta}}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X^f_t}{X^h_t} \right)^{\frac{1}{\eta}} A_{t,f}.
\]

Similarly, the FOC with respect to \(X^h_{t,f}\) is given by

\[
\beta' u'(X^h_t) \frac{dX^h_t}{dX^h_{t,f}} + \mu \beta' \left[ -u''(X^f_t) X^f_t \frac{dX^f_t}{dX^h_{t,f}} - u'(X^f_t) \frac{dX^f_t}{dX^h_{t,f}} \right] + \left( u''(X^f_t) \left( \frac{dX^f_t}{dX^h_{t,f}} \right)^2 + u'(X^f_t) \frac{d^2X^f_t}{d(X^h_{t,f})^2} \right) A_{t,f} = 0.
\]

(28)

which may be written as equation (19) in the text:

\[
\frac{(X^h_t)^{\frac{1}{\eta} - \frac{1}{\eta}} (X^h_{t,f})^{-\frac{1}{\eta}}}{\mu (X^f_t)^{\frac{1}{\eta} - \frac{1}{\eta}} (X^f_{t,f})^{-\frac{1}{\eta}}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X^f_{t,f}}{X^h_t} \right)^{-\frac{1}{\eta}} A_{t,f} + \frac{1}{\sigma X^h_t} A_{t,f}.
\]
The left-hand side of equation (18) is the relative marginal utilities of Home and Foreign from consumption of the home good in period $t$. Therefore the left-hand side of (18) may be replaced with $\frac{X^h_{t,h}}{\mu^h_{t,h}} \frac{1}{1 + \tau_{t,h}}$. Similarly, the left-hand side of (19) may be written as $\frac{X^h_{t,h}}{\mu^h_{t,h}} (1 + \tau_{t,f})$. Substituting these values for the left-hand side of the FOCs and dividing the FOCs of each period yields the tax formula in the text.

E Proof of Proposition 3

Let us consider the optimality conditions associated with the planning problem $(P')$:

$$
\frac{(X^h_{t,f})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^h_{t,h})^{-\frac{1}{\sigma}}}{\mu (X^f_{t,f})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^f_{t,h})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X^f_{t,f}}{X^f_t} \right)^{-\frac{1}{\sigma}} \frac{A_t}{X^f_t} \tag{29}
$$

$$
\frac{(X^h_{t,h})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^h_{t,f})^{-\frac{1}{\sigma}}}{\mu (X^f_{t,f})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^f_{t,h})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X^f_{t,h}}{X^f_t} \right)^{-\frac{1}{\sigma}} \frac{A_t}{X^f_t} + \frac{1}{\sigma} \frac{A_{t,f}}{X^f_t} \tag{30}
$$

As we have shown in proposition 2, the solution to the above is only a function of $A_{t,h}/A_{t,f}$. Now, in order to prove our monotonicity result, we consider an increase in $A_{t,f}$ while we keep $A_{t,h}$ constant. We show the claim by showing that $X^f_{t,f}$ increases in $A_{t,f}$. Our first claim is that when this happens, $X^f_{t,h}$ must increase. To show this, suppose to the contrary that it does not and it decreases. This decrease implies that $X^h_{t,f}$ must increase. Therefore, holding $X^h_{t,h}$ constant, the RHS of the above equations increases while its LHS decreases. In order for the above to hold, we must thus have that $X^f_{t,h}$ increases. This is because, ceteris paribus, an increase in $X^f_{t,h}$ increases $X^f_t$ which then reduces the RHS – because $\sigma > 1$ – and increases the LHS.

Now, if we divide the two equations, we have

$$
\frac{(X^h_{t,f})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^h_{t,h})^{-\frac{1}{\sigma}}}{(X^f_{t,f})^{-\frac{1}{\eta} + \frac{1}{\sigma}} (X^f_{t,h})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X^f_{t,f}}{X^f_t} \right)^{-\frac{1}{\sigma}} \frac{A_t}{X^f_t} + \frac{1}{\sigma} \frac{A_{t,f}}{X^f_t} \frac{A_{t,f}}{X^f_t} + \frac{1}{\sigma} \frac{A_{t,f}}{X^f_t}
$$

which leads to the equation

$$
\left( \frac{X^f_{t,f}}{X^h_{t,f}} \right)^{\frac{1}{\sigma}} \left( \frac{X^h_{t,h}}{X^f_t} \right)^{\frac{1}{\sigma}} = 1 + \frac{\frac{1}{\sigma} A_{t,f}}{X^f_t} \frac{A_{t,f}}{X^f_t} \frac{A_{t,f}}{X^f_t} + \frac{1}{\sigma} \frac{A_{t,f}}{X^f_t}
$$

Given our assumptions and the arguments above, the LHS of this equation decreases. This is because $X^f_{t,f}$ declines (which by feasibility implies that $X^f_{t,f}$ increases) and $X^f_{t,h}$ increases ($X^h_{t,h}$...
increases by feasibility). We argue that the RHS of the above increases under our assumptions which is a contradiction. To see this, note that we can write

\[
\frac{1}{\sigma} \frac{A_{t,f}}{X_{t,f}} \left( 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) (1 - \alpha) \left( \frac{X_{t,f}}{X_{t,f}^f} \right)^{1-\frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} \right)
\]

\[
= \frac{1}{\sigma} \frac{d}{d\left( \frac{A_{t,f}}{X_{t,f}^f} \right)} 
\]

\[
= \frac{\frac{1}{\sigma} A_{t,f} d\left( \frac{A_{t,f}}{X_{t,f}^f} \right) \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}}{X_{t,f}^f} \right)^{1-\frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} }{\left[ 1 - \frac{1}{\eta} + \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}}{X_{t,f}^f} \right)^{1-\frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} \right]^{2}}
\]

where in the above the operator \( d(\cdot) \) represents the infinitesimal change in a variable. We therefore have

\[
\frac{1}{\sigma} \frac{d}{d\left( \frac{A_{t,f}}{X_{t,f}^f} \right)} \left( \frac{1}{\eta} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}}{X_{t,f}^f} \right)^{1-\frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} 
\]

\[
= \frac{1}{\sigma} \left( 1 - \frac{1}{\eta} \right) d\left( \frac{A_{t,f}}{X_{t,f}^f} \right) 
\]

Note that in the above expression \( d\left( \frac{A_{t,f}}{X_{t,f}^f} \right) > 0 \) – since \( A_{t,f} / X_{t,f}^f \) is increasing. At the same time \( d\left( \frac{X_{t,f}^f}{X_{t,f}} \right) < 0 \), since \( X_{t,f}^f \) decreases and \( X_{t,h} \) increases. This implies a contradiction and thus we have established that \( X_{t,f}^f \) must increase in response to an increase in \( A_{t,f} \).
F Derivation of the Hat-Algebra Model and Calculation of Sufficient Statistics

We refer to status quo allocation, those in the competitive economy without any taxes, as \( \bar{X}_{t,k}^j \) and those under arbitrary policy as \( \tilde{X}_{t,k}^j \). As we have mentioned in section 3, any allocation that satisfies feasibility and implementability constraint is derived from a competitive equilibrium for some tax policy. Therefore, \( \{ \tilde{X}_{t,k}^j \} \) and \( \{ X_{t,k}^j \} \) must satisfy these constraints, namely

\[
X_{t,h}^h + d_{t,h}^f X_{t,h}^f = A_{t,h}
\]

\[
X_{t,f}^f + d_{t,f}^h X_{t,f}^h = A_{t,f}
\]

\[
\sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta_{s}^{j} \left( X_{t}^{f} \right)^{1 - \frac{1}{\eta}} = \sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta_{s}^{j} \left( X_{t}^{f} \right)^{1 - \frac{1}{\eta}} \left( X_{t}^{f} \right)^{- \frac{1}{\sigma}} A_{t,f}
\]

where in the above \( X_{t}^{f} \) is the CES aggregator. Moreover, since \( X_{t,k}^j \) is derived from a competitive equilibrium, prices \( p_{t,h} \) and \( p_{t,f} \) – for the output of foreign and domestic country – exists so that

\[
X_{t,k}^j = \left( \frac{p_{t,k} d_{t,k}^j}{p_{t,h} d_{t,h}^j + p_{t,f} d_{t,f}^j} \right)^{-\sigma} \left( \prod_{s=0}^{t} \beta_{s}^{j} \right)^{\eta} \left[ \left( p_{t,h} d_{t,h}^j \right)^{1-\sigma} + \left( p_{t,f} d_{t,f}^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \prod_{l=0}^{\infty} \left( \prod_{s=0}^{l} \beta_{s}^{j} \right)^{\eta} \left[ \left( p_{l,h} d_{l,h}^j \right)^{1-\sigma} + \left( p_{l,f} d_{l,f}^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tau^j
\]

where \( \tau^j \) is the time-0 wealth of the country. Note that given above, total expenditure in period \( t \) is given by

\[
p_{t,h} d_{t,h}^j X_{t,h}^j + p_{t,f} d_{t,f}^j X_{t,f}^j = \left( \prod_{s=0}^{t} \beta_{s}^{j} \right)^{\eta} \left[ \left( p_{t,h} d_{t,h}^j \right)^{1-\sigma} + \left( p_{t,f} d_{t,f}^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tau^j
\]

Thus, the price index in period \( t \), \( P_t^j \) is given by

\[
P_t^j = \left[ \left( p_{t,h} d_{t,h}^j \right)^{1-\sigma} + \left( p_{t,f} d_{t,f}^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

and therefore, we can write

\[
p_{t,k} d_{t,k}^j X_{t,k}^j = \left( \frac{p_{t,k} d_{t,k}^j}{P_t^j} \right)^{-\sigma} \left( \prod_{s=0}^{t} \beta_{s}^{j} \right)^{\eta} \left( P_t^j \right)^{1-\eta} \tau^j
\]
Given the above, we can write

\[ X_j^t = \left[ (p_{t,h}d_{t,h}^j)^{1-\sigma} + (p_{t,f}d_{t,f}^j)^{1-\sigma} \right]^{\sigma_{-1}} \frac{\left( \prod_{s=0}^{t} \beta_s^j \right)^{\eta}}{\sum_{l=0}^{\infty} \left( \prod_{s=0}^{t} \beta_s^j \right)^{\eta}} \left( P_t^j \right)^{\sigma_{-1}} \left( P_l^j \right)^{1-\sigma} \]

Thus \( X_j^t \) is the total expenditure at \( t \) from the perspective of time 0. Thus, the share of time-\( t \) expenditure as a fraction of time-0 wealth is given by

\[ \alpha_j^t = \frac{X_j^t P_t^j}{T^j} = \frac{\left( \prod_{s=0}^{t} \beta_s^j \right)^{\eta}}{\left( P_l^j \right)^{1-\sigma}} \left( \frac{P_t^j}{P_t^j} \right)^{1-\sigma} \]

As a result, we can write

\[ \prod_{s=0}^{t} \beta_s^j \left( X_j^t \right)^{\frac{n-1}{\eta}} = \left( T^j \right)^{\frac{1-\frac{1}{\eta}}{1}} \frac{\left( \prod_{s=0}^{t} \beta_s^j \right)^{\eta}}{\left( P_t^j \right)^{1-\eta}} \left( P_l^j \right)^{1-\eta} \]

Moreover, the intratemporal expenditure share of good \( k \) as a fraction of total expenditure, \( \lambda_{j,k}^t \) is given by

\[ \lambda_{j,k}^t = \frac{p_{t,k}d_{t,k}^j X_j^t}{P_t^j X_t^j} = \frac{\left( p_{t,k}d_{t,k}^j \right)^{1-\sigma}}{\left( P_t^j \right)^{1-\sigma}} \]

Therefore,

\[ \left( \frac{X_j^t}{X_t^j} \right)^{\frac{1-\sigma}{\sigma}} = \left( \frac{p_{t,k}d_{t,k}^j}{P_t} \right)^{\frac{1-\sigma}{\sigma}} \]

Finally note that the share of \( j \)’s output that is consumed domestically, \( \pi_{i,j}^t \), is simply given by

\[ \pi_{i,j}^t = \frac{X_{i,j}^j}{A_{i,j}} \]

Now, consider an arbitrary allocation, \( \tilde{X}_{i,j}^j \) that satisfies the feasibility and implementability constraints. If we define \( \tilde{X}_{i,j}^j = \frac{\tilde{X}_{i,j}^j}{X_t^j} \) and \( \tilde{X}_j^t = \frac{\tilde{X}_j^t}{X_t^j} \), then we can write the implementability con-
Similarly, we can write the utility of the representative consumer in country \( h \) as

\[
\sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta_s^h (X_t^{f})^{1-\frac{1}{\eta}} (\hat{X}_t^{f})^{1-\frac{1}{\eta}} = \sum_{t=0}^{\infty} \prod_{s=0}^{t} \beta_s^h (X_t^{f})^{1-\frac{1}{\eta}} (\hat{X}_{t,f}^{f})^{1-\frac{1}{\eta}} (\hat{X}_t^{f})^{1-\frac{1}{\eta}} (\hat{X}_{t,f}^{f})^{1-\frac{1}{\eta}} A_{t,f}
\]

Thus, implementability constraint becomes

\[
\sum_{t=0}^{\infty} \alpha_t f (\hat{X}_t^{f})^{1-\frac{1}{\eta}} = \sum_{t=0}^{\infty} \alpha_t \frac{\lambda_t f}{\pi_t} (\hat{X}_t^{f})^{1-\frac{1}{\eta}} (\hat{X}_{t,f}^{f})^{1-\frac{1}{\eta}}
\]

where in the above

\[
\hat{X}_t^{f} = \left[ \left( \frac{X_{t,h}^{f}}{X_t^{f}} \right)^{1-\frac{1}{\eta}} + \left( \frac{X_{t,f}^{f}}{X_t^{f}} \right)^{1-\frac{1}{\eta}} \right]^{\frac{1}{1+\frac{1}{\eta}}}
\]

Similarly, we can write the utility of the representative consumer in country \( h \) as

\[
\sum_{t=0}^{\infty} \alpha_t h (\hat{X}_t^{h})^{1-\frac{1}{\eta}}
\]

where \( \hat{X}_t^{h} \) is defined similarly as \( \hat{X}_t^{f} \).

Finally, the feasibility constraints can be written as

\[
X_{t,j} + d_{t,j} X_{t,j}^{-j} = A_{t,j}
\]

\[
\hat{X}_{t,j}^{j} \frac{X_{t,j}^{j}}{A_{t,j}} + \frac{d_{t,j}}{A_{t,j}} \hat{X}_{t,j}^{j} - 1 = 1
\]

\[
\pi_t^{j} \hat{X}_{t,j}^{j} + \left( 1 - \pi_t^{j} \right) \hat{X}_{t,j}^{j} = 1
\]

This completes the derivation of the optimal policy problem under hat-algebra.
Calibration of the Model

As we have described in the text, the values of $\alpha_{jt}$, $\lambda_{jt,k}$, and $\pi_{jt}$ can be directly calculated from the data for $t = 1995, \ldots, 2016$. For their values in the periods $t \geq 2017$, we use the fact that the model is stationary. In particular, as we have shown above

\[
\frac{\alpha_{jt+1}}{\alpha_{jt}} = \beta \left( \frac{X_{jt+1}}{X_j} \right)^{1-\frac{1}{\eta}} = \frac{X_{jt+1}}{X_j} \beta \left( \frac{X_{jt+1}}{X_j} \right)^{-\frac{1}{\eta}}
\]

\[
= \frac{X_{jt+1}}{X_j} \frac{P_{jt+1}}{P_j} = \frac{1 + g}{R}
\]

where $g$ is the growth rate of the world economy for $t \geq 2017$ and $R$ is the gross interest rate in our baseline calibration $g = 0.015$ and $R = 1.02$. By definition, $\alpha_{jt}$’s must sum to 1 and therefore, we have

\[
\alpha_{j1995} + \cdots + \alpha_{j2016} + \alpha_{j2017} \frac{1}{1 - \frac{1+g}{R}} = 1
\]

The above equation pins down $\alpha_{j2017}$. Note also that since the implementability constraint must hold for $X_{jt,k} = 1$, we must have that

\[
\alpha_{j1995} \lambda_{j1995,j} + \cdots + \alpha_{j2016} \lambda_{j2016,j} + \alpha_{j2017} \lambda_{j2017,j} \frac{1}{1 - \frac{1+g}{R}} = 1
\]

The above equation determines $\lambda_{j2017,j}/\pi_{j2017}$. Finally, two more relationships are required to determine $\lambda_{j2017,j}$. We use two sources of information: first, the fact that trade costs are stationary after 2016 implies that

\[
\frac{\lambda_{b2017,h}}{1 - \lambda_{b2017,h}} = \frac{\lambda_{b2016,h}}{1 - \lambda_{b2016,h}}
\]

\[
\frac{\lambda_{f2017,f}}{1 - \lambda_{f2017,f}} = \frac{\lambda_{f2016,f}}{1 - \lambda_{f2016,f}}
\]
Moreover,

\[ 1 - \pi_t^j = \frac{\partial_t^j X_{t,j}}{A_{t,j}} = \frac{\partial_t^j p_{t,j} X_{t,j}}{p_{t,j} A_{t,j}} = \left(1 - \lambda_{t-j}^j\right) \left(\alpha_t^j \beta_t^j - \pi_t^j\right). \]

Evaluating above at \( t = 2017 \) for \( j = h \) gives the second equation that needs to be solved. This determines all the statistics required to solve the optimal policy problem.

**Derivation of Trade Flows**

Note that before tax exports as a share of GDP in the home country is given by

\[ \frac{EX_t^h}{GDP_t^h} = \frac{X_{t,h}^f}{A_{t,h}} = \frac{\tilde{X}_{t,h}^f}{A_{t,h}} = \left(1 - \pi_t^h\right) \hat{X}_{t,h}^f. \]

Similarly for imports, we can write

\[ \frac{IM_t^h}{GDP_t^h} = \frac{d_{t,f}^h \hat{p}_{t,f} \hat{X}_{t,f}^h}{\hat{p}_{t,h} A_{t,h}} = \frac{d_{t,f}^h \hat{p}_{t,f} \tilde{X}_{t,f}^h \hat{p}_{t,f} \hat{X}_{t,f}^h}{\hat{p}_{t,h} A_{t,h}} = \left(1 - \lambda_t^h\right) \left(\alpha_t^h \beta_t^h - \pi_t^h\right). \]

where in the last step we use the optimality condition of the consumer in country \( h \).

**G Other Figures**

Figure 10 depicts our estimates of real interest rates (in the U.S. and the rest of the world) and their comparisons with the yield on 10-year TIPS.
Figure 10: Real Interest Rates; The blue solid line is the difference between yield on 10-year treasury notes and CPI inflation. The green dashed line is GDP-weighted average of long-term nominal rate less CPI inflation. The averaging is done for 16 advanced countries using the data provided by Jordà et al. (2019). The red dashed line depicts the yield on 10-year U.S. TIPS.