Optimal Trade Policy with Trade Imbalances

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Abstract

Trade imbalances are a salient feature of international trade, yet we know little about their implications for optimal trade policy. For example, does a government have more incentives to restrict imports when the country runs a larger trade deficit? Are optimal import tariffs counter-cyclical? Does capital control obviate or mitigate the need to manipulate trade policies in response to trade shocks? To answer these questions, we characterize optimal trade policy under a dynamic trade model in which trade imbalances are generated endogenously. Our key finding is that optimal import taxes are counter-cyclical and optimal export taxes are pro-cyclical, and the size or direction of trade imbalances have no bearing on optimal trade policy. Nevertheless, under certain growth paths, optimal trade policy and equilibrium trade deficit are correlated. Finally, we find that the optimal policy will discourage (encourage) the accumulation of foreign debt when the country is expected to grow faster (slower) than the rest of the world. This aspect of optimal policy may be implemented using capital control.

1 Introduction

A salient feature of international trade is the presence of trade imbalances. The level and direction of these imbalances may be affected by various policies including trade policy and capital controls. How should governments conduct their trade policy under trade imbalances? The literature on trade policy has largely avoided this question by focusing on static models in which trade is balanced by assumption and, thus, various dynamic aspects of trade policies are overlooked. Our goal in this paper is to explore the relationship between inter-temporal trade and optimal trade policy.

The presence of inter-temporal dynamics poses several important questions regarding the optimal conduct of trade policy. First, in the presence of international capital flows, which make trade imbalances possible, governments could supplement trade policy with capital controls to manipulate the flow of goods and services across borders. The potential interdependencies between

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capital controls and trade policies may have important implications about the design and benefits of trade agreements. For example, following negotiated trade liberalizations, governments may have an incentive to use capital controls more actively to manipulate their terms of trade, thereby frustrating the intent of trade agreements to some degree. To what extent are such arguments valid? As a step toward addressing these questions, in this paper we characterize the interdependence of capital controls and trade policy.

A second question is related to the pattern of optimal tariff over business cycles. Are optimal tariffs related to the size of the economy or to its growth rate? A country that is expecting a high growth rate in the future may find it optimal to run a deficit at the current period to smooth its consumption over time. Does the growth rate of the economy have any bearing on optimal conduct of trade policy?

Our point of departure is two previous papers on optimal trade policy (Beshkar and Lashkaripour 2019, henceforth BL) and optimal capital control (Costinot, Lorenzoni, and Werning 2014, henceforth CLW). BL find the structure of optimal import and export taxes under a static general-equilibrium Ricardian model in which trade is balanced by assumption and, hence, inter-temporal considerations are absent. To focus on capital control, CLW assume free trade and adopt a dynamic endowment model in which the endowments are subject to exogenous changes over time. Building on these works, we study optimal trade and capital control policies in a dynamic model.

We work within a two-country Ricardian model with time-varying labor productivity. The variation in productivity over time creates a role for international lending and borrowing for consumption-smoothing purposes. We assume that the policy instruments at the disposal of the government include import and export tax/subsidy as well as a tax on foreign borrowing and lending of domestic households.

First, we show that under general intra-period preferences, the optimal structure of trade policy shares some similarities to those prescribed by static models. In particular, as shown by BL and Costinot et al. 2015, optimal import tariffs are uniform across products under a static Ricardian model. We show that this result can be extended to our dynamic Ricardian model for imports within a period. That is, imports within a period must be taxed uniformly. Tariffs, however, can be used for manipulation of intertemporal terms of trade. Therefore, they can vary over time.

To obtain an intuition about this result, note that the optimality of uniform tariffs in a static Ricardian model follows because, due to the assumption of constant unit-labor requirement, import tariffs do not affect the relative prices of home imports or the relative prices of home exports.

Next, in order to focus on inter-temporal trade and the pattern of optimal policy over time, we restrict our attention to CES preferences, which yields a uniform optimal import and export tax in each period. Therefore, the problem of optimal trade policy under a multi-product model with CES preferences reduces to the problem of optimal trade policy for a two-good economy. Under this environment, we find that, in the absence of capital control, both import and export taxes/subsidies are necessary for the implementation of optimal policy. This is in contrast to a static two-good model in which one of the trade tax instruments are redundant.

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1It is notable that shortly after its accession to the World Trade Organization, China was frequently accused of manipulating its exchange rate to affect the flow of goods and services.

2A key difference between tariffs and capital controls is that tariffs could be imposed on trade flows at the sectoral level, while capital flow taxes or exchange rate manipulations cannot replicate a sector-specific tax on trade flows. In other words, compared to sector-specific tariffs, these policies are more blunt instruments that affect aggregate trade flows across all sectors.
We find that the productivity of the home country relative to the rest of the world is the key parameter that explains the variation in optimal trade taxes across periods. In particular, we find that import taxes are counter-cyclical and export taxes are pro-cyclical. Intuitively, this result is obtained because the government is interested in manipulating its inter-temporal terms of trade by reducing exports in booms and limiting imports in recessions. In other words, from the government’s point of view, the households save too much in booms and, thus, they consume too much of the foreign good in recessions. The government could, therefore, achieve its desired inter-temporal allocation by a higher import tax in low-productivity periods and a higher export tax in high-productivity periods.

Finally, we turn to a quantitative exercise to provide a rough measure of optimal trade policy. To do so, we extend the “exact hat algebra” methodology of Dekle et al. (2008) to the primal version of our dynamic Ramsey problem. This allows us to use only data on trade shares to quantify the model and perform our optimal policy exercise. We quantify our model using data on production and data from 1995 to 2008. Approximating the US trade policy with free trade during this period, we conduct three counterfactual analyses. First, we calculate the optimal level of capital control for the United States given its free trade obligations under the WTO. This exercise sheds light on the degree to which capital control taxes, which are not restricted under the WTO agreement, could substitute for the lost policy space due to the constraints imposed by the WTO on trade taxes. We then quantify the welfare effect of unilaterally optimal trade policy in the US. In addition to shedding light on the magnitude of the welfare effects of protectionism, this exercise will also reveal the degree to which optimal trade taxes vary over time in response to business cycles. Finally, we calculate optimal import tariffs for each year assuming that export and capital control taxes are unavailable.³

We estimate the optimal level of tariffs for the US to be around 25%. Our quantitative analysis reveals that even though in theory the optimal trade and capital control policies are variable over time, in practice the degree of this variation is quite small and, hence, the optimal policy is approximately acyclical.

**Literature**  
Staiger and Sykes (2010) take on the issue of interdependence between trade policy and currency manipulation and ask if governments could frustrate the intent of trade agreements by manipulating the value of their currency. They conclude that the trade effects of such policies could not be identified well-enough to make a judgement about whether these policies frustrate the intent of trade agreements. Bagwell and Staiger (1990) consider trade wars and self-enforcing trade agreement in a dynamic environment in which the countries’ endowments are subject to shock, but no inter-temporal trade takes place. The dynamics in the model come from the fact that governments could exchange trade policy concessions over time. Keeping the assumption of balanced trade in each period, Bagwell and Staiger (2003) extend their previous work to study trade policy over persistent business-cycle shocks.

Under the assumption of free trade, Schmitt-Grohé and Uribe (2017) study optimal capital control policy for a small open economy. Similar to CLW, Schmitt-Grohé and Uribe (2017) also find that optimal capital control would reduce borrowing during recessions, causing a counter-cyclical

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³The assumption that export and capital control taxes are unavailable reflects the institutional constraints and the general policy approach of the United States since the Bretton Woods agreement. In particular, export taxes are banned by the US constitution. Moreover, since the Bretton Woods agreement, the United States has strongly supported a policy regime that generally refrains from capital controls.
protection pattern. Unlike our model, the main reason to use capital control in their model is to correct an inefficiency arising from the interaction of the collateral constraint with the price of the non-tradable goods. This is in contrast with our model where the domestic government can reduce price of imports and inflate price of exports using trade and capital control policies. Empirical evidence on the intertemporal pattern of trade policy is rare and the existing studies deliver diverging results. As discussed in Lake and Linask (2016), the conventional wisdom is that import protection is counter cyclical, which is supported by various observations on the use of temporary trade barriers within the framework of the WTO (e.g., Bown and Crowley 2013). However, more recent works such as Lake and Linask (2016) suggest that import protection, particularly in developing countries, are pro-cyclical. Our theoretical results, which are based on a terms of trade framework, are more in line with the conventional wisdom mentioned above. Nevertheless, our quantitative results mostly point to a negligible variation in optimal trade policy over the business cycle.

In Section 2, we present the basics of the model. In Section 3 we present the planner’s problem and establish our first result about the pattern of optimal trade policy under a dynamic trade model. In Section 4, we derive optimal trade policy and characterize its variation over time. In Section 4, we also consider capital control taxes as an additional policy instrument at the government’s disposal and discuss its interdependence with trade taxes. In Section 5, we consider a simple growth path and compute the optimal trade policy and the equilibrium levels of deficit and surplus. In Section 6, finally, in Section 7, we conclude by discussing some of the potential implications of our analysis as well as further questions for future research.

2 The Basic Model

We use a multi-period, two-country, $K$-industry, model in which consumption and production takes place in each period of time, $t$. Time, $t$, is assumed to be in $\{1, \cdots, T\}$ where $T$ is a finite natural number or $\infty$. Each country, $i,j \in \{h,f\}$, produces a distinct variety in each industry. We let $p_{t,i,k}$ and $x_{t,i,k}$ denote, respectively, period-$t$ price and quantity of consumption in country $j$ of country $i$’s variety of product $k$. With appropriate interpretation of subscripts and superscripts, bold-faced variables denote vectors and capitalized variables denote aggregate values.

Consumers can trade a one-period bond on the world capital market. In period $t$, the consumer in $j$ country can buy a claim for consumption in period $t+1$, denoted by $b_{t+1}$, at a price of $q_t$ per unit.

Producers’ Problem We adopt a Ricardian framework in which each goods is produced using labor as the only input to production. Labor productivity in industry $k$ of country $i$ at time $t$, denoted by $a_{t,i,k}$, is independent of the quantity of production. Labor is perfectly mobile across industries within the same country. The population of labor in each country is assumed to be constant over time, and we normalize the population in each country to 1. Producers are perfectly competitive and, thus, their price is equal to marginal costs of production:\footnote{Absent domestic taxes, producer and consumer prices are equal $p_{t,i,k} = p_{t,i,k}^t$.}

$$p_{t,i,k} = \frac{w_{t,i}}{a_{t,i,k}}$$

where $w_{t,i}$ is the wage in country $i$ in period $t$.\footnote{Absent domestic taxes, producer and consumer prices are equal $p_{t,i,k} = p_{t,i,k}^t$.}
Consumers’ Problem  The lifetime utility of the representative consumer in country \( j \) is given by

\[
T \sum_{t=0}^{\infty} \beta^t u \left( g \left( x^j_t \right) \right),
\]

where \( x^j_t \) is country \( j \)'s vector of consumption in period \( t \), \( g \left( x^j_t \right) \) is the aggregate consumption (i.e., utility) in period \( t \) and \( u \left( \cdot \right) \) is concave function. The consumer’s per-period budget constraint in country \( j \) is given by

\[
p^j_t x^j_t + q^j_t b^j_{t+1} = w_{t,j} + b^j_t - T^j_t,
\]

where \( T^j_t \) is a lump-sum tax paid by the consumers in country \( j \) – note that \( T^j_t \) can be negative in which case the government is making a transfer to the representative consumer in country \( j \) in period \( t \). The optimization problem of the consumer in country \( j \) is to choose a vector of consumption and bonds for each period \( \{ x^j_t \} \) to maximize its lifetime utility (1) subject to its per-period budget constraints (2). Letting \( \lambda^j_t \) denote the Lagrange multiplier on the budget constraint of country \( j \) consumers in period \( t \), the optimal consumer choice implies

\[
\beta^t \frac{du \left( g \left( x^j_t \right) \right)}{dx^j_t} = \lambda^j_t p^j_t, \forall i, k
\]

and

\[
q^j_t = \frac{\lambda^j_{t+1}}{\lambda^j_t}.
\]

Combining these conditions and noting that price index in the home country is given by \( P^h_t = \frac{p^h_t}{\left. \frac{dg \left( x^h_t \right)}{dx^h_t} \right|_{x^h_t}} \) for any \( j, k \), the Euler equation for the home consumer may be written as

\[
q^h_t = \beta \frac{u' \left( g \left( x^h_{t+1} \right) \right)}{u' \left( g \left( x^h_t \right) \right)} \frac{P^h_t}{P^h_{t+1}}.
\]

Policy Instruments  We assume that the home government is policy active, while the foreign government takes a laissez faire approach. The policy instruments at the disposal of the home government include import and export tax/subsidy as well as a tax on international borrowing and lending of domestic households. Together with iceberg transport costs, \( d^j_{t,i,k} \), trade taxes create a wedge between home and foreign prices. To be specific, the ad valorem export tax on good \( k \) in the home country, \( \tau^e_{t,h,k} \), is implicitly given by:

\[
p^f_{t,h,k} \equiv (1 + \tau^e_{t,h,k}) d^f_{t,h,k} p^h_{t,h,k}.
\]

Similarly, the import tax, \( \tau^i_{t,f,k} \), is implicitly given by:

\[
p^h_{t,f,k} \equiv (1 + \tau^i_{t,f,k}) d^h_{t,f,k} p^f_{t,f,k}.
\]
Finally, the home government imposes a capital control tax, $\tau_{t,b}$, which is a tax on the home consumer’s holding of foreign assets, namely,

$$q^h_t \equiv (1 + \tau_{t,b}) q^f_t. \quad (8)$$

Given these policies, $\tau_{t,h,k}, \tau_{t,f,k}, \tau_{t,b}, T^h_t$ in the home country and the fact that the foreign country is passive (for which all these values are set to zero), an equilibrium in this economy is defined as follows:

1. Consumers maximize their utility in 1 subject to the budget constraint 2 while taking as given prices and policies.
2. Firms maximize profits, taking as given prices and policies.
3. Home government’s budget constraint is satisfied

$$T^h_t = \sum_k \tau_{t,h,k} d^h_{t,h,k} P^h_{t,h,k} x^f_{t,h,k} + \sum_k \tau_{t,f,k} d^h_{t,f,k} P^f_{t,f,k} x^h_{t,h,k} + \tau_{t,b} q^f_t b^h_t$$

4. All markets, labor, goods and international bonds, clear. Note that bond market clearing is given by $b^f_t + b^h_t = 0$.

**Marginal Utility Wedges** In the absence of intra- and inter-temporal trade costs, which may include transportation costs and any policy-induced costs, the wedge between the foreign and home marginal utilities will be equal to the relative shadow value of income in the two countries. Trade and capital control taxes create additional wedges between marginal utilities. In particular, letting $\theta_{t,f,k} \equiv \frac{du(g(x^h_t))}{dx^h_t} / \frac{du(g(x^f_t))}{dx^f_t}$ denote the wedge between home and foreign marginal utility from the consumption of the foreign goods, the import tax will satisfy the following condition:

$$1 + \tau_{t,f,k} \equiv \frac{P^f_{t,f,k}}{P^h_{t,f,k}} = \frac{\lambda^f_t}{\lambda^h_t} \theta_{t,f,k}. \quad (9)$$

Similarly, intra-temporal wedge for the export good, $\theta_{t,h,k} = \frac{du(g^f(x^h_t))}{dx^h_t} / \frac{du(g^f(x^f_t))}{dx^f_t}$, is related to export tax in the following way:

$$1 + \tau_{t,h,k} \equiv \frac{P^h_{t,h,k}}{P^f_{t,h,k}} = \frac{\lambda^h_t}{\lambda^f_t} \theta_{t,h,k}. \quad (10)$$

Finally, the inter-temporal wedge, $\theta_{t,b} \equiv \frac{u'(g(x^h_{t+1}))/u'(g(x^f_{t+1}))}{u'(g(x^h_t))/u'(g(x^f_t))}$ is related to capital control tax as follows:

$$1 + \tau_{t,b} \equiv \frac{q^h_t}{q^f_t} = \frac{\lambda^h_t}{\lambda^f_t} = \frac{\lambda^h_t}{\lambda^f_t} \frac{P^h_t/P^h_{t+1}}{P^f_t/P^f_{t+1}}. \quad (11)$$

We should note that the level of the wedges on its own do not represent distortions. Their changes, on the other hand, measures the distortions generated by a tax system and are thus informative of deviations of the allocation from those of an economy without distortions.
3 Optimal Trade Policy

To find the optimal policy of the home government, we use the *primal* approach as in Lucas Jr and Stokey (1983), by solving a planning problem in which the equilibrium quantities are directly chosen by the government. We then find the set of trade and capital control taxes that implement the optimal allocation. This approach is also used by CLW to characterize the optimal capital control taxes under free trade. Later, we will also consider scenarios in which the government’s policy space is subject to varying degrees of constraints imposed by trade agreements and domestic rules.5

Under the primal approach, the planner’s problem is to choose the vector of allocations for all periods, \( x^h_t \), to maximize the welfare of the representative consumer in the home country, i.e.,

\[
\max_{\{x^h_t\}_{t=0}^T} \sum_{t=1}^{T} \beta^t u \left( g \left( x^h_t \right) \right),
\]  

subject to

1. Per-period labor-market clearing conditions:6

\[
\left( x^h_{t,h} + d^f_{t,h} \odot x^f_{t,h} \right) \cdot \frac{1}{a_{t,h}} = 1, 
\]

\[
\left( d^f_{t,h} \odot x^h_{t,f} + x^f_{t,f} \right) \cdot \frac{1}{a_{t,f}} = 1, \quad (12)
\]

2. Implementability condition:

\[
\sum_{t=1}^{T} \beta^t \nabla u \left( g \left( x^f_t \right) \right) \cdot x^f_t = \sum_{t=0}^{T} \beta^t \left[ \nabla u \left( g \left( x^f_t \right) \right) \right] x^f_{t,f} \cdot a_{t,f}. \quad (13)
\]

This condition requires that the total expenditure allocated to foreign consumers by the home planner is equal to the value of their income.

3. No domestic distortion in either country (given that the available tax instruments are levied only on international exchanges), which implies that marginal utilities from consumption of the domestically-produced goods in each country are proportional to their input requirement:

\[
\frac{\partial g \left( x^f_{t,f} \right)}{\partial x^f_{t,f,k}} = \lambda^f_{t,f,k} \cdot a_{t,f,k}, \quad (14)
\]

\[
\frac{\partial g \left( x^h_{t,h} \right)}{\partial x^h_{t,h,k}} = \lambda^h_{t,h,k} \cdot a_{t,h,k}, \quad (15)
\]

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5 CLW and Costinot, Donaldson, Vogel, and Werning (2015) have demonstrated the benefits of applying this approach to international trade policy problems.

6 Recall that \( d \) denotes trade costs. Moreover, we use \( \odot \) to denote element-wise multiplication of vectors.
A few features of this optimal policy problem are worth mentioning. First, since the government distributes the revenue from its tax revenues in a lump-sum fashion, the budget constraint of the domestic consumer imposes no extra constraint on the planning problem. Second, since taxes are only imposed on international flows, production is domestically efficient. Equation (14) specifies that for any two goods produced in each country, marginal rate of substitution must be equal to marginal rate of transformation or, equivalently, the relative productivities.

The first order condition of the planner’s problem \(\Pi\) with respect to the allocation of the import good, \(x_{t,f,k}^h\), may be written as:

\[
\beta_t \frac{\partial u(g(x_{t,f,k}^h))}{\partial x_{t,f,k}^h} = \frac{\eta_f^f}{a_{t,f,k}},
\]

where \(\eta_f^f\) is the Lagrange multiplier on the resource constraint of the foreign country ((14)). Moreover, substituting the equilibrium price, \(p_{t,f,k} = \frac{w_{t,f}}{a_{t,f,k}}\), in the foreign consumer’s optimality condition we obtain

\[
\beta_t \frac{\partial u(g(x_{t,f,k}^f))}{\partial x_{t,f,k}^f} = \frac{\lambda_f^f w_{t,f}}{a_{t,f,k}}.
\]

The above two equations imply that

\[
\theta_{t,f,k} \equiv \frac{\beta_t \frac{\partial u(g(x_{t,f,k}^h))}{\partial x_{t,f,k}^h}}{\beta_t \frac{\partial u(g(x_{t,f,k}^f))}{\partial x_{t,f,k}^f}} = \frac{\eta_f^f}{\lambda_f^f w_{t,f}}.
\]

The left-hand side of this equation is the optimal wedge between the home and foreign’s marginal utility of consuming the \(t, f, k\) variety, while the term on the right-hand side includes only economy-wide variables. Therefore, we confirm that the optimality of uniform tariffs under a static Ricardian model with balanced trade (BL; CLW) carries over to a dynamic Ricardian model with trade imbalances.

**Proposition 1.** Under a dynamic Ricardian model: (i) The optimal import tariffs are uniform across products but generally differential across periods. (ii) If intra-temporal preferences, \(g\), takes a CES form, then the optimal export taxes are also uniform across products.

This Proposition states that, generally, optimality requires uniform import tariffs and differential export taxes, while optimal export taxes will be also uniform in the case of CES preferences. The intuition behind the optimality of uniform tariff within each period is the same as the one discussed in BL: Since unit-labor requirement is independent of quantity of production and wages are equal across all sectors within each period, import tariffs have no effect on the relative prices of home imports or the relative prices of home exports within each period. Therefore, the terms-of-trade effects of differential import tariffs could be replicated by a uniform tariff on all products. Additionally, uniform tariffs create less distortions for domestic consumption. This implies that tariffs must be uniform within a period.

Optimal tariffs, however, are not uniform across periods. This result follows because wages are not equalized across periods and, thus, time-varying tariffs can affect the relative inter-temporal prices of imports and exports. Finally, as stated in part (ii) of Proposition 1, under CES preferences,
optimal export taxes are also uniform across products. However, for more general preferences, export taxes are differential across products.\textsuperscript{7}

4 Intertemporal Structure of Optimal Trade Policies

In comparison to static trade policy analyses, the problem of optimal policy under a dynamic setting has at least two novel features. First, trade policy could fluctuate over time, which creates the possibility for tariffs to affect household savings in expectation of future changes in trade policy.\textsuperscript{8} For instance, if governments announce a commitment to gradually reduce import tariffs over time, households are induced to decrease their current consumption in order to save for consumption at more favorable prices in the future. A second feature of trade policy in dynamic settings, which will be discussed in the next section, is the emergence of an additional tax instrument, namely, an inter-temporal trade or capital control tax, which may be a complement or substitute for trade policy.

CES Preferences  In order to focus on the inter-temporal patterns of trade policy, we henceforth restrict our attention to CES preferences within each period. As we know from part (ii) of Proposition 1, under CES preferences, optimal import and export taxes are uniform. This result implies that under CES preferences the home government’s optimal policy problem reduces to choosing taxes on aggregate import and export volumes, rather than on individual traded varieties. Therefore, adopting CES preferences suppresses the variation of optimal trade taxes across products.

Formally, we assume that

\[
g (x^j_t) \equiv X^j_t \equiv \left[ \sum_k \alpha_k \left( x^j_{t,h,k} \right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},
\]

and we let \(X^j_{t,i}\) denote the aggregate volume of goods exported from \(i\) to \(j\) in period \(t\). We can similarly define aggregate productivity as

\[
A_{t,i} = \left[ \sum_k (\alpha_k)^{\sigma} \left( a_{t,i,k} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.
\]

Moreover, we assume that intertemporal preferences are also characterized by a CES function, namely,

\[
u (X^j_t) = \frac{\gamma}{\gamma-1} \left( X^j_t \right)^{\frac{\gamma-1}{\gamma}},
\]

where, \(\gamma\) is the intertemporal elasticity of substitution.

\textsuperscript{7}This result reflects the terms-of-trade motive of the government, which is to induce a monopoly markup over the competitive exporters’ price. The optimal markup for a product may be achieved by levying an export tax that reflects the import demand elasticity for that product in the foreign country.

\textsuperscript{8}For example, a surge in the United States’ trade deficit in 2018 was attributed to a looming trade war with China and other countries. Other examples include increased investment in the export sector in expectation of free trade agreements (McLaren, 1994), or increased imports in anticipation of tariff hikes (Alessandria et al., 2019).
Under CES preferences, therefore, we can rewrite the planner’s problem \((P)\) using aggregate values only, namely,

\[
\max \left\{ \sum_{t=1}^{T} \beta_t u \left( X_t^h \right) \right\} \tag{P'}
\]

subject to

\[
X_{t,h} + d_{t,h} X_{t,h} = A_{t,h}, \tag{16}
\]

\[
d_{t,h} X_{t,f} + X_{t,f} = A_{t,f},
\]

\[
\sum_{t=1}^{T} \beta_t \left( \frac{du(X_{t,f}^f)}{dX_{t,f}^f} X_{t,f} + \frac{du(X_{t,h}^h)}{dX_{t,h}^h} X_{t,h} \right) = \sum_{t=1}^{T} \beta_t \frac{du(X_{t,f}^f)}{dX_{t,f}^f} A_{t,f}. \tag{17}
\]

Before introducing the necessary conditions for optimality, note that the necessary conditions would be also sufficient if our programming problem is convex. This is guaranteed when the constraint set is strictly convex and objective of the maximization problem is strictly concave. Standard concavity properties of the CES function together with concavity of \(u\) guarantee that the objective is strictly concave. To ensure the convexity of the constraint set, we make the following assumption:

**Assumption 1.** \(\sigma \geq \gamma \geq 1\).

In other words, we assume that goods are more substitutable within a period than across periods. This assumption guarantees that the following conditions hold:

1. \(\sum_{t=0}^{T} \beta_t u' \left( X_t^f \right) X_t^f \equiv \sum_{t=0}^{T} \beta_t \left( X_t^f \right)^{1-\frac{1}{\gamma}} \) is concave in \(X_1^f, X_2^f, \ldots\).

2. \(\sum_{t=0}^{T} \beta_t u' \left( X_t^f \right) \frac{dX_t^f}{dX_{t,f}} A_{t,f} \equiv \sum_{t=0}^{T} \beta_t \left( X_t^f \right)^{\frac{1}{\sigma} - \frac{1}{\gamma}} \left( X_{t,f}^f \right)^{-\frac{1}{\gamma}} A_{t,f} \) is convex in \(X_1^f, X_2^f, \ldots\) and \(X_{t,f}^f, X_2^f, \ldots\).

Under Assumption 1, the implementability condition (17) can be replaced by the inequality

\[
\sum_{t=1}^{T} \beta_t \left( \frac{du(X_{t,f}^f)}{dX_{t,f}^f} X_{t,f} + \frac{du(X_{t,h}^h)}{dX_{t,h}^h} X_{t,h} \right) - \sum_{t=1}^{T} \beta_t \frac{du(X_{t,f}^f)}{dX_{t,f}^f} A_{t,f} \geq 0
\]

Moreover, if two allocations satisfy the above inequality, then their convex combination also satisfies it. Thus standard arguments – see Luenberger (1997) – imply that we have a convex optimization problem and that first-order conditions are necessary and sufficient.

Using \(\mu\) to denote the Lagrange multiplier of the implementability constraint 17, the FOC with respect to \(X_{t,h}^h\) may be written as

\[
\frac{du(X_{t,h}^h)}{dX_{t,h}^h} = \mu \left[ \frac{du(X_{t,f}^f)}{dX_{t,f}^f} - \frac{d^2 u(X_{t,f}^f)}{dX_{t,f}^f dX_{t,h}^h} X_{t,f}^h + \frac{d^2 u(X_{t,h}^h)}{dX_{t,h}^h dX_{t,h}^h} \left( A_{t,h} - X_{t,h}^h \right) \right]. \tag{18}
\]
The LHS of this equation, which is the marginal utility of home consumers from consumption of home output, is decreasing in \(X_{t,h}^h\). The RHS of this equation, which is the marginal cost of consuming the home good for the home consumer, is increasing in \(X_{t,h}^h\). Similarly, the FOC with respect to \(X_{t,h}^h\) may be written as

\[
\frac{du(X_t^h)}{dX_{t,f}^h} = \mu \left[ \frac{du(X_t^f)}{dX_{t,f}^f} - \frac{d^2u(X_t^f)}{d(X_{t,f}^f)^2} X_t^h + \frac{d^2u(X_t^f)}{dX_{t,h}^h dX_{t,f}^f} \left( A_{t,h} - X_{t,h}^h \right) \right].
\] (19)

Using the properties of the CES preferences, equation (18) may be rewritten to obtain the following condition for optimal marginal utility wedge for the export good:

\[
\frac{1}{\mu \theta_{t,h}} = 1 - \frac{1}{\gamma} + \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X_t^f}{X_t^h} \right)^{-\frac{1}{\sigma}} A_{t,f} + \frac{1}{\sigma} X_{t,f}^f.
\] (20)

where, \(\theta_{t,h} \equiv \left( \frac{X_t^f}{X_t^h} \right)^{-\frac{1}{\sigma} - \frac{1}{\gamma}} \left( \frac{X_t^h}{X_t^h} \right)^{-\frac{1}{\gamma}}\) and \(\mu\) is the Lagrange multiplier on the implementability condition (17). Similarly, equation (19) yields the following optimality condition for marginal-utility wedge for the import good:

\[
\frac{\theta_{t,f}}{\mu} = 1 - \frac{1}{\gamma} + \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X_t^f}{X_t^f} \right)^{-\frac{1}{\sigma}} A_{t,f} + \frac{1}{\sigma} X_{t,f}^f,
\] (21)

where, \(\theta_{t,f} \equiv \left( \frac{X_t^f}{X_t^f} \right)^{-\frac{1}{\sigma} - \frac{1}{\gamma}} \left( \frac{X_t^f}{X_t^f} \right)^{-\frac{1}{\gamma}}\).

Interestingly, the above equations allow us to write wedges in terms of trade shares and elasticities to provide a better intuition for determinants of optimal policy. In particular, we can write

\[
\frac{1}{\mu \theta_{t,h}} - \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) = \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \frac{1}{\pi_{t,f}^f} X_{t,f}^f
\] (22)

\[
\frac{\theta_{t,f}}{\mu} - \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) = \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \frac{1}{\pi_{t,f}^f} + \frac{1}{\sigma} \frac{1}{\pi_{t,f}}
\] (23)

where in the above \(\pi_{t,f}\) is the share of foreign output that is consumed in the foreign country while \(\lambda_{t,f}\) is the share of foreign expenditure that is spent on foreign output. The above formulas illustrate the main determinants of optimal wedges (and taxes as we describe below). First, as expected, it highlights the role of trade (im)balances. Note that with trade imbalance, expenditure in the foreign country must be equal to income and thus \(\lambda_{t,f} = \pi_{t,f}\). As an example, in periods when \(\pi_{t,f}\) is high relative to \(\lambda_{t,f}\), i.e., when the foreign country’s expenditure is high relative to its output, the wedge on exports must be high. This is resulting from the increased monopoly power of the home government when the foreign country is buying a higher share of home output. Second, both intra and inter-temporal elasticities matter for optimal trade policies. Finally, the home government has extra motive to create a wedge on imports – captured by the term \(\frac{1}{\sigma} \pi_{t,f}\).
in (23). Mathematically, this is coming from the fact that taxing imports, has the extra effect of reducing the level of income in the foreign country – the right hand side of (17).

Using the relationship between wedges and trade policy (9, 10), we obtain the following result.

**Proposition 2.** The optimal level of total trade restrictions in period \(t\) is given by

\[
(1 + \tau_{t,f}) (1 + \tau_{t,h}) = 1 + \frac{1}{\sigma} \frac{A_{t,f}}{X_{t,f}}.
\]

(24)

The individual level of optimal import and export taxes in any period depend on trade and capital control taxes in the previous period. In particular,

\[
1 + \tau_{t,f} = \frac{1 + \tau_{t-1,f}}{1 + \tau_{t-1,b}} \frac{1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X_{t-1,f}}{X_{t-1}^{f}}\right)^{-\frac{1}{\sigma}} A_{t,f}}{1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X_{t-1,f}}{X_{t-1}^{f}}\right)^{-\frac{1}{\sigma}} A_{t-1,f}},
\]

(25)

and

\[
1 + \tau_{t,h} = \frac{1 + \tau_{t-1,h}}{(1 + \tau_{t-1,b}) (1 + \tau_{t-1,h})} \frac{1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X_{t-1,f}}{X_{t-1}^{f}}\right)^{-\frac{1}{\sigma}} A_{t,f}}{1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X_{t-1,f}}{X_{t-1}^{f}}\right)^{-\frac{1}{\sigma}} A_{t-1,f}}.
\]

(26)

In contrast to the static version of the model in which the import and export taxes are perfectly substitutable (and, hence, one of the policy instruments is redundant), this Proposition shows that under a dynamic model with no capital controls, both import and export taxes are generally necessary for the implementation of the optimal policy. The capital control tax, however, could substitute one of the trade policy instruments.\(^9\)

To characterize the time-variation of optimal trade taxes, it is useful to rewrite the optimality conditions using fraction of outputs consumed in each country. Letting \(\pi_{t,i} = \frac{X_{t,i}}{A_{t,i}}\) and \(z_{t} = \frac{A_{t,h}}{A_{t,f}}\), the FOCs (20-21) may be written as:

\[
\frac{(\pi_{t,h})^{-\frac{1}{\sigma}} \left(\left(\pi_{t,h} z_{t}\right)^{1-\frac{1}{\sigma}} + (1 - \pi_{t,f})^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\mu \left(1 - \pi_{t,h}\right)^{-\frac{1}{\sigma}} \left(\left((1 - \pi_{t,h}) z_{t}\right)^{1-\frac{1}{\sigma}} + (\pi_{t,f})^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}} \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \frac{(\pi_{t,f})^{-\frac{1}{\sigma}}}{\left((1 - \pi_{t,h}) z_{t}\right)^{1-\frac{1}{\sigma}} + (\pi_{t,f})^{1-\frac{1}{\sigma}}}.
\]

\(^9\)This indeterminacy reflects Bond’s (1990) argument that the solution to optimal trade policy determines all tariff levels relative to a numeraire.
and

\[
\frac{(1 - \pi_{t,f})^{-1/\sigma} \left( \left[ (\pi_{t,h} z_t)^{1-1/\sigma} + (1 - \pi_{t,f})^{1-1/\sigma} \right] \sigma z_t - 1 \right)^{\sigma - 1/\gamma}}{\mu (\pi_{t,f})^{-1/\sigma} \left( \left[ ((1 - \pi_{t,h}) z_t)^{1-1/\sigma} + (\pi_{t,f})^{1-1/\sigma} \right] \sigma z_t - 1 \right)^{\sigma - 1/\gamma}} = 1 - \frac{1}{\gamma} + \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \frac{1}{((1 - \pi_{t,h}) z_t)^{1-1/\sigma} + (\pi_{t,f})^{1-1/\sigma}} + \frac{1}{\sigma \pi_{t,f}}.
\]

Therefore, in each period, the fraction of home and foreign production that is consumed at home is pinned down by the relative productivity, \( z_t \), in that period. The effect of future and past productivities on the current allocation operates only through the time-invariant Lagrange multiplier, \( \mu \). This observation also implies that

**Corollary 1.** Up to a normalization, the optimal import and export taxes/subsidies in period \( t \) are uniquely determined by the relative productivities in period \( t \), i.e., \( z_t \).

This proposition implies that the size and direction of current trade balance has no bearing on the current optimal trade policy. That is because two periods with equal relative productivities—and hence identical optimal policy—could have very different levels of trade imbalances. Therefore, in general, there is no relationship between optimal trade policy and trade balance in a given period.

To describe the behavior of taxes over time, we start from a simple case of an economy where the elasticity of substitution between and across periods are equal, i.e., \( \sigma = \gamma \). In this case, equations (20) and (21) reduce, respectively, to

\[
\frac{(\pi_{t,h})^{-1/\sigma}}{\mu (1 - \pi_{t,h})^{-1/\sigma}} = 1 - \frac{1}{\sigma},
\]

and

\[
\frac{(1 - \pi_{t,f})^{-1/\sigma}}{\mu (\pi_{t,f})^{-1/\sigma}} = 1 - \frac{1}{\sigma} + \frac{1}{\sigma \pi_{t,f}}.
\]

As it can be observed, in this case, \( \pi_{t,i} \)'s are independent of \( z_t \). This implies that when \( \sigma = \gamma \), under the optimal policy of home, the fraction of output that is consumed in each country remains constant over time.

**Corollary 2.** If \( \sigma = \gamma \), the optimal trade policy is invariant to variations in \( z_t \).

To provide intuition for this result, note that in this setup, the home government’s main goal is to diminish the foreign country’s factor income by manipulating the demand for foreign output. When \( \sigma = \gamma \), the relative prices of the foreign country’s production is independent of their imports. This would imply that home exports should not be taxed, i.e., their relative prices should stay the same over time. Furthermore, homogeneity of the problem implies that import taxes are independent of \( z_t \) and thus constant over time.
The intuition provided above for determinants of wedges suggest that when $z_t$ is low and income in the foreign country is relatively high, it should be optimal to have high export taxes and low import tariffs. Unfortunately, we cannot show this analytically. We can, however, show that the fraction of foreign country’s income that is consumed in the foreign country, $\pi_f$, is decreasing in $z_t$. This suggests that the forces for procyclical export taxes and countercyclical import tariffs is strong.\(^{10}\) We have the following proposition:

**Proposition 3.** The share of foreign income that is spent on foreign consumption, $\pi_{t,f}$, is increasing in $z_t$.

This Proposition implies that there are forces towards making import taxes counter-cyclical and export taxes are pro-cyclical. In other words, optimal trade policy encourages a pro-cyclical consumption pattern. This finding is similar to the key finding of CLW for the case of capital control taxes under free trade.

To obtain intuition about this result, note that the government is interested in reducing the national saving rate during booms, in order to reduce the country’s demand for imports during recessions, thereby achieving a better *inter-temporal* term-of-trade.\(^{11}\) This goal may be achieved by a higher import tariff during recessions or a higher export tax in booms.

**Capital Control Policy** Our analysis so far shows that trade policy instruments alone are sufficient to implement the optimal allocation and, thus, capital control taxes are redundant when the government has unconstrained access to trade policy instruments. Now suppose that export taxes/subsidies are exogenously set to zero for all periods, but the home government has access to import tariffs and capital control taxes. Proposition ((2)) makes it clear that the government could achieve the optimal allocation with these two policy instruments.\(^{12}\) In particular, after eliminating export tax/subsidy, the government will respond with an adjustment in import tariffs that preserves the terms of trade within each period. Moreover, the government uses capital control taxes/subsidies to preserve the inter-period terms of trade. Therefore, Propositions 2 and 3 establish the following result:

**Corollary 3.** Suppose that the home government can only choose import and capital control taxes (no export policy instrument). Then, for a given relative productivity, the optimal tax on foreign asset positions in period $t$ is decreasing in the home country’s expected relative growth rate. Moreover, with zero relative growth rate in the next period, optimal capital control tax is zero.\(^{13}\)

This proposition implies that if the size of the economy is larger (smaller) in the next period, the optimal capital control policy is to tax (subsidize) the accumulation of foreign debt.

**Corollary 4.** It is optimal to discourage (encourage) the accumulation of foreign debt when the country is expected to grow faster (slower) than the rest of the world.

\(^{10}\)In all of our simulations, this result holds.

\(^{11}\)In other words, from the perspective of the government, under free trade, consumers consume too much of the foreign good in recessions and too little of the domestic good in booms. Relatedly, the saving rate of the consumers in booms (i.e., periods with high relative productivities) are too high from the government’s point of view.

\(^{12}\)Optimal policy implies a set of relative prices within and across periods. To set intra-period relative prices, one trade policy instrument (i.e., import or export tax) is sufficient. To set inter-period relative prices, we can either use a capital control tax which creates a wedge between current and future consumptions, or a combination of import and export taxes.

\(^{13}\)This qualitative result on capital control is also valid under free trade.
5 Economic Growth, Trade Imbalances and Optimal Trade Policy

In this section, we discuss the implications of economic growth on trade imbalances and optimal trade policy. To this end, we study some likely exogenous growth scenarios within our model and compute optimal trade taxes and equilibrium trade imbalances over time.

In light of proposition 1, under the unilaterally optimal allocation, the households in the home country consume a greater fraction of domestic production in periods in which home output is relatively larger. Moreover, during these periods, the fraction of foreign production that is consumed at home decreases. As we illustrate in Section 3, this trade-off leads to an import tariff that decreases with relative endowment and an export tax that increases with relative endowment.

This implies that over a period of high growth, one in which the productivity of the home country increases relative to the foreign country, we must have that export taxes increase while import tariffs decrease. This is illustrated in Figure 1. In this figure, we consider two countries that start at the same level of endowment while the home country grows at an annual constant rate of 4% while the foreign country grows annually at rate 2%. We assume that this lasts for 10 years and afterwards, the two countries have constant endowments. We normalize export tax to 0 at time 0. As we see, over this period the home country increases its export tax while at the same time it reduces its import tariffs. Moreover, the combined change in export and import taxes implies greater restriction in trade in periods with greater relative productivity.

As we have shown, the difference between inter-temporal and intra-temporal elasticity of substitution is the main determinant of the variation in tax policies. In our calculations, we have assumed that $\gamma = 1.1$ while we allow $\sigma$ to vary. The values of $\sigma$ we consider are 5–15. Note that $\sigma$ is equivalent to the trade elasticity in an Armington model – see Caliendo and Parro (2014) – and its estimated values are in this range. As $\sigma$ increases, we see that the level of export subsidies declines while its variation increases. This illustrates a trade-off between intra- and inter-temporal terms of trade manipulation. As $\sigma$ increases, imports become very elastic relative to import and thus within period terms of trade manipulation is not very beneficial to the home government. Furthermore, the benefits of inter-temporal terms of trade manipulation increases and thus both export subsidies and import taxes change more.
In Figure 2, we plot trade deficit in the home country over time and as it varies with $\sigma$. While in our model, there is no particular relationship between deficit and trade policies, since home country finds it optimal to borrow – due to the fact that its income is growing relative to its trading partner – as deficit decreases export subsidies increase and import taxes decrease. Finally, Figure 3 depicts the intra-temporal relative price of imports to exports in the home country. The fact that this relative price is higher than 1 reflects the bias towards export at home, i.e., the fact that intra-temporal wedge is positive. Moreover, this relative price increases over time. In other words, as the home country becomes richer, its incentive for intra-temporal terms of trade manipulation of its imports becomes stronger.
6 Quantitative Analysis

To evaluate the quantitative importance of trade and capital control taxes, we apply our theoretical results to the case of the United States for the first 13 years of its WTO membership from 1995 to 2010. Given that the United States adopted historically low tariffs after the inception of the WTO in 1995, we approximate the trade policy of the US in this period to be free trade. We then use the annual data on the output, import and export of the U.S. during this period and calculate the optimal trade and capital control taxes under various scenarios. While the exercise is not very quantitatively sophisticated, it provides a rough and back of the envelope calculation of optimal trade taxes. Moreover, we think that the methodology developed here – using hat-algebra in optimal policy analysis – can be used in quantitative models of optimal trade policy.

First, we calculate the optimal level of capital control for the United States given its free trade obligations under the WTO. This exercise sheds light on the degree to which capital control taxes, which are not restricted under the WTO agreement, could substitute for the lost policy space due to the constraints imposed by the WTO on trade taxes.

Second, we consider a counterfactual scenario in which the U.S. could unilaterally choose its trade policy and calculate the optimal import and export taxes for each year in our sample. In addition to shedding light on the magnitude of the welfare effects of protectionism, this exercise will also reveal the degree to which optimal trade taxes vary over time in response to business cycles.

In our third quantitative exercise, we calculate optimal import tariffs for each year assuming that export and capital control taxes are unavailable. These assumptions are motivated by institutional constraints in the United States. In particular, export taxes are banned by the US constitution. Moreover, since the Bretton Woods agreement, the United States has strongly supported a policy regime that excludes capital controls.

Data We use World Bank’s data on imports and exports and GDP for the United States and the rest of the world for the time period of 1995–2008. During these years, the US economy grew at an average annual rate of 3.6%, while the rest of the world grew at a rate of 1.7%. Overall, the share of the US economy from the world output varied between 25.43% (in 1999) and 23.31% (in 2008). The US ran a trade deficit every year during this period with imports exceeding exports by 13% ($120 billion) in 1995 and a maximum of 59% ($849 billion) in 2005.

Hat Algebra for a Dynamic Model To evaluate the quantitative effects of trade and capital control policy, we extend the hat algebra methodology to our dynamic Ramsey problem. Hat algebra eliminates the need to estimate various policy-invariant parameters of the model such as transportation costs and productivity parameters. Furthermore, as we show below, it highlights the sufficient statistics that are required to be measured in the data in order to solve for optimal policies.

Consider the Planner’s problem (P'). Referring to allocations with a bar as the free-trade allocations, and using the hat-algebra notation, \( \hat{y} \equiv \frac{y}{t} \) for any variable \( y \), we can write the resource constraints (16) as

\[
\begin{align*}
X^h_{t,h} \hat{X}^h_{t,h} + d^f_{t,h} X^f_{t,h} \hat{X}^f_{t,h} &= A_{t,h}, \\
X^f_{t,f} \hat{X}^h_{t,h} + d^h_{t,f} X^h_{t,f} \hat{X}^h_{t,f} &= A_{t,f}.
\end{align*}
\]
Note that in the above, $\hat{\pi}_{t,j}^j \hat{X}_{t,j}^i = X_{t,i}^j$. Letting $\pi_{t,j} \equiv \frac{\pi_{t,j}}{\pi_{t,j}}$ denote the fraction of country $j$ output that is consumed in $j$, these conditions may be written as

$$
\pi_{t,h} \hat{X}_{t,h}^h + (1 - \pi_{t,h}) \hat{X}_{t,h}^f = 1,
\pi_{t,f} \hat{X}_{t,f}^f + (1 - \pi_{t,f}) \hat{X}_{t,f}^h = 1.
$$

Similarly, we can rewrite the implementability constraint (17) in hat-algebra form:

$$
\sum_{t=0}^{T} \beta_t \left( X_t^f \right)^{1-\frac{1}{\gamma}} \left( \hat{X}_t^f \right)^{1-\frac{1}{\gamma}} = \sum_{t=0}^{T} \beta_t \left( X_t^f \right)^{\frac{1}{\alpha} - \frac{1}{\gamma}} \left( X_{t,f}^f \right)^{-\frac{1}{\alpha}} A_{t,f} \left( \hat{X}_t^f \right)^{\frac{1}{\alpha} - \frac{1}{\gamma}} \left( \hat{X}_{t,f}^f \right)^{-\frac{1}{\alpha}}
$$

Letting $\alpha_{t}^j$ denote the share of $j$'s total $T$-period income allocated to period $t$, and $\lambda_{t,i}^j$ denote the share of $j$'s expenditure on $i$'s output in period $t$, the planner's problem, may be rewritten as follows

$$
\max_{\{\hat{X}_{t,j}^i\}_{t,j},\hat{X}_t^f} \sum_{t=0}^{T} \alpha_{t}^j \left( \left[ \sum_j \lambda_{t,j}^h \left( \hat{X}_{t,j}^h \right)^{\frac{1}{\alpha} - \frac{1}{\gamma}} \right] \frac{1}{1 - \frac{1}{\gamma}} \right)^{1-\frac{1}{\gamma}}
$$

subject to

$$
\sum_{t=0}^{T} \alpha_{t}^f \left( \hat{X}_t^f \right)^{1-\frac{1}{\gamma}} \sum_{t=0}^{T} \alpha_{t}^f \lambda_{t,f}^f \left( \left( \hat{X}_t^f \right)^{\frac{1}{\alpha} - \frac{1}{\gamma}} \right)^{-\frac{1}{\alpha}} \left( \hat{X}_{t,f}^f \right)^{-\frac{1}{\alpha}} = \hat{X}_t^f
$$

$$
\pi_{t,h} \hat{X}_{t,h}^h + (1 - \pi_{t,h}) \hat{X}_{t,h}^f = 1,
\pi_{t,f} \hat{X}_{t,f}^f + (1 - \pi_{t,f}) \hat{X}_{t,f}^h = 1
$$

$$
\left[ \sum_j \lambda_{t,j}^h \left( \hat{X}_{t,j}^h \right)^{1-\frac{1}{\gamma}} \right]^{\frac{1}{\gamma - \alpha}} = \hat{X}_t^f
$$

Note that the above formulation of the optimal policy problem highlights the main main statistics that are required to be measured in the data in order to solve for optimal policy. It, thus, implies that direct measurement of output or trade costs does not directly affect optimal policies and only through the sufficient statistics. These sufficient statistics are given by $\alpha_{t}^j$, the share of total income earned in period $t$, trade shares, $\lambda_{t,i}^j$, and share of country $i$'s output consumed in country $i$. We construct $\alpha_{t}^j$, $\lambda_{t,i}^j$, and $\pi_{t}^j$ based on the data on total output (GDP) of United States and the rest of the world and import and export volumes. The hat values are then chosen to maximize the above problem. Finally, assuming that the status quo is free trade, we can write import and export

---

\[14\] In order to keep the analysis simple, we assume that consumption is domestically equal to GDP less Net Exports.
taxes using the hat-algebra notation:

\[ 1 + \tau_{t,h} = \frac{(\hat{X}_{t,h}^f)^{\frac{1}{\sigma}} (\hat{X}_{t}^f)^{\frac{1}{\sigma} - \frac{1}{\gamma}}}{(\hat{X}_{t,h}^h)^{\frac{1}{\sigma}} (\hat{X}_{t}^h)^{\frac{1}{\sigma} - \frac{1}{\gamma}}} \tag{27} \]

Similarly, we can write import taxes as

\[ 1 + \tau_{t,f} = \frac{(\hat{X}_{t,f}^h)^{\frac{1}{\sigma}} (\hat{X}_{t}^h)^{\frac{1}{\sigma} - \frac{1}{\gamma}}}{(\hat{X}_{t,f}^f)^{\frac{1}{\sigma}} (\hat{X}_{t}^f)^{\frac{1}{\sigma} - \frac{1}{\gamma}}} \tag{28} \]

In addition to the problem above, we also consider two alternative policy exercises: one in which the home government (U.S.) is constrained by trade agreements to keep intra-temporal trade undistorted and one in which the home government can only change import tariffs. These assumption imply that their associated wedges, intertemporal wedge and intratemporal wedge on home goods, must be zero. In the associated optimal policy problems, we impose these constraints.

Finally, we choose parameter values for \( \sigma \) and \( \gamma \) in line with the macro and trade literatures. We assume that \( \sigma = 5 \) and \( \gamma = 2 \) in our baseline calculations. We also consider variations in these parameters to understand their effect on optimal taxes, namely we consider two other cases with parameter values given by \((\sigma, \eta) = (10, 2)\) and \((\sigma, \eta) = (2, 2)\).

**Results** Our first set of results is related to the case in which the United States remains committed to its free-trade obligations but adopts capital control taxes to manipulate its intertemporal terms of trade. The first column of Table 1 shows the welfare effect of this policy as well as the size of the optimal tax on the stock of foreign assets in the US. Compared to complete free foreign exchange policy (i.e., zero trade and capital control taxes), the welfare of the representative consumer in United States barely changes in the period of study as a result of adopting optimal capital control taxes. Moreover, as the table illustrates, optimal capital controls are also fairly negligible.

To quantify the degree to which capital control taxes could substitute for the lost policy space due to the WTO agreement, we calculate the U.S. welfare under unilaterally optimal trade policy. As can be seen in Table 1, the change in welfare resulting from unconstrained optimal dynamic trade policy is 0.7% in consumption equivalent units. Comparing this value to the gains of optimal capital control policy, we see that trade agreements puts very stringent restrictions on optimal policy in a way that optimal capital controls are not able to replicate them. In line with the result on optimal capital controls, we see that import and export taxes do not vary much over the period considered.

Finally, we study the case in which import tariffs are the only policy instruments at the US government’s disposal. As we discussed earlier, due to domestic constraints on export taxes, unilateral trade policy of the US would mostly comprise of import tariffs. Following the same intuition as above, optimal import tariffs can basically generate all of the gains achieved by unrestricted optimal trade policy. Moreover, the level of optimal tariffs do not change much over time.

To understand the effect of changes in elasticity parameters, we also repeat the above exercise for \((\sigma, \eta) = (10, 3)\) and \((\sigma, \eta) = (2, 2)\). While the levels of optimal taxes and welfare effects change, the basic insights remain the same. First, capital controls cannot generate any welfare
## 7 Conclusion

In this paper, we analyzed unilaterally-optimal trade policy under a dynamic model with one factor of production. We find that the relative productivity of the home country to the rest of the world is the key time-varying parameter that determines the fluctuations in the optimal policy.

We characterized the interdependence of capital control and trade policy for a simple two-good model. In particular, we find that after entry in a trade agreement that constrains trade taxes, the government could use capital control to restore a fraction of its lost policy space. An interesting question that could be addressed in subsequent research is whether capital control could serve a useful purpose as a *flexibility* mechanism in trade agreements. Flexibility may be a desired feature for trade agreements for at least two reasons. First, if political economy preferences are subject to shocks in the future (as in Beshkar 2010, Maggi and Staiger 2011, and Beshkar and Bond 2017, among others) governments will negotiate an agreement that includes a mechanism for policy flexibility such as the WTO Agreement on Safeguards. Second, similar to Bagwell and Staiger (1990), if trade agreements must be self-enforcing, flexibility in capital control policies could reduce the governments’ incentive to renege on the agreement at times when a surge in imports or a widening trade deficit increases temptations to leave an international agreement.

The possibility of resorting to the use of capital control after negotiating a trade agreement could complicate the negotiation process especially if the governments’ ability to use capital controls is asymmetric. For example, due to differences in policymaking institutions, in some countries (e.g., the United States) capital control policies are harder to implement than other countries (e.g.,

### Table 1: Optimal Trade and Capital Control Policies

<table>
<thead>
<tr>
<th>Available Instruments</th>
<th>Only Capital Control</th>
<th>Only Import Tax</th>
<th>Import and Export Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare to Gain from protection (% of GDP)</td>
<td>≈ 0</td>
<td>0.70%</td>
<td>0.70%</td>
</tr>
<tr>
<td><strong>Year</strong></td>
<td><strong>Capital Control Tax</strong></td>
<td><strong>Import Tax</strong></td>
<td><strong>Export Tax</strong></td>
</tr>
<tr>
<td>1995</td>
<td>25.09%</td>
<td>14.53%</td>
<td>9.45%</td>
</tr>
<tr>
<td>1996</td>
<td>-0.0029%</td>
<td>25.30%</td>
<td>14.56%</td>
</tr>
<tr>
<td>1997</td>
<td>0.0535%</td>
<td>25.44%</td>
<td>14.58%</td>
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Gains while import tariffs almost replicate the gains from unrestricted policies. Moreover, the variations in optimal taxes are negligible.
Moreover, the potency of capital controls as an instrument to manipulate terms of trade depends on the magnitude of trade imbalances, which could vary substantially across countries. It may be, therefore, argued that giving up trade policy space is more costly for the former type of countries. As a result, the calculus of ‘balanced concessions’ in trade deals becomes a more complicated issue when countries differ in their ability to use capital control to manipulate their terms of trade.

References


McLaren, J. (1994). Size, sunk cost and judge bowker’s objection to free trade. 9


Appendix

A Proof of Proposition 1

Consider the optimal taxation problem in primal form

$$\max \sum_{t=0}^{T} \beta^t u \left( g \left( x_t^h \right) \right)$$

subject to

$$\sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k,j} x_{t,j,k} \frac{\partial}{\partial x_{j,k}} g \left( x_t^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k} \frac{\partial}{\partial x_{f,k}} g \left( x_t^f \right) y_{t,f,k}$$

$$\sum_{k} y_{t,j,k} = 1$$

$$x_{t,j,k}^h + x_{t,j,k}^f = y_{t,j,k}$$

$$\frac{\partial}{\partial x_{t,j,k}} g \left( x_t^f \right) = \frac{\lambda_t^f}{a_{t,j,k}}$$

First, we can simplify the above by realizing that $y_{t,j,k}$ can be substituted out. This is as follows: $y_{t,h,k}$ does not affect neither the implementability constraint nor the marginal utility restriction. Therefore, we can replace the resource constraint with

$$\sum_{k} \frac{x_{t,j,k}^h + x_{t,j,k}^f}{a_{t,j,k}} = 1$$

For the foreign country, we can use the marginal utility constraint and replace it in the implementability constraint to get

$$\sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k,j} x_{t,j,k} \frac{\partial}{\partial x_{j,k}} g \left( x_t^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \sum_{k} \frac{\lambda_t^f}{a_{t,f,k}} y_{t,f,k}$$

$$= \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \lambda_t^f \sum_{k} \frac{y_{t,f,k}}{a_{t,f,k}}$$

$$= \sum_{t=0}^{T} \beta^t u' \left( g \left( x_t^f \right) \right) \lambda_t^f$$

Therefore, the above problem becomes

$$\max \sum_{t=0}^{T} \beta^t u \left( g \left( x_t^h \right) \right)$$
subject to

\[ \sum_{t=0}^{T} \beta^t u' \left( g \left( x_i^t \right) \right) \sum_{k,j} x_{t,j,k}^f \frac{\partial}{\partial x_{j,k}^f} g \left( x_i^f \right) = \sum_{t=0}^{T} \beta^t u' \left( g \left( x_i^f \right) \right) \lambda_i^f \]

\[ \sum_k \frac{x_{t,j,k}^h + x_{t,j,k}^f}{a_{t,j,k}} = 1 \]

\[ \frac{\partial}{\partial x_{t,j,k}^f} g \left( x_i^f \right) = \frac{\lambda_i^f}{a_{t,j,k}} \]

Note that the last constraint is slack for the home country. This is because with and without the home government would like to set marginal utilities for domestic production proportional to the inverse of productivity. Formally, when we remove these constraints – for the home country, the solution must satisfy

\[ \beta^t u' \left( g \left( x_i^h \right) \right) \frac{\partial}{\partial x_{t,h,k}^h} g \left( x_i^h \right) = \frac{\gamma_i^h}{a_{t,h,k}} \]

This implies that the constraint must be satisfied, since by setting \( \lambda_i^h = \frac{\gamma_i^h}{\beta^t u'(g(x_i^h))} \), the above becomes the constraint.

Next, consider the first order condition of the above problem with respect to \( x_{t,f,k}^h \):

\[ \beta^t u' \left( g \left( x_i^h \right) \right) \frac{\partial}{\partial x_{t,f,k}^h} g \left( x_i^h \right) = \frac{\gamma_i^f}{a_{t,f,k}} \]

Comparing this to the marginal utility constraint associated with the foreign country, we see that taxes on imports should be uniform. To see that, note that we have

\[ \beta^t u' \left( g \left( x_i^h \right) \right) \frac{\partial}{\partial x_{t,f,k}^h} g \left( x_i^h \right) = \frac{\gamma_i^f}{a_{t,f,k}} \]

\[ \frac{\partial}{\partial x_{t,f,k}^h} g \left( x_i^h \right) = \frac{\lambda_i^f}{a_{t,f,k}} \]

The above implies that

\[ \frac{\beta^t u' \left( g \left( x_i^h \right) \right) \frac{\partial}{\partial x_{f,k}^h} g \left( x_i^h \right)}{\beta^t u' \left( g \left( x_i^f \right) \right) \frac{\partial}{\partial x_{f,k}^f} g \left( x_i^f \right)} = \frac{\beta^t u' \left( g \left( x_i^h \right) \right) \frac{\partial}{\partial x_{f,k}^h} g \left( x_i^h \right)}{\beta^t u' \left( g \left( x_i^f \right) \right) \frac{\partial}{\partial x_{f,k}^f} g \left( x_i^f \right)} \]

for any two sectors \( k, k' \).

### B Proof of Proposition 2

The FOC with respect to \( X_{t,h}^h \):
\[ u'(X^h_t) \frac{dX^h_t}{dX^h_{t,h}} + \mu \left[ -u''\left( X^f_t \right) X^f_t \frac{dX^f_t}{dX^f_{t,f}} - u'\left( X^f_t \right) \frac{dX^f_t}{dX^f_{t,h}} \right] + \left( u''\left( X^f_t \right) \frac{dX^f_t}{dX^f_{t,f}} \frac{dX^f_t}{dX^f_{t,h}} + u'\left( X^f_t \right) \frac{d^2X^f_t}{dX^f_{t,f}dX^f_{t,h}} \right) A_{t,f} = 0 \quad (29) \]

Noting that 
\[ u'(X^h_t) = \left( X^h_t \right)^{-\frac{1}{\gamma}}, \quad u''(X^h_t) = -\frac{1}{\gamma} \left( X^h_t \right)^{-1-\frac{1}{\gamma}}, \quad \frac{dX^f_t}{dX^f_{t,h}} = \left( \frac{X^f_t}{X^f_{t,h}} \right)^{\frac{1}{\sigma}}, \quad \text{and} \quad \frac{d^2X^f_t}{dX^f_{t,h}dX^f_{t,f}} = \left( \frac{X^f_t}{X^f_{t,h}} \right)^{-\frac{1}{\sigma}} \frac{1}{\sigma X^f_t}, \]

this can be written as
\[ \left( \frac{X^f_t}{X^f_{t,h}} \frac{X^f_{t,f}}{X^f_t} \right)^{\frac{1}{\sigma}} \frac{1}{\sigma X^f_t} \]

The FOC with respect to \( X^h_{t,f} \) is given by
\[ \beta^t u'(X^h_t) \frac{dX^h_t}{dX^h_{t,f}} + \mu \beta^t \left[ -u''\left( X^f_t \right) X^f_t \frac{dX^f_t}{dX^f_{t,f}} - u'\left( X^f_t \right) \frac{dX^f_t}{dX^f_{t,h}} \right] + \left( u''\left( X^f_t \right) \left( \frac{dX^f_t}{dX^f_{t,f}} \right)^2 + u'\left( X^f_t \right) \frac{d^2X^f_t}{d\left( X^f_{t,f} \right)^2} \right) A_{t,f} = 0. \tag{30} \]

which may be written as
\[ \left( \frac{X^h_t}{X^h_{t,f}} \right)^{\frac{1}{\sigma} - \frac{1}{\gamma}} \left( \frac{X^h_t}{X^h_{t,f}} \right)^{-\frac{1}{\sigma}} = 1 - \frac{1}{\gamma} + \left( \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X^f_t}{X^f_{t,h}} \right)^{\frac{1}{\sigma}} \frac{1}{\sigma X^f_t} \frac{A_{t,f}}{X^f_t} + \frac{1}{\sigma X^f_t} A_{t,f}. \]

The left-hand side of equation (20) is the relative marginal utilities of Home and Foreign from consumption of the home good in period \( t \). Therefore the left-hand side of (20) may be replaced with \( \frac{X^h_t}{\mu M} \frac{1}{1+\gamma h} \). Similarly, the left-hand side of (21) may be written as \( \frac{X^h_t}{\mu L} (1 + \tau_{t,f}) \). Substituting these values for the left-hand side of the FOCs and dividing the FOCs of each period yields the tax formula in the proposition.
C Proof of Proposition 3

Let us consider the optimality conditions associated with the planning problem \((P')\):

\[
\frac{(X^h_t)^{\frac{1}{\gamma} + \frac{1}{\sigma}}}{\mu (X^f_t)^{\frac{1}{\gamma} + \frac{1}{\sigma}} (X^h_{t,h})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}} A_{t,f} X^f_t
\]

(31)

\[
\frac{(X^h_t)^{\frac{1}{\gamma} + \frac{1}{\sigma}}}{\mu (X^f_t)^{\frac{1}{\gamma} + \frac{1}{\sigma}} (X^h_{t,f})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}} A_{t,f} + \frac{1}{\sigma} A_{t,f} X^f_t
\]

(32)

As we have shown in proposition 1, the solution to the above is only a function of \(A_{t,h}/A_{t,f}\). Now, in order to prove our monotonicity result, we consider an increase in \(A_{t,f}\) while we keep \(A_{t,h}\) constant. We show the claim by showing that \(X^f_{t,f}\) increases in \(A_{t,f}\). Our first claim is that when this happens, \(X^f_{t,h}\) must increase. To show this, suppose to the contrary that it does not and it decreases. This decrease implies that \(X^h_{t,f}\) must increase. Therefore, holding \(X^f_{t,h}\) constant, the RHS of the above equations increases while its LHS decreases. In order for the above to hold, we must thus have that \(X^f_{t,h}\) increases. This is because, ceteris paribus, an increase in \(X^f_{t,h}\) increases \(X^f_t\) which then reduces the RHS – because \(\sigma > 1\) – and increases the LHS.

Now, if we divide the two equations, we have

\[
\frac{(X^h_{t,f})^{-\frac{1}{\sigma}}}{(X^f_{t,f})^{-\frac{1}{\sigma}}} = 1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}} A_{t,f} X^f_t
\]

which leads to the equation

\[
\left(\frac{X^f_{t,f}}{X^h_{t,h}}\right)^{\frac{1}{\sigma} \left(\frac{X^h_{t,f}}{X^f_{t,f}}\right)^{\frac{1}{\sigma}}} = 1 + \frac{\frac{1}{\sigma} A_{t,f} X^f_t}{1 - \frac{1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\sigma}\right) \left(\frac{X^f_{t,f}}{X^f_t}\right)^{-\frac{1}{\sigma}} A_{t,f} X^f_t}
\]

Given our assumptions and the arguments above, the LHS of this equation decreases. This is because \(X^f_{t,f}\) declines (which by feasibility implies that \(X^h_{t,f}\) increases) and \(X^f_{t,h}\) increases \((X^h_{t,h}\) increases by feasibility). We argue that the RHS of the above increases under our assumptions.
which is a contradiction. To see this, note that we can write
\[
\begin{align*}
\frac{1}{\sigma} \frac{A_{t,f}}{X_{t,f}^f} &= \frac{1}{\sigma} \left( \frac{1}{\gamma} + \frac{1}{\gamma} - \frac{1}{\sigma} \right) (1 - \alpha) \left( \frac{X_{t,f}^f}{X_t^f} \right)^{1 - \frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} \\
&= \frac{1}{\sigma} \left( \frac{1}{\gamma} + \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}^f}{X_t^f} \right)^{1 - \frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f}
\end{align*}
\]

where in the above the operator \( d (\cdot) \) represents the infinitesimal change in a variable. We therefore have
\[
\begin{align*}
d \left( \frac{1}{\sigma} \frac{A_{t,f}}{X_{t,f}^f} \right) &= \frac{1}{\sigma} \left( \frac{1}{\gamma} + \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}^f}{X_t^f} \right)^{1 - \frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f} \\
&= \frac{1}{\sigma} \left( \frac{1}{\gamma} + \frac{1}{\gamma} - \frac{1}{\sigma} \right) \left( \frac{X_{t,f}^f}{X_t^f} \right)^{1 - \frac{1}{\sigma}} \frac{A_{t,f}}{X_{t,f}^f}
\end{align*}
\]

Note that in the above expression \( d \left( \frac{A_{t,f}}{X_{t,f}^f} \right) > 0 \) – since \( A_{t,f} \) is increasing. At the same time \( \left( \frac{dX_{t,f}^f}{X_t^f} \right) < 0 \), since \( X_{t,f}^f \) decreases and \( X_{t,f}^h \) increases. This implies a contradiction and thus we have established that \( X_{t,f}^f \) must increase in response to an increase in \( A_{t,f} \).