Interdependence of Trade Policies in General Equilibrium*

Mostafa Beshkar Ahmad Lashkaripour
Indiana University Indiana University

First version: June 2016
This version: May 5, 2019

Abstract

Many of the major results from theories of international trade are obtained within a General Equilibrium (GE) framework, but our understanding of trade policy is still largely limited to partial equilibrium analyses. We characterize optimal policy and policy interdependencies in a multi-industry GE model that features factor market, input-output, and cross-demand linkages, and show how GE considerations change the analysis of trade policies both quantitatively and qualitatively. We find that: (i) The variation in optimal trade taxes are substantially dampened when GE factor-market effects are taken into account; (ii) Input-output linkages introduce a new channel of international externality by affording governments the ability to levy a tax on value-added generated and consumed outside its jurisdiction; (iii) Negotiated tariff cuts in a subset of industries lead to unilateral cuts in other industries; and (iv) A free trade agreement may lead to the adoption of wasteful trade barriers by a welfare-maximizing government. Fitting our model to trade data for 15 major economic regions, we show that these effects are quantitatively significant.

*The first draft of this paper entitled “Trade Policy with Inter-sectoral Linkages” was presented at the SITE Summer Workshop (June 2016). For their helpful comments and discussions, we are grateful to Pol Antras, Costas Arkolakis, Kyle Bagwell, Eric Bond, Lorenzo Caliendo, Angela Campbell, Arnaud Costinot, Svetlana Demidova, Farid Farrokhi, Filomena Garcia, Grey Gordon, Michael Kaganovich, Sajal Lahiri, Nuno Limao, Volodymyr Lugovskyy, Kaveh Majlesi, Giovanni Maggi, Monika Mrázová, Marcelo Olarreaga, Frederic Robert-Nicoud, Andres Rodriguez-Clare, Ali Shourideh, Anson Soderbery, Tommaso Tempesti, Ben Zissimos and participants at various seminars and conferences. We also thank Mostafa Tanhayi Ahari for his feedback and research assistance.
1 Introduction

The analysis of trade policies is often complicated by general-equilibrium linkages across industries. Consider, for example, the recent tariffs imposed by the United States on steel imports. In addition to its local effects on steel producers and consumers, such a policy has two general ramifications on the rest of the economy. First, by reallocating resources across industries and modifying demand and trade patterns, steel tariffs may affect the cost of inputs (labor, capital, intermediate inputs, etc.) as well as the intensity of import competition in the rest of the economy. A second complication arises due to the interdependence of trade policies across industries: In response to the general equilibrium effects of steel tariffs, the government may be compelled to adjust its trade policy across all industries, thereby creating further welfare consequences.

The consequences of policy interdependence across sectors have largely escaped notice in the optimal policy literature. To avoid the complications resulting from general-equilibrium interactions, most of the trade policy literature has focused on partial equilibrium models. Several authors, including Ossa (2014) and Caliendo and Parro (2014), have advanced the analysis of trade policy in general equilibrium by developing and adopting a computational approach. Beyond these studies, the analytics of optimal policy within general equilibrium multi-industry models remain largely unknown. An exception is Costinot, Donaldson, Vogel, and Werning (2015), who study optimal policy under Dornbusch, Fischer, and Samuelson’s (1977) version of the Ricardian model.

Our objective in this paper is two-fold. First, we analytically characterize the optimal trade policy in the presence of various cross-industry linkages, including general-equilibrium factor price linkages, input-output linkages, and cross-demand effects. Second, we study various trade policy interdependencies that arise due to these general equilibrium linkages. Achieving this second objective involves characterizing the optimal policy under various external constraints on the government’s policy space.\footnote{Constraints on the policy space may be imposed by incomplete trade agreements or political and institutional considerations, for example.}

In characterizing the optimal trade policy, we consider a competitive general-equilibrium model that features general (non-parametric) consumer preferences
and production technologies. In this general setup, we characterize the optimal industry-level export and import taxes as a function of two sufficient statistics: (i) own- and cross-price elasticities of demand, and (ii) trade tax pass-throughs net of wage effects. We use our analytical characterization to study the structure of optimal trade policy in three special cases.

We first consider a general Ricardian economy without input-output linkages. We show that, in this particular setup, the optimal import taxes are uniform across products, but the optimal export taxes are differential and vary with the own- and cross-price elasticities of foreign demand for the exported products. Our uniformity result extends the result in Costinot et al. (2015) to environments that feature a general demand system that admits any arbitrary pattern of cross-substitutability between products. We also extend our analysis to a multiple-country case and show that the optimal import tariffs discriminate among exporting countries but remain uniform across products imported from the same country.

Second, we consider a Ricardian economy that features Input-Output Linkages. We first show that, here, the entire matrix of tax passthroughs is fully determined by the global Input-Output (IO) matrix. Correspondingly, the optimal trade tax schedule can be fully characterized in terms of reduced-form demand elasticities and input-output shares. The resulting optimal tax formula indicates that optimal import taxes are uniform only across final goods or intermediate goods that are not re-exported. However, optimal import taxes are differential across imported goods that are destined for re-exporting.

To understand this result, note that any tax levied on re-exported intermediate goods is effectively a tax on a transaction among foreign entities, because such components are produced and eventually consumed abroad. Therefore, input-output linkages provide the government with additional taxing power beyond its jurisdiction, which we call the extraterritorial taxing power. The departure from uniform tariffs on re-exported intermediate goods reflects the government’s desire to exercise its extraterritorial taxing power on such trade flows.

---

2To be more precise the optimal trade tax schedule also depends on observable expenditure and revenue shares.

3As Yi (2003) points out in a model with vertically-fragmented production, the effect of endogenous trade costs are amplified when countries specialize in different stages of production, and intermediate goods cross national borders multiple times. Our analysis complements Yi (2003) by considering the effect of IO linkages on endogenous trade costs.
It is worth highlighting the difference between the externality generated by extraterritorial taxing power and the standard terms-of-trade externality emphasized in the prior literature. The standard terms-of-trade argument concerns the price of an exchange between a consumer and a producer, when only one of them is located in a foreign country. The extraterritorial taxing argument, however, concerns the government’s ability to tax a value-added that is produced and consumed outside its jurisdiction. Accordingly, the presence of IO linkages could amplify the unilateral gains and the externalities associated with trade taxes.

Third, we consider the case where the economy features industry-specific factors of production. In this case, the marginal product of labor is diminishing in the industry-level output. Hence, due to general equilibrium demand linkages, a tax on one industry could alter the productivity of labor in all other industries. As a result, import tariffs are differential across products and their variation depends on the entire schedule of home and foreign’s industry-level supply elasticities as well as the cross-demand elasticities between industries.

Our second set of results characterize the interdependence of trade policies across industries in the Ricardian model. In summary, we find that: (i) Import policy is an imperfect substitute for export policy; (ii) Under mild conditions, import tariffs across industries are complementary; (iii) Non-Revenue Trade Barriers (NRTBs), also known as wasteful trade barriers, may be optimal in the absence of revenue-raising trade policy instruments such as tariffs.

Our result about the interdependence of import and export policies is akin to—but distinct from—the Lerner’s (1936) Symmetry Theorem. We find that, in general, import policy is only an imperfect substitute for export policy. In the Ricardian model, this imperfect substitutability takes a sharper form: the equilibrium obtained under optimal import tariffs can be exactly replicated with a set of export policies, but no set of import tariffs could replicate the equilibrium under the optimal export taxes. Under reasonable scenarios, an important implication of this result is that the elimination of export subsidies would lead to an increase in trade volume.4 This insight is in contrast to one obtained under a partial equilibrium analysis in which the elimination of export subsidies will necessarily reduce trade

---

4Within our model, this result is valid under a scenario in which the government could not use export taxes due to political or institutional constraints such as the constitutional ban on export taxes in the United States.
volumes.\footnote{This result also provides a novel perspective on the GATT/WTO’s ban on export subsidies. As reviewed by Lee (2016), the terms-of-trade literature has found it “quite difficult to justify the prohibition of export subsidies given the trade-volume-expanding nature of export subsidies.” Our general equilibrium analysis provides a potential explanation for this puzzle, because we show that the elimination of export subsidies will spur unilateral tariff cuts to a degree that leads to an overall increase in trade volumes.}

To obtain a general intuition about the cross-industry tariff complementarity, it is instructive to consider the following scenario. Suppose that, initially, export policy instruments are unavailable but the government could freely choose import tariffs. Then, suppose the home government enters an incomplete trade agreement that restricts import tariffs in a subset of industries. Our tariff complementarity result indicates that, under this partial restriction, it is optimal for the home government to voluntarily lower its import tariffs on unrestricted industries.\footnote{This finding is in line with Martin and Ng’s (2004) observation that after entering the WTO, many developing countries started cutting their tariffs beyond their obligations under the agreement. Baldwin (2010) also highlights these unilateral tariff liberalizations, but provides an alternative explanation based on the fragmentation of the production processes.}

Our result concerning the optimality of NRTBs sheds fresh light on measures such as import bans and inefficient customs regulations (i.e., red tapes at the border) that discourage imports but do not generate revenues. These measures are quite prevalent in practice. For example, in the wake of negotiated tariff cuts, many countries have opted for non-tariff barriers that do not generate any revenues for the governments (Goldberg and Pavcnik 2016). From the perspective of the standard terms-of-trade analysis, the adoption of NRTBs is hard to explain because such measures reduce trade without compensating the resulting consumption losses with a better terms of trade. Under a multi-industry general-equilibrium framework, however, NRTBs could improve a country’s welfare because restricting imports in one industry improves a country’s terms of trade in \textit{all other industries} by depressing foreign factor rewards. Therefore, if the consumption loss due to import restriction in an industry is sufficiently small, imposing an NRTB in that industry could be welfare-improving. We show that this condition is satisfied in relatively homogenous sectors where imported varieties could be easily substituted with domestic counterparts.

Finally, we provide a quantitative assessment of our findings by fitting our model to trade and production data from 15 regions (spanning 40 countries) across 16 industries. In this process we also demonstrate how our theory simplifies the
conduct of quantitative analysis of trade policy.

Our quantitative exercise indicates that unilateral gains from optimal trade policy are (one average) around 1% in terms of real GDP. However, the gains from unilateral policy would be considerably lower (around 0.35% of real GDP) if the government was restricted from using export policy. This finding gives further perspective on our proposition regarding the imperfect substitutability of import and export taxes. The gains from NRTBs, when revenue-raising taxes are absent, are smaller but not negligible. For Taiwan and Mexico, for instance, we estimate that the gains from optimal NRTBs are around 0.11% and 0.12% of their GDPs, respectively.

The unilateral gains from optimal policy are considerably larger in the presence of input-output linkages. Specifically, accounting for global input-output linkages, the average unilateral gains from optimal trade policy increases 60% (from 1% to 1.6% of GDP). The higher gains from trade policy are partly driven by the extraterritorial taxing power effect identified by our theory. That is, with IO linkages, countries can generate revenues by taxing transactions between consumers and producers in the rest of the world.

To illustrate the role of policy interdependencies, we conduct a counterfactual analysis corresponding to a hypothetical gradual trade agreement that is reminiscent of the constraints introduced over time by the GATT and the WTO. Starting from the home government’s unconstrained optimal policy equilibrium, we introduce a sequence of partial restrictions on the government’s policy space and quantify its optimal response with respect to unrestricted industries. We conduct our counterfactual analysis twice, once where the United States is treated as the home country and another where the European Union is treated as home.

For our no-agreement baseline, we adopt an import tariff equal to the average Smoot-Hawley tariffs of 59%, and calculate the optimal export policy. The first sequence of liberalization that we consider is a ban on export policies, while import tariffs are left at the discretion of the home government. This scenario is in line with the GATT and WTO’s relatively more stringent conditions on export subsidies than import tariffs. We calculate that the restriction on export policy will induce the US and EU governments to decrease their import tariffs uniformly from 59% to around 30%. As a result of restrictions on export policy, the gains from unilateral trade policy for the US and EU are reduced by more than fifty percent.
The second sequence of liberalization retains the ban on all export taxes, but also restricts import tariffs in half of the traded industries. For both the EU and the US, we compute that such a restriction would induce the government to lower its import tariffs by a third in unrestricted industries. This strong tariff complementarity could have important implications for the optimal sequencing of trade liberalization.

The paper is organized as follows. After discussing the related literature in the subsequent Section, we begin by laying down our general framework in Section 3. In Section 4, we derive the unconstrained optimal tax/subsidy schedule under a general Ricardian model. In Section 5, we extend our analysis of optimal policy to environments with input-output linkages and specific factors of production. In Section 6, we analyze the interdependence of trade policies and the optimality of NRTBs by introducing constraints to the optimal tax/subsidy problem. Section 7 presents our quantitative analyses. Finally, in Section 8, we provide concluding remarks including a discussion on the implications of policy interdependencies for trade negotiations.

2 Related Literature

In this Section, we review the literature on general equilibrium analysis of trade policy and policy interdependence, and discuss the relevance of our contributions to these previous studies.

The Literature on Optimal Trade Policy

Our results regarding the optimal schedule of trade taxes cover the previous general-equilibrium characterizations of the optimal trade policy including Costinot et al. 2015, Opp 2010, and Itoh and Kiyono 1987. While Opp 2010 focuses on import tariffs and Itoh and Kiyono 1987 focus on export subsidies, Costinot et al. 2015 consider the simultaneous choice of import and export policies and show the optimality of uniform import tariffs for the case where trade elasticities are the same across sectors and preferences are additively separable. We show that these results continue to hold in an environment with heterogenous trade elasticities across sectors and a general (not necessary separable) preference structure.
Using the primal approach and assuming additively-separable utilities, CDVW show that the messy problem of optimal trade policy in a multi-sector general-equilibrium Ricardian model (Dornbusch, Fischer, and Samuelson 1977, henceforth, DFS) reduces to an elegant cell-problem in which the optimal trade policy for each product may be found in isolation from the policy of other sectors. CDVW show that the cell-approach is specially useful in dealing with two difficulty in the problem of optimal trade taxes under DFS’s Ricardian model, namely, the infinite-dimensionality of the optimal tax problem (since there is a continuum of goods) and non-smoothness (since the world production frontier has kinks.)

The simplicity and elegance afforded by the cell-problem come at a cost since by imposing constraints on preferences and technologies, this approach limits the type of general equilibrium linkages that can be analyzed. We suggest an alternative method to deal with these difficulties, which involves laying out a more general model with countable products that are imperfectly substitutable. The assumption of imperfect substitutability of products obviates the need to worry about the effect of policy on the extensive margin of trade. In the limit, when the number of products and the degree of substitutability of products tend to infinity, the model delivers DFS as a special case.

The idea that optimal trade policy for a product should depend on the elasticity of its supply and demand was proposed by Bickerdike (1906) and was later popularized by others including Kahn (1947), who calculated the exact formula for optimal import tariff to be equal to the inverse of the foreign export supply elasticity. This approach came under criticism due to its disregard for general-equilibrium effects (Graaff 1949; Horwell and Pearce 1970; Bond 1990). Nevertheless, those criticisms were mostly suggestive and did not provide a practical framework to evaluate general-equilibrium effects of trade policy. The subsequent literature, perhaps for practical reasons, adopted Bickerdike’s “elasticity approach” to study the variation in sectoral trade policies (e.g., Grossman and Helpman 1995; Broda et al. 2008; Bagwell and Staiger 2011; and Beshkar et al. 2015). In this paper, by providing an analytical characterization of optimal trade policy (both constrained

---

7If a problem can be formulated as a cell-problem, the Lagrange multiplier provides a sufficient statistic for the effect of the rest of the economy on each cell.

8The existing general-equilibrium analyses of trade policy are either conducted for a small open economy (as in the tariff reform literature cited below), or a two-sector economy with only one import good and one export good (e.g., Bagwell and Staiger 1999, Limão and Panagariya 2007).
and unconstrained), we offer a practical way to analyze trade policy in general equilibrium.

We are unaware of any previous work that views NRTBs as a beggar-thy-neighbor policy. The existing studies of non-tariff barriers as policy variables—e.g., Berry, Levinsohn, and Pakes (1999), Harrigan and Barrows (2009), and Maggi, Mrázová, and Neary (2017)—view them implicitly as an instrument to transfer wealth to interest groups without generating any welfare gains at the national level. Similar to our study of NRTBs, Maggi, Mrázová, and Neary (2017) analyze the use of wasteful trade barriers when the governments’ policy space is constrained by a trade agreement. They show that if tariff commitments could not be fully contingent on political realizations, the extent of tariff liberalization is limited by the need to prevent such wasteful behavior. Our framework offers a complementary perspective on NRTBs as instruments that could be potentially used to improve a country’s terms of trade in expense of foreign countries.

Within a partial equilibrium framework and assuming free trade in intermediate inputs, Blanchard et al. (2016) study the effect of IO linkages on the optimal final-goods tariffs. In addition to accounting for general equilibrium effects, we allow for the imposition of trade taxes on intermediate inputs as well as final goods. We find that the optimal trade restriction on intermediate inputs could be even higher than the optimal import tariffs on imported final goods. Therefore, our analysis offers a caveat about the assumption of free trade in intermediate inputs.

A growing literature, including Demidova and Rodríguez-Clare (2009), Felbermayr, Jung, and Larch (2013), Haaland and Venables (2016), Costinot, Rodríguez-Clare, and Werning (2016), and Caliendo, Feenstra, Romalis, and Taylor (2015) analyzes trade policy under the monopolistically competitive framework of Melitz (2003). All of these papers focus on models with a single tradable sector and, thus, their results are not readily comparable to our findings regarding the optimal policy across multiple sectors. A partial exception is Costinot, Rodríguez-Clare, and Werning (2016) who study firm-specific policies and find that within the same sector, optimal firm-specific tariffs are increasing in the productivity of the foreign firms.

Our theory contributes to a growing literature that attempts to quantify the trade policy equilibrium of optimizing governments (Perroni and Whalley, 2000; Ossa, 2011, 2012, 2014). This literature, which is aptly discussed by Ossa (2016)
and Costinot and Rodríguez-Clare (2014), uses numerical optimization to find the tariff choice of optimizing governments. Numerical optimization is often plagued with the curse of dimensionality when many sectors are involved. Applying our theory to trade data, we show that our analytical formulas facilitate the computational task in such cases. Moreover, we take a first step towards highlighting the empirical significance of cross-price elasticity effects in the design of optimal policy.

Our characterization of optimal taxes, which can be interpreted as optimal markups in a monopoly problem, contributes to the analysis of multi-product monopolies as studied by Armstrong and Vickers (2018) and Amir et al. (2016), among others. In comparison to the monopoly problem, the problem of optimal trade policy introduces additional nuances that are caused by general-equilibrium effects on wages, productivities, and income, which do not emerge in standard multi-product monopoly problems. Moreover, with the introduction of input-output linkages, which affords governments an extraterritorial taxing power, the trade policy problem can no longer be cast purely as a multi-product monopoly-monopsony problem.

The Literature on Policy Interdependence

The existing literature is mostly silent about trade policy interdependencies due to its focus on “optimal” policy—rather than the tradeoffs that policymakers face outside the optimum—and partial equilibrium, which precludes interrelations across sectors. Partial exceptions include the literature on incomplete trade agreements and the literatures on tariff complementarity in Free Trade Areas and the Piecemeal Tariff Reforms, which we now discuss.

In a model of incomplete trade agreements, Horn, Maggi, and Staiger (2010) show that governments will have an incentive to use domestic subsidies in response to negotiated tariff cuts. The increase in domestic subsidies after entering a trade agreement tends to partially offset the benefits from negotiated trade liberalization.

Note that the problem of optimal trade policy in the absence of IO linkages resembles a multi-product monopoly problem (on the export side) and a multi-product monopsony problem (on the import side). In particular, our finding that the sectoral variation in optimal export (import) policies are determined by demand-side (supply-side) parameters, is reminiscent of the solution to the monopolist’s (monopsonist’s) problem.
There is a literature on tariff complementarity in Free Trade Areas (FTA). While we find that tariffs across sectors within a country are complementary, Richardson (1993), Bond, Riezman, and Syropoulos (2004) and Ornelas (2005) find that for members of a Free Trade Area (FTA), internal and external tariffs are complementary. In particular, they find that as a response to tariff cuts within an FTA, the member countries will voluntarily reduce their tariffs on imports from non-members. Similarly, in a North-South model, Zissimos (2009) considers tariff complementarities across countries within a region that compete for imports from the rest of the world.

The theory of piecemeal tariff reform (Hatta 1977; Fukushima 1979; Anderson and Neary 1992, 2007; Ju and Krishna 2000) is another strand of the literature that touches on the issue of policy interdependence. This literature is primarily concerned with welfare-enhancing tariff reforms that are revenue-neutral (or revenue-enhancing) in a small open economy. A general finding of the piecemeal reform literature is that compressing the variation of existing tariffs in developing countries—by reducing the highest tariff rates and increasing the lowest ones—could increase welfare without decreasing revenues. Although we focus on an entirely different problem in this paper, our finding about the optimality of uniform tariffs resonates with this literature’s recommendation for tariff reforms.

As in this paper, Bagwell and Lee (2015) provide a perspective on the WTO’s ban on export subsidies. Within a heterogenous-firm model, Bagwell and Lee (2015) show that if import tariffs (as well as transportation costs) are very low, then an export subsidy may benefit a country at the expense of its trading partners. Their finding suggests that a ban on export subsidies is useful only after substantial liberalizations have been reached through previous negotiations. By contrast, our analysis suggests that a ban on export subsidies is useful even without any restrictions on import tariffs.

Another related literature studies issue linkages in international relations. This literature considers various conditions under which there might be an interdependence between trade policies and non-trade policies—such as environmental policies (Ederington, 2001, 2002; Limão, 2005), production subsidies (Horn, Maggi, and Staiger, 2010), and intellectual property protection. These papers draw conclusions about whether these non-trade issues should be linked to trade agreements (see Maggi 2016 for a review).
The Economic Environment

The global economy consists of $i = 1, \ldots, N$ countries (with $C$ denoting the set of countries) and $k = 1, \ldots, K$ industries (with $\mathcal{K}$ denoting the set of industries). With the exception of Section 4.2, our analysis focuses on a two-country case, i.e., $C = \{ h, f \}$, where $h$ is referred to as the Home country and $f$ is referred to as the Foreign country representing an aggregate of the rest of the world.

Each country $i$ is populated with $L_i$ workers that are perfectly mobile across industries but immobile across countries. In the non-Ricardian extension of the model, each industry $k$ in country $i$ is also endowed with $S_{i,k}$ units of an industry-specific factor of production, which is combined with labor in production.

In a typical industry $k$, country $j \in C$ produces a differentiated variety for each destination market, $i \in C$, which we denote by $ji,k$ (supplier $j$–destination $i$–industry $k$). Since no restrictions are imposed on the size or the number of industries, our framework can be alternatively viewed as one concerning product-level taxes.

3.1 Preferences

The consumers in country $i$ choose the vector of consumption quantities, $q_i \equiv \{ q_{ji,k} \}$, to maximize a general utility function, $U_i(q_i)$, subject to the budget constraint. The optimal choice of the consumers yields an indirect utility function,

$$V_i(Y_i, \hat{p}_{i,k}) \equiv \max_{q_i} U_i(q_i)$$

subject to

$$\sum_{k \in \mathcal{K}} \sum_{j \in C} (\hat{p}_{ji,k} q_{ji,k}) = Y_i,$$

which is the demand function $D_i(\hat{p}_i, Y_i)$, which summarizes the demand-side of the economy as a function of total income $Y_i$ and the vector of consumer prices $\hat{p}_i \equiv \{ \hat{p}_{ji,k} \}$ in country $i$. We define the price and income elasticities associated with demand function $D_i(.)$ as follows:

D1. [Marshallian Demand Elasticities]

(i) [own price elasticity] $\varepsilon_{ji,k} \equiv \partial \ln q_{ji,k} / \partial \ln \hat{p}_{ji,k}$;
(ii) [cross-price elasticity] $\varepsilon_{ji,g} \equiv \partial \ln q_{ji,k} / \partial \ln \hat{p}_{ji,g}$ for $j, g \neq ji, k$;
(iii) [income elasticity] $\eta_{ji,k} \equiv \partial \ln q_{ji,k} / \partial \ln Y_i$. 

12
Throughout the paper, we restrict our attention to well-behaved demand functions that are continuous and locally non-satiated. We also assume that demand for each traded variety exhibits an elastic region where $|\varepsilon_{ji,k}| > 1$. As in monopoly problems, this condition will be necessary for obtaining a bounded solution for optimal trade taxes.

### 3.2 Technology

We assume that firms are competitive and technologies exhibit constant returns to scale. The general case of our framework allows for production to employ (i) labor, (ii) intermediate inputs, and (iii) industry-specific factors of production. Accordingly, a cost-minimizing producer supplying good $ji,k$ faces the following non-parametric marginal cost function, which pins down their competitive “producer” price, $p_{ji,k}$, as a function of input prices and output level:

$$ p_{ji,k} = C_{ji,k}(w_j, \tilde{p}_j^T; q_j). \quad (2) $$

To elaborate, the marginal cost is a function of the labor wage in economy $j$, $w_j$; the vector of intermediate input prices employed by producers in country $j$, $\tilde{p}_j^T \equiv \{\tilde{p}_{j,\ell}^T\}_{\ell,g}$; and the producer’s output schedule, $q_j \equiv \{q_{j,\ell}\}_{\ell,g}$. This last argument accounts for (i) the presence of industry-specific factors of production, which leads to a marginal cost that is increasing in output, as well as (ii) a finite elasticity of transformation between output produced for different markets—we discuss the micro-foundation underlying Equation 2 in more detail in Section 5.2.

We begin our analysis in Section 4 with the basic Ricardian case of the above structure. In that case, $C_{ji,k}(.) = a_{ji,k}w_j$, with $a_{ji,k}$ being a constant (policy-invariant) unit labor cost. Then, in Section 5, we consider the most general case of our model that admits both input-output linkages and industry-specific factors of production.

### 3.3 Policy Instruments

The government in country $i$ has access to a full set of industry-level export tax/subsidy instruments (denoted by $x_{ij,k}$) and import tax/subsidy instruments (denoted by $t_{ji,k}$). Moreover, the government could impose Non-Revenue Trade
Barriers (NRTBs, denoted by $\tau_{ji,k}$) which are frictions in the form of iceberg transport costs that impede imports without raising revenues. NRTBs account for policies such as red-tape barriers at the border, frivolous regulations, and other policies that act as coveted protectionism. Together, these policy instruments create a wedge between the consumer price, $\tilde{p}_{ji,k}$, and the producer price, $p_{ji,k}$, of each good $ji,k$ as follows:

$$\tilde{p}_{ji,k} = (1 + t_{ji,k}) (1 + x_{ji,k}) (1 + \tau_{ji,k}) p_{ji,k},$$

where $t_{ji,k}$ denotes the import tariff applied by country $i$ on good $ji,k$; $x_{ji,k}$ denotes the export tax applied by country $j$ on good $ji,k$; and $\tau_{ji,k}$ denotes the NRTB applied by country $i$ on good $ji,k$. The combination of these tax instruments raises the following tax revenue for the government in country $i$:

$$R_i = \sum_{k \in K} \sum_{j \in C} [t_{ji,k} (1 + x_{ji,k}) p_{ji,k} q_{in,k} + x_{ij,k} p_{ij,k} q_{ij,k}].$$

Throughout this paper, we assume that domestic policies are unavailable, i.e., $t_{ii,k} = x_{ii,k} = \tau_{ii,k} = 0$ for all $i$ and $k$. We also focus on cases where “only” the Home country, indexed $h$, sets trade taxes. All other countries (denoted as Foreign) are assumed to follow a passive Laissez-Faire policy. In the two-country case, for instance, this assumption entails that $x_{fh,k} = t_{hf,k} = \tau_{hf,k} = 0$ for all $k$.

### 3.4 Equilibrium

Now, we define equilibrium in the two-country case where $C = \{h,f\}$, noting that an analogous definition applies to the multi-country extension. Provided that the equilibrium is unique, a combination of policies imposed by the Home country, namely, $x_{hf} \equiv \{x_{hf,k}\}$, $t_{fh} \equiv \{t_{fh,k}\}$, and $\tau_{fh} \equiv \{\tau_{fh,k}\}$, is consistent with only one equilibrium wage vector, $w \equiv \{w_i\}$. Since Foreign does not impose taxes by assumption, we can uniquely characterize policy outcomes in terms of $x_{hf}$, $t_{fh}$, $\tau_{fh}$, and $w$ (with $w$ itself implicitly depending on the trade taxes). Considering this, we formally define the set of feasible wage-policy combinations as follows.

**D2. [Feasible Wage-Policy Combinations]**

Suppose Foreign does not impose taxes (i.e., $x_{fh} = t_{hf} = \tau_{hf} = 0$). A vector of Home
taxes and wages \( A \equiv (x_{hf}, t_{fh}, \tau_{fh}; w) \) constitutes a feasible combination if (i) the producer price for any good \( ji, k \) is characterized by

\[
p_{ji,k} = C_{ji,k}(w_j; \hat{p}_{j,i}; q_j);
\]

(ii) the consumer price for any good \( ji, k \) is given by

\[
\hat{p}_{ji,k} = (1 + x_{ji,k}) (1 + t_{ji,k}) (1 + \tau_{ji,k}) p_{ji,k};
\]

(iii) consumption choices are a solution to 1 in country \( i \in \{h, f\} \):

\[
q_i = D_i(Y_i, \tilde{p}_i);
\]

(iv) factor markets clear in country \( i \in \{h, f\} \)

\[
w_i L_i + \Pi_i = \sum_n \sum_k p_{in,k} q_{in,k};
\]

where \( \Pi_i \) denotes the surplus paid to the industry-specific factors; and (v) Total income equals factor income plus tax revenue in country \( i \in \{h, f\} \)

\[
Y_i = w_i L_i + \Pi_i + R_i,
\]

where \( R_i \) is given by Equation 3.

We hereafter denote the set of all feasible wage-policy combinations by \( A \). Since only home imposes taxes, we can simplify the notation by letting \( t_k \equiv t_{fh,k} \) and \( x_k \equiv x_{hf,k} \) and \( \tau_k \equiv \tau_{fh,k} \) denote Home’s trade taxes and NRTBs, with \( t, x, \tau \) denoting the corresponding vectors. Relatedly, for any feasible wage-policy combination, \((x, t, \tau; w) \in A\), we use

\[
W_h(x, t, \tau; w) \equiv V_h(Y_h(x, t, \tau; w), \hat{p}_h(x, t, \tau; w))
\]

to denote Home’s welfare under that policy. In the two-country case of our model, we recurrently appeal to the Lerner symmetry theorem. So, to fix minds, we present a formal statement of the Lerner symmetry in the following.

Lemma 1. [The Lerner Symmetry] \( A = (x, t, \tau; w) \) and \( A' = (x', t', \tau; 1) \) represent
identical equilibria iff

\[
\begin{cases}
1 + x' = (1 + x) \frac{w_f}{w_h} \\
1 + t' = \frac{w_h}{w_f} (1 + t)
\end{cases}
\]

An immediate corollary of the Lerner symmetry is that when both export and import taxes are available, we can normalize wages in both economies and still identify one of the multiple optimal wage-policy combinations. This result, however, follows only if a full set of export and import tax instruments are available to the Home government. As we will see in Section 6, once the policy space is restricted, we can no longer normalize both \( w_h \) and \( w_f \). Therefore, under partial restrictions on the policy space, general equilibrium changes in \( w_h/w_f \) become consequential to the structure of optimal policy.

4 Optimal Trade Policy in a Ricardian Model

We begin our analysis by characterizing the optimal trade policy of the Home country under Ricardian technologies in a two-country setup where \( C = \{ h, f \} \). We then transition to the multiple-country case of the Ricardian model in Subsection 4.2. In the Ricardian model, the set of feasible wage-policy combinations, \( \mathbb{A} \), is given by D2, with the specific restriction that labor is the only factor of production and the unit labor cost is constant. That is, \( \Pi_i = 0 \) for all \( i \), and

\[ p_{ji,k} = a_{ji,k} w_j, \quad \forall j, i \in C; \forall k \in K \]

where \( a_{ji,k} \) is invariant to policy. Correspondingly, Home’s unconstrained optimal trade policy is a solution to the following problem:

\[
\max_{(x,t,\tau;w) \in \mathbb{A}} W_h (x, t, \tau; w)
\] (4)

It follows trivially that, when revenue-raising taxes are available, the optimal NRTB is zero, \( \tau^* = 0 \). To solve for the optimal revenue-raising trade taxes \( (x^* \) and \( t^*) \), we treat Foreign labor as the numeraire (i.e., \( w_f = 1 \)) and invoke demand-side envelop conditions. Doing so, we show that \( x^* \) and \( t^* \), should simultaneously
solve the $2 \times K$ system of First Order Conditions (FOC), corresponding to $K$ import tax instruments,

$$
\sum_g \left[ t_g r_{fh,g} \frac{d \ln q_{fh,g}(x, t, \tau; w)}{d \ln (1 + t_k)} \right] = \frac{d \ln w_h}{d \ln (1 + t_k)} \quad \forall k \in K,
$$

and $K$ export tax instruments\(^{10}\)

$$
\sum_g \left[ \left( \frac{1}{1 + x_g} - 1 \right) \lambda_{hf,g} \frac{d \ln q_{hf,g}(x, t, \tau; w)}{d \ln (1 + x_k)} \right] = \lambda_{hf,k} + \bar{\tau} \frac{d \ln w_h}{d \ln (1 + x_k)} \quad \forall k \in K.
$$

In the above expressions, $\lambda_{hf,k} \equiv \tilde{p}_{hf,k} q_{hf,k} / Y_f$ denotes the share of country $f$’s expenditure on variety $hf,k$; $r_{fh,k} \equiv p_{fh,k} q_{fh,k} / w_f L_f$ denotes the share of country $f$’s output generated from sales of product $fh,k$; and $\tilde{\tau} \equiv \left( \frac{\partial W_h}{\partial \ln w_h} / \frac{\partial V_f}{\partial Y_f} \right) L_f^{-1}$ is an aggregate term that accounts for effect of $w_h$ on aggregate welfare.

The left-hand side of both equations accounts for the marginal revenue loss holding prices constant. This effect is regulated by two distinct linkage between industries. First, cross-price elasticity effects, whereby a tax on variety $fh,k$ (or $hf,k$) can modify the demand for all other varieties and the tax revenues collected in all other industries. Second, income effects, whereby the tax revenue collected from good $fh,k$ (or $hf,k$) can alter the entire demand schedule, $q_h = D_h(Y_h, \tilde{p}_h)$, through its effect on aggregate income, $Y_h$.

The right-hand side of FOCs 5 and 6 accounts for the terms-of-trade gains from policy. In the case of import taxes, the only source of terms-of-trade gains are wage effects. That is, Home can raise its wage (relative to Foreign) by exercising its collective export/import market power. Export taxes, meanwhile, allow Home to exercise its export market power in a more local fashion. That is, using export taxes, Home can directly extract monopoly markups from foreign consumers—an effect that is picked up by the additional term, $\lambda_{hf,k}$, on the right-hand side of Equation 6.

Given that import taxes can only improve the terms-of-trade through their ef-

\(^{10}\)The number of export and import tax instruments need not to be equal, as our model allows for some tax instruments to be redundant. Suppose Home exports in only $K'$ industries, with $\lambda_{hf} = 0$ in the remaining $K - K'$ industries. In that case for a non-exported industry $k$, if $\partial \lambda_{hf,k} / \partial 1 + x_k = 0$, then $1 + x_k$ is a redundant instrument. Similar arguments apply to import taxes.
fect on the aggregate wage rate, it follows immediately that the optimal import
tax should be uniform across industries. Moreover, since Equation 5 describes the
optimal import tax for any given vector of applied export taxes, the optimality of
uniform import taxes holds irrespective of what export taxes are applied.\textsuperscript{11}

\textbf{Lemma 2.} \textit{Under Ricardian technologies, for an arbitrary vector of applied export taxes, the optimal import taxes are uniform across industries.}

To provide further intuition, note that in a Ricardian economy, the marginal cost
net of wage is invariant to policy. So, import taxes cannot affect neither consumer
prices nor producer prices in the Foreign market beyond wage effects. Put differ-
ently, Home’s import policy does not generate a \textit{local-price externality} in Foreign,
beyond wage effects. As a result, the optimal import policy is a uniform import
tax that allows Home to manipulate its collective market power while introducing
minimal allocative inefficiency in the local economy—we elaborate more on this
point in Section 6.\textsuperscript{12}

To further simplify the FOCs 5 and 6, we can appeal to the Lerner sym-
metry. That is, following Lemma 1, we can set $\frac{d \ln w_h}{d \ln (1 + t_k)} = \frac{d \ln w_h}{d \ln (1 + x_k)} = 0$ in FOCs 5 and 6 to identify one of the multiple optimal
policy combinations. Once this particular solution is identified, the remaining
solutions can be determined with an across the board shift in all import and ex-
port taxes. Taking these steps delivers the following theorem, which characterizes
Home’s optimal trade policy as a function of reduced-form demand elasticities and
expenditure shares.

\textbf{Theorem 1.} \textit{The unconstrained optimal policy under Ricardian technologies consists of
zero NRTBs, a uniform tariff, $\bar{\bar{t}}^*$, and a variable industry-level export tax,

\begin{align*}
(1 + x^*_k) (1 + \bar{\bar{t}}^*) = & \frac{\varepsilon_{hf,k}}{1 + \epsilon_{hf,k} + \xi_{hf,k}}. \\
\end{align*}

(7)

where $[\xi_{hf,k}]_k = \left[ \Xi^{-1} - I_K \right]$ accounts for cross-price elasticity effects between indus-

\textsuperscript{11}A formal proof is provided in Appendix A.
\textsuperscript{12}The above intuition is similar to that highlighted by Costinot \textit{et al.} (2015). However, as we
note later in Section 5, in more general environments, the intuition behind the uniformity of import
taxes is more nuanced. For example, optimal import tariffs can be non-uniform even when the
Foreign economy has a Ricardian production structure but the Home economy does not.
tries, with \( \Xi \equiv \left[ \frac{\lambda_{hf.g} \varepsilon_{hf.g} h f, k}{\lambda_{hf.g} \varepsilon_{hf,g}} \right]_{k \times g} \). [13]

As noted earlier, due to the Lerner symmetry, the optimal policy is indeterminate and unique only up to a uniform tariff, \( \bar{t}^* \). The optimal policy may, for instance, consist of a high uniform import tax and industry-level export subsidies, or a low uniform import tax and industry-level export taxes. Also, note that with zero cross-substitutability between industries (i.e., \( \Xi = I_K \iff \xi_{hf,k} = 0 \)), the optimal export tax formula reduces to the familiar single-product optimal monopoly markup, \( \varepsilon_{hf,k} / (1 + \varepsilon_{hf,k}) \).

An attractive feature of Theorem 1 is that it characterizes the optimal policy in terms of estimable and observable statistics. In the words of Piketty and Saez (2013), such sufficient statistic formulas have two broad merits. First, “this approach allows us to understand the key economic mechanisms behind the formulas.” Second “the ‘sufficient statistics’ formulas are also often robust to changing the primitives of the model.” In the present context, the formula characterized by Theorem 1 can be empirically evaluated with readily-available trade statistics. Moreover, as shown later in the Section 7, the above theorem greatly simplifies the quantitative analysis of trade policy in Ricardian gravity models.

4.1 Special Cases of the Ricardian Model

Two canonical models in international trade, namely, the multi-industry competitive gravity model and Dornbusch, Fischer, and Samuelson (1977), are special cases of the general Ricardian framework discussed above. In this subsection, we use Theorem 1 to derive the optimal trade tax formula for each of these special cases.

(i) The Multi-Industry Gravity Model (Costinot et al. 2011). Suppose that \( U_i = \prod_k Q_{ik}^{e_{ik}} \), where \( Q_{ik} = \left( \sum_{j=h,f} \chi_{ji,k} \eta_{ij,k}^{\rho_k} \right)^{1/\rho_k} \). It immediately follows that \( \varepsilon_{hf,k} = -1 - \varepsilon_k \lambda_{ji,k} / e_{i,k} \), where \( e_k \equiv \rho_k / (1 - \rho_k) \). The Cobb-Douglass assumption eliminates cross-price elasticity effects, so that \( \xi_{hf,k} = 0 \). Plugging these values into the optimal tax formula (Equation 7) yields

\[
(1 + x_k^*) \left( 1 + F^* \right) = 1 + e_{i,k} / \varepsilon_k \lambda_{hf,k}.
\]

\[13\] denotes \( K \times 1 \) vector of ones, while \( \Xi \) is a \( K \times K \) matrix.
That is, the optimal trade tax consists of a uniform tariff, $t^*$, and a industry-specific export tax that varies primarily with the industry-level trade elasticity, $\epsilon_k$. If the economy is modeled as a single industry, the above formula reduces to Gros’ (1987) optimal tariff formula, $t^* = 1/\epsilon\lambda_{ff}$.

(ii) Dornbusch et al. (1977) The Dornbusch et al. (1977) (DFS) model analyzed in Costinot et al. (2015) is a limiting case of the gravity model, but with a CES upper-tier utility aggregator. That is, $U_i = \left( \sum_k Q_{i,k}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$, where $Q_{i,k} = \lim_{\rho_k \to 1} \left( \sum_{j=h,f} q_{ji,k}^{\rho_k} \right)^{1/\rho_k}$. As shown in Appendix A.1, our optimal trade tax formula (7) implies the following limit-pricing solution for the DFS model:

$$(1 + x_k^*) (1 + t^*) = \begin{cases} \frac{\sigma}{\sigma-1} & \text{if } \frac{\sigma}{\sigma-1} \leq \frac{a_{ff,k}w_f}{\bar{a}_{ff,k}w_h}, \\ \frac{a_{ff,k}w_f}{\bar{a}_{ff,k}w_h} & \text{if } \frac{\sigma}{\sigma-1} > \frac{a_{ff,k}w_f}{\bar{a}_{ff,k}w_h}. \end{cases}$$

That is, the optimal export tax is equal to the optimal monopoly markup in the case of strong comparative advantage industries, and a limit-pricing markup in the case of weak comparative advantage industries. Correspondingly, the optimal export tax is weakly increasing in the degree of comparative advantage, $a_{ff,k}w_f / a_{ff,k}w_h$, which is the pattern emphasized in Costinot et al. (2015). On a related note, since $1/\lambda_{ff,k}$ is increasing in the degree of comparative advantage, $a_{ff,k}w_f / a_{ff,k}w_h$, the positive association between optimal export taxes and comparative advantage is also implicit in the gravity model. However, in that case, the importance of comparative advantage for optimal policy diminishes the lower the trade elasticity, $\epsilon_k$.

4.2 Multiple Countries

Now, we turn to the case where the world economy consists of $N > 2$ countries. That is, the Home country trades with multiple partners, and can impose discriminatory taxes on goods imported from or exported to different countries. The problem facing the Home country is similar to 4; but now the Home economy sets $(N-1) \times K$ import tax instruments, $t = \{t_{jh,k}\}_{j,k}$, and $(N-1) \times K$ export tax instruments, $x = \{x_{hi,k}\}_{i,k}$.

Since we are dealing with more than 2 countries, we can no longer appeal to the Lerner symmetry to normalize the vector of wages. Instead, Home’s import
taxes can improve the terms-of-trade through their effect on the wage rate in $N - 1$ different economies. As a result, Home’s import policy has a local-price externality across goods produced in different foreign countries.\footnote{See Bagwell and Staiger’s (1999) discussion of local-price externality for the case of a two-good multiple-country model.} However, Home is still unable to change the relative prices of goods produced from the same country, i.e., it can impose no local price externality across products from the same supplier. That being the case, the optimal tariff remains uniform across goods imported from the same country, but can vary across different countries. The following proposition, which is formally proven in Appendix \ref{sec:appendix_d}, outlines this claim.

**Proposition 1.** In a multi-country Ricardian model, the optimal tariff on products imported from any given country $i$ is uniform: $t^*_{jih,k} = t^*_{jih}$ for all $k \in K$. However, differential tariffs may be optimal on products imported from different countries.

The degree to which import taxes discriminate between exporters depends on the size and the openness of the Home country. A small economy’s trade tax, for instance, has a negligible effect on the relative wage of other countries. As a result, for such a country, the optimal import tax will be uniform and the optimal export tax will be characterized by Theorem \ref{thm:optimal_t}. For a large economy, however, the degree of import tax discrimination across trading partners can be significant.

Proposition \ref{prop:uniform_t} can facilitate the quantitative analysis of tariffs in a multi-country setup. As we will elaborate later, solving computationally for optimal import taxes in a multi-country model involves a non-linear optimization over $(N - 1) \times K$ tariff rates. If the number of countries and industries is large, the computation will be hindered by the curse of dimensionality. Proposition \ref{prop:uniform_t}, however, will allow researchers to shrink the state space by a factor of $K$ (i.e., solve for $N - 1$ tariffs instead of $(N - 1) \times K$).

## 5 Optimal Policy with Cross-Product Price Linkages

We now turn to characterizing the optimal trade policy in our general model, which allows for input-output linkages and industry-specific factors of production. Following the discussion in Section \ref{sec:imports}, competitive producer prices in this
general setup are given by Equation 2. That is,

\[ p_{ji,k} = C_{ji,k}(w_j, \tilde{p}^T_j; q_j), \]

where \( \tilde{p}^T_j = \{ \tilde{p}^T_{ij,k} \} \) is a vector describing the price of all intermediate inputs available to producers in economy \( j \), and \( q_j = \{ q_{ji,k} \} \) is a vector describing economy \( j \)'s output across all industries. Also, note that, in this general setup, each economy \( i \) exhibits a surplus, \( \Pi_i \), that is paid to the specific factors of production in that economy.

This general case features a rich set of cross-product price linkages that were absent in baseline Ricardian model. To be specific, in the Ricardian model, a tax on a given product affected the price of other products only through its impact on country-level wages. Here, however, a change in the price of a product may also have a more direct effect on the price of other products either through input-output (IO) linkages or through its effect on the demand schedule, \( q_j \).

To handle cross-price linkages, we characterize the optimal policy in terms of (i) reduced-form demand elasticities and (ii) trade tax passthroughs, both of which are estimable statistics. To fix minds, define the passthrough of taxes on to consumer prices (net of wage effects) as follows:

\[ \sigma_{ji,k}^{\mu,g} = \frac{\partial \ln \tilde{p}_{ji,k}(t, x; w)}{\partial \ln (1 + t_{ji,g})} = \frac{\partial \ln \tilde{p}_{ji,k}(t, x; w)}{\partial \ln (1 + x_{ji,g})}. \]

To elaborate, \( \sigma_{ji,k}^{\mu,g} \) captures the passthrough of a trade tax on good \( \mu \), \( g \) to the “consumer” price of good \( ji,k \), netting out the effect of that tax on country-level wages. Analogously, we use \( \sigma_{j,h,g}^{h,k} = \sigma_{j,h,g}^{h,k} - 1 \{ g = k \} \) to denote the passthrough of taxes onto “producer” prices. Recall that in the Ricardian model, \( \sigma_{ji,k}^{\mu,g} = 1 \) if \( ji,k = \mu, g \), while \( \sigma_{ji,k}^{\mu,g} = 0 \) if \( ji,k \neq \mu, g \). But, here, the own-passthrough of trade taxes may be incomplete and cross-passthrough of trade taxes may be non-zero.

As in the Ricardian case, the Lerner Symmetry implies that if \( (1 + t^*, 1 + x^*) \) is a vector of optimal trade taxes, then vector \( ((1 + t^*) (1 + \bar{i}), (1 + x^*) / (1 + \bar{i})) \), where \( \bar{i} \in \mathbb{R}_+ \), also constitutes an optimal policy combination. Considering this and for notational convenience, we express the optimal trade policy formula in
terms of $T_k$ and $X_k$, which are defined as

$$
\begin{align*}
1 + T_k & \equiv 1 + t_k^* / (1 + \bar{t}), \\
1 + X_k & \equiv 1 / [(1 + \bar{t}) (1 + x_k^*)].
\end{align*}
$$

Using the above definition and invoking demand- and supply-side envelop conditions, we can write the FOC corresponding to the import tax on industry $k$ as follows,

$$
\sum_g \left( -T_g r_{fh,g} \frac{\partial \ln q_{fh,g} (t, x; w, Y)}{\partial \ln (1 + t_k)} + \lambda_{fh,g} \frac{\partial \ln q_{fh,g} (t, x; w, Y)}{\partial \ln (1 + t_k)} \right) = \frac{\partial TOT_h}{\partial \ln (1 + t_k)} \quad \forall k,
$$

where the right-hand side denotes the terms-of-trade gains from the import tax, which we formally define as,

$$
\frac{\partial TOT_h}{\partial \ln (1 + t_k)} \equiv \sum_g \left( \lambda_{fh,g} \frac{\partial \ln \tilde{p}_{fh,g}}{\partial \ln (1 + t_k)} - r_{fh,g} \frac{\partial \ln p_{fh,g}}{\partial \ln (1 + t_k)} \right)
$$

$$
= \sum_g \left( \sigma_{fh,g} \lambda_{fh,g} - \sigma_{fh,g}^* r_{fh,g} \right).
$$

Importantly, the terms-of-trade gains (net of wage effects) are fully characterized by the sub-matrix of import tax passthroughs, $\sigma_{fh} \equiv \left[ \sigma_{fh,k}^{i,j} \right]_{i,j,k}$. The left-hand side of Equation 9, meanwhile, represents the trade volume loss from the import tax. Given that

$$
\frac{\partial \ln q_{ji,g} (t, x; w, Y)}{\partial \ln (1 + t_k)} = \sum_s \sum_{\ell} \varepsilon_{ji,s} \sigma_{fh,k}^{\ell,i} s_{ji,g},
$$

the trade volume loss can also be fully characterized in terms of pass-throughs, $\sigma_{fh} \equiv \left[ \sigma_{fh,k}^{i,j} \right]_{i,j,k}^g$, as well as import and export demand elasticities, $\varepsilon_{fh} \equiv \left[ \varepsilon_{fh,k}^g \right]_{i,j,k}$ and $\varepsilon_{hf} \equiv \left[ \varepsilon_{hf,k}^g \right]_{i,j,k}$.

---

15Recall that $\sigma_{fh,k}^{i,j} = \partial \ln p_{fh,g} / \partial \ln (1 + t_k)$ denotes the pass-through of taxes onto “producer” prices.
Similarly, the FOC corresponding to the export tax on industry $k$ is given by
\[
\sum_g \left( T_g r_{fh,g} \frac{\partial \ln q_{fh,g}(t,x;w,Y)}{\partial \ln (1 + x_k)} - \lambda_{hf,g} \frac{\partial \ln q_{hf,g}(t,x;w,Y)}{\partial \ln (1 + x_k)} \right) = \frac{\partial TOT_h}{\partial \ln (1 + x_k)} \quad \forall k,
\]  
(10)
where, as before, the right-hand side denotes the terms-of-trade gains from the export tax:
\[
\frac{\partial TOT_h}{\partial \ln (1 + x_k)} \equiv \sum_g \left( \lambda_{hf,g} \frac{\partial \ln \tilde{p}_{hf,g}}{\partial \ln (1 + x_k)} - r_{fh,g} \frac{\partial \ln p_{hf,g}}{\partial \ln (1 + x_k)} \right) 
\]
(11)
\[
= \sum_g \left( \sigma_{hf,g}^{k} \lambda_{hf,g} - \sigma_{fh,g}^{k} r_{fh,g} \right).
\]
Also, as with the case of import taxes, the left-hand side of Equation 10 represents the trade volume loss from export tax, $x_k$, which again can be fully characterized in terms of the passthroughs and demand elasticities. Combining Equations 9 and 10, the following theorem provides an analytical characterization of optimal policy as a function of trade shares, $\lambda$, revenue shares, $r$, demand elasticities, $\varepsilon$, and tax passthroughs, $\sigma$. The former two statistics are directly observable, while the latter two can be locally estimated.

**Theorem 2.** [Optimal Trade Taxes under General Price Linkages]

The optimal trade tax schedule $\mathcal{T}, \mathcal{X}$ is implicitly given by
\[
\begin{bmatrix}
-r_{fh} \circ \varepsilon_{fh} \sigma_{fh} & \lambda_{hf} \circ \varepsilon_{hf} \sigma_{fh}\\
r_{fh} \circ \varepsilon_{fh} \sigma_{fh} & -\lambda_{hf} \circ \varepsilon_{hf} \sigma_{hf}
\end{bmatrix}
\begin{bmatrix}
\mathcal{T} \\
\mathcal{X}
\end{bmatrix}
= \begin{bmatrix}
\nabla \ln_{1+t} TOT_h \\
-\nabla \ln_{1+x} TOT_h
\end{bmatrix},
\]
where $\varepsilon_{ji} = \begin{bmatrix} e_{ji,k} \end{bmatrix}_{k,j,g}$ is a $K \times 4K$ sub-matrix of demand elasticities and $\sigma_{ji} = \begin{bmatrix} \sigma_{ji,k}^{g} \end{bmatrix}_{j,g,k}$ is a $4K \times K$ sub-matrix of tax passthroughs.\(^\dagger\)

Based on the above theorem, the optimal trade tax on any subset of industries is uniform if the tax has a zero passthrough (net of wage effects) onto Foreign prices, $\{p_{fh,k}\}$ and $\{\tilde{p}_{hf,k}\}$. This result applies to both export and import taxes, and is simply reflective of the fact that in the zero passthrough case, the only purpose for

\(^{\dagger}\)Note that $\sigma_{ji}$ where $ji \in C \times C$ are different block elements of the pass-through matrix: $\sigma = [\sigma_{ji}]_{ji}$. Also, $\circ$ denotes the Hadamard or entry-wise product.
trade taxes is to increase Home’s labor wage relative to Foreign. This particular objective is best achieved through a uniform import/export tax.\(^{17}\)

Relatedly, the Ricardian model studied in Section 4 is a special case of the above theorem, where the own passthrough trade taxes is complete, but the cross passthrough of trade taxes is zero. As a result, \(\nabla \ln(1+t)T_{OTh} = 0\) and \(\nabla \ln(1+x)T_{OTh} = \lambda_{hf}\), which leads to a uniform import tax, and a differential export tax that is proportional to the inverse of the foreign country’s import demand elasticity for each product.

In the next two subsections, we derive the matrix of pass-throughs,

\[
\sigma = \begin{bmatrix}
\sigma_{hh} & \sigma_{fh} & \sigma_{hf} & \sigma_{ff}
\end{bmatrix},
\]

and use Theorem 2 to characterize the optimal policy in two special cases. First, a Ricardian model with general input-output linkages. Second, a generalized specific-factors model where the marginal cost of production (net of labor wage) is increasing in output.

Before moving forward, however, it is worth noting that when export taxes are unavailable, Theorem 2 implies the following formula for optimal import tariffs,

\[
1 + t^* = (1 + \bar{t}) \left[ 1 - (r_{fh} \circ \varepsilon_{fh}(\sigma_{fh})^{-1} \nabla \ln(1+t)T_{OTh}) \right],
\]

where the industry-specific component (the term in bracket) accounts for Home’s industry-level monopsony power, while the uniform term accounts for Home’s aggregate monopsony power due to general equilibrium wage effects. Unlike the benchmark case, however, the exact value of \(\bar{t}\) is critical here, and is determined by the following equation:\(^{18}\)

\[
1 + \bar{t} = \left( \hat{\varepsilon}_{hf} - \sum_{g} (t_{g} - \bar{t}) r_{fh,g} \hat{e}_{fh,g} \right) / (1 + \hat{\varepsilon}_{hf}),
\]

\(^{17}\)Theorem 2 also indicates that optimal trade taxes do not explicitly depend on neither the aggregate labor demand elasticities nor the income elasticities of demand. This outcome is a direct byproduct of the Lerner symmetry theorem. In particular, accounting for general equilibrium income and wage effects leads to a uniform shift in all export or import taxes. But given the Lerner symmetry, when both export and import taxes are available, a uniform shift in either tax instrument is redundant.

\(^{18}\)See Appendix E for a formal derivation.
where \( \tilde{\epsilon}_{hf} = \sum_k \sum_g \left( \frac{r_{hf}\, k}{\tilde{\epsilon}_{hf}\, g} \right) \) denotes the elasticity of Foreign’s demand for Home labor—see Section 6 for a formal definition. Considering this, the above expression corresponds to the optimal markup on Home’s wage rate, with a correction for non-uniformity. Moreover, based on the above, when export taxes are restricted, general equilibrium wage effects dampen the cross-industry heterogeneity in optimal import tariffs. So, even though tariffs are non-uniform, they are less heterogeneous than traditional theories would suggest.

5.1 Input-Output Linkages

Below, we derive the passthrough matrix and the optimal trade tax schedule in a Ricardian model with input-output linkages. In this case, the production of each good employs labor and intermediate inputs from (possibly) all industries. Stated formally, the production of good \( ji, k \) (export \( j \)-importer \( i \)-industry \( k \)) is characterized by \( q_{ji,k} = q_{ji,k} (L_{ji,k}, q_{ji,k}^T) \), where \( L_{ji,k} \) is the amount of labor employed in the production of good \( ji, k \); and \( q_{ji,k}^T \equiv \{ q_{ji,k}^{gi} \}_{i=1}^g \) is the vector of intermediate input quantities, with \( q_{ji,k}^{gi} \) denoting the quantity the intermediate input \( i \) uses in the production of good \( ji, k \).\(^{19}\) Such a production setup implies that the competitive price set by cost-minimizing firms (net of taxes) should exhibit the following formulation, which is a special case of Equation 2:

\[
p_{ji,k} = C_{ji,k} \left( w_j, \bar{p}_j^T \right) \quad \forall j, i \in C; \ k \in \mathbb{K}
\]

That is, the producer of good \( ji, k \) is a function of the local wage rate, \( w_j \), and the (tax-inclusive) price of all intermediate inputs, \( \bar{p}_j^T \equiv \{ \bar{p}_{ji,k} \}_{i=1}^g \). For notational convenience, we assume that the price of a product in a given market is the same whether it is used as an intermediate input (indexed \( I \)) or a consumption good (indexed \( C \)), namely, \( \bar{p}_i^T = \bar{p}_i^C = \bar{p}_i \) for \( i = h, f, h, f \).\(^{20}\) Correspondingly, the consumer price of goods \( ji, k \) can be determined exclusively as a function of the taxes and the

---

\(^{19}\)In the above notation, \( q_{ji,k}^{gi} = 0 \) if \( i \neq j \), by construction. For instance, variety \( fh, g \) which is sold by foreign firms to the home country cannot be directly employed as an input by foreign firms, but variety \( ff, g \) that is sold in the foreign market can be.

\(^{20}\)By the choice of parameters in the preferences, this structure still allows for different prices of intermediate and consumption goods, which may occur due to, for example, differential taxation of intermediate and consumption goods.
wages as follows:

\[ \tilde{p}_{ji,k}(t, x; w) = (1 + t_{ji,k}) (1 + x_{ji,k}) C_{ji,k}(w_j, \tilde{p}_j) \quad \forall j, i \in C; \ k \in K \]  

(13)

where, by assumption, \( x_{fh,k} = t_{fh,k} = 0 \), while \( t_k \equiv t_{fh,k} \) and \( x_k \equiv x_{hf,k} \) for all \( k \).

To characterize the entire \( 4K \times 4K \) matrix of pass-throughs, \( \sigma \equiv [\sigma_{ji,k}^{gj,i}]_{jik, jg} \), we apply the implicit function theorem to Equation 13. We also appeal to Shepard’s lemma that \( \partial \ln p_{ji,k}/\partial \ln p_{ji,k} = \tilde{p}_{ji,k} q_{ji,k}/p_{ji,k} q_{ji,k} \), where the right-hand side \( \lambda_{ji,k}^{gj,i} \equiv \tilde{p}_{ji,k} q_{ji,k}/p_{ji,k} q_{ji,k} \) denotes the \( jik \times jg \)th element of the global \( 4K \times 4K \) IO matrix, \( A \equiv [\lambda_{ji,k}^{gj,i}]_{jik, jg} \). Following these two steps, we can produce the following lemma, which states that (beyond wage effects) the pass-through of taxes on to consumer prices is fully determined by the IO matrix.

**Proposition 2.** In a Ricardian Model with input-output linkages, the matrix of trade tax pass-throughs is exclusively characterized by the global input-output matrix:

\[ \sigma = (I - A)^{-1}. \]

Note that \( \sigma = [\sigma_{hh}, \sigma_{fh}, \sigma_{hf}, \sigma_{ff}] \) is closely related to the Domar weights characterized by Baqaee and Farhi (2017).\(^{21}\) That \( \sigma \) does not depend on the demand-side of the economy is an artifact of the Ricardian supply structure. Based on this assumption, trade taxes affect the unit labor cost only through the their effect on input prices. Once we relax the Ricardian supply structure, which is done in the following section, \( \sigma \) depends on the entire schedule of demand elasticities.

**Optimal Tariff on Intermediate vs. Final Products**

We now use the pass-through matrix \( \sigma \) and Theorem 2 to determine how optimal trade taxes differ across intermediate and final goods. To fix minds, note that our general IO structure allows us to have two versions of the same good: a final good version and an intermediate input version. Thus, it can accommodate the case where differential taxes are applied to the same product, depending on the intended final use.

\(^{21}\)Since there are no misallocations in our Ricardian economy, the *revenue-based* and *cost-based* input-output matrixes and the corresponding Domar weights are identical in our setup.
Our main claim is that the optimal import tax on any given good depends crucially on whether it will be re-exported as part of another product. To see this, suppose Home imposes an import tax \( t_k \) on good, \( fh,k \), but does not export goods that employ \( fh,k \) as an intermediate input. In that case, Proposition 2 implies that \( \sigma_{fh,g}^{fh,k} = \sigma_{fh,g}^{fh,k} = 0 \). That is, \( t_k \) cannot influence consumer and producer prices in Foreign beyond general-equilibrium wage effects. Correspondingly, \( \partial \ln TOT_h / \partial \ln (1 + t_k) = 0 \) and the trade volume losses due to \( t_k \) are only direct effects, i.e., \( \partial \ln q_{fh,k} / \partial \ln (1 + t_g) = \epsilon_{fh,g}^{fh,k} \) and \( \partial \ln q_{fh,k} / \partial \ln (1 + t_g) = 0 \). Considering this, Theorem 2 immediately implies that \( T_k = 0 \) for product \( fh,k \) that is not re-exported through input-output linkages.22

The following proposition formally outlines this claim.

**Proposition 3.** In a Ricardian model with input-output linkages, the optimal import tariff is uniform across all imported final goods and intermediate goods that are not re-exported.

Now, consider imported intermediate inputs that are re-exported as part of another (more downstream) product. In this case, an import tax on an intermediate input \( fh,k \) could influence the consumer and producer prices in the rest of the world beyond general-equilibrium wage-effects. In particular, the consumer price of any exported good \( hf,g \) (namely, \( \tilde{p}_{hf,g} \)) that uses \( fh,k \) as an input will be affected by \( t_k \). Similarly, the producer price of a foreign-produced good \( fh,s \), which uses \( hf,g \) as an intermediate input, will be affected indirectly by \( t_k \). As a result, \( \partial \ln TOT_h / \partial \ln (1 + t_k) \neq 0 \), which leads to non-uniformity in import taxes.

Following up on the above argument, re-exporting gives the Home government the power to effectively tax transactions outside its territory. This extraterritorial taxing power creates a policy externality that is distinct from the terms-of-trade and local-price effects identified in the prior literature ((Bagwell and Staiger, 1999)). To elaborate, suppose Home imposes an import tax on tires (good \( fh,k \)) and exports them back as part of a fully-assembled vehicle to Foreign (good \( hf,g \)). The price of this exported vehicle can be decomposed into a domestic value-added component, \( \tilde{p}^{VA}_{hf,g} \) (i.e., the price without the tires), and a foreign-produced

---

22Another special case where taxes are identical to the Ricardian case, is one where (i) the input-output structure is symmetric \( a_{j,i}^{j,k} = a_k \) for all \( j, i, k \) and \( g \). In that case, \( \sigma_{j}^{j,i,k} = \sigma_k \) if \( i = j \) and \( \sigma_{j}^{j,i,k} = \sigma_i^{j} \) if \( i \neq j \), with \( \sigma_k - \sigma_i^{j} = a_k \), and (ii) preferences are quasi-linear and additively separable (i.e., \( \eta_{i}^{j,g} = 0 \) if \( g \neq k \) and \( \epsilon_{fh,k} = -\epsilon_{fh,k}^{bh,k} \)). For this special case, it is straightforward to show that \( T_k = 1 \) and \( X_k = 1/\epsilon_{fh,k} \).
and foreign-consumed component, \( \hat{p}_{F,k} \) (i.e., the price of tires). Stated formally, \( \hat{p}_{hf,k} = \hat{p}_{VA,k} + \hat{p}_{F,k} \). Hence, supposing that the tires or the vehicle are not used as intermediate inputs in any other good, the terms-of-trade effect of such an import tax \( (t_k) \) can be stated as

\[
\frac{\partial TOT_h}{\partial \ln (1 + t_k)} = \lambda^{VA}_{hf,g} \frac{\partial \ln \hat{p}_{hf,g}}{\partial \ln (1 + t_k)} + \lambda^{F}_{hf,g} \frac{\partial \ln \hat{p}_{F,k}}{\partial \ln (1 + t_k)}
\]

where \( \lambda^{VA}_{hf,g} \equiv \lambda_{hf,g} \left( \hat{p}_{VA,k}/\hat{p}_{hf,g} \right) \) and \( \lambda^{F}_{hf,g} \equiv \lambda_{hf,g} \left( \hat{p}_{F,k}/\hat{p}_{hf,g} \right) \). The second term in the above equation corresponds to rents accruing to the Home government through taxing a transaction between tire producers and consumers who are located outside of its territory. If tires were instead assembled on the car in Foreign, the Home government would not have such extraterritorial taxing power. Such extraterritorial taxing power can perhaps explain why the gains from policy are significantly larger in the presence of IO linkages—a claim we formally document in Section 7.

### 5.2 Generalized Specific-Factors Model

Now we consider a generalized specific factors model in which producer prices are given by the following special case of Equation 2:

\[
p_{ji,k} = C_{ji,k} \left( w_{ji}; q_{j,k} \right) \quad i, j \in C, \quad k \in K.
\]

That is, the competitive price of goods \( ji,k \) is a function of country \( j \)'s output level in industry \( k \), namely, \( q_{j,k} = \{q_{ji,k}\}_i \). Before characterizing the passthrough matrix in this case, let us briefly discuss the micro-foundation that rationalizes the above equation.

Generally speaking, the upward-sloping supply curve underlying Equation 14 may arise due to \( (i) \) a finite elasticity of transformation between different output varieties from the same country, and/or \( (ii) \) industry-specific factors of production. To elaborate, suppose that country \( j \)'s composite output in industry \( k \) is characterized by \( Q_{j,k} = Q_{j,k} (q_{jh,k}, q_{jf,k}) \), which allows for a finite elasticity of transformation between output produced for different markets. A well-
known special case of this specification is the constant elasticity of transformation (CET) production possibility frontier popularized by Powell and Gruen (1968):

\[ Q_{j,k} = \left( \sum_{i=h,f} X_{ji,k} f_{ji,k} \rho_k \right)^{1/\rho_k}, \]

where \( \rho_k > 1 \). The composite output \( Q_{j,k} \) is itself produced using labor, \( L_{j,k} \), that is perfectly mobile across industries, and an industry \( k \)-specific factor, \( S_{j,k} \), that is immobile, i.e., \( Q_{j,k} = F_{j,k}(L_{j,k}, S_{j,k}) \), where \( F_{j,k}(\cdot) \) is a non-parametric production function.

Given the above production structure, cost-minimizing firms set a competitive price that is a function of their output schedule and the local wage rate, as in Equation 14. A classic special case covered by Equation 14, is the the standard Ricardo-Viner model, in which the output produced for domestic and foreign markets are perfectly substitutable (i.e., \( \rho_k \rightarrow 1 \) in the CET case).

Noting that \( C_{j_i,k}(\cdot) \) describes the supply curve of variety \( j_i,k \), the supply-side of this economy can be fully summarized in terms of the following reduced-form (inverse) supply elasticities.

**D3. [Supply Elasticities]**

[own-supply elasticity] \( \gamma_{ji,k} \equiv \partial \ln C_{j_i,k}(w_j; q_{j_i,k}) / \partial \ln q_{ji,k} \)

[cross-supply elasticity] \( \gamma_{ji,k}^{j_i,k} \equiv \partial \ln C_{j_i,k}(w_j; q_{j_i,k}) / \partial \ln q_{ji,k}^{j_i,k} \).

To attain further perspective on the above elasticities, note two special cases. First, the standard Ricardo-Viner model where \( \gamma_{fh,k} = \gamma_{ff,k}^{fh,k} \) simply equals the inverse of Foreign’s export supply (or excess supply) elasticity. Second, the CET model without industry-specific factors of production where the supply elasticity assumes a more structural interpretation, \( \gamma_{ji,k} = \left(1 - \tilde{r}_{ji,k}\right)/\left(\rho_k - 1\right) \) for all \( ji,k \).

Considering D3, we now proceed to characterizing the passthrough matrix in the generalized specific factors model. To this end, we follow similar steps to those taken in the case of IO linkages. We apply the implicit function theorem to the following equation, which characterizes consumer prices:

\[ \tilde{p}_{ji,k}(t, x; w) = (1 + t_{ji,k}) \left(1 + x_{ji,k}\right) C_{j_i,k}(w_j; q_{j_i,k}) \quad \forall j, i \in \mathbb{C}; \ k \in \mathbb{K}. \]

Doing so, yields a \( 4K \times 4K \) matrix of pass-throughs,

\[ \sigma = (I - \Sigma)^{-1} \quad (15) \]

\( ^{23}\tilde{r}_{ji,k} = r_{ji,k} / \sum_i r_{ji,k} \) denotes the within-industry output share.
where \( \Sigma = \left[ \gamma_{j_i,k}^{f_h,g} \kappa_{f_h,g}^{f_h,k} \right]_{j_i,k} \) is fully determined by reduced-form demand and supply elasticities. Noting that \( \sigma = [ \sigma_{hh} \sigma_{hf} \sigma_{ff} ]_j \), we can plug \( \sigma_{hh} \) and \( \sigma_{hf} \) calculated using the above equation into Theorem 2 to calculate the optimal trade tax schedule exclusively as a function of reduced-form supply and demand elasticities as well as observable expenditure and revenue shares.

In the standard Ricardo-Viner case, where there is zero cross-substitutability between industries and \( \gamma_{ji,k} = \gamma_{j_i,k}^{f_h,k} = \gamma_{j_i,k}^{f_h,g} \) for all \( j, i, \) and \( k \), Equation 15 implies that \( \sigma_{ji,k}^{f_h,k} = 1/(1 - \epsilon_{ji,k} \gamma_{ji,k}) \) if \( j, k = j, g \) and \( \sigma_{ji,k}^{f_h,g} = 0 \) if \( j, k \neq j, g \). Plugging these expressions into Theorem 2, implies that \( \gamma_{f_h,k} = \gamma_{f_h,k}^{f_h,k} \) and \( X_k = 1/\epsilon_{h_f,k} \), which in turn yields the following familiar-looking optimal tax formula for the Ricardo-Viner case:

\[
1 + t_k^* = (1 + \bar{t}) (1 + \gamma_{f_h,k}), \\
(1 + x_k^*) (1 + \bar{t}) = \epsilon_{h_f,k} / (1 + \epsilon_{h_f,k}).
\] (16)

What is perhaps notable about the above formula, is that it can be obtained without abstracting from either general equilibrium wage effects or general equilibrium income effects. The only necessary assumption is that cross-demand elasticities be zero between industries and all trade tax instruments be available to the government.

Importantly, Equation 2 and Theorem 2 indicate that cross-substitutability between Ricardian and non-Ricardian industries can lead to the non-uniformity of optimal import tariffs even across Ricardian industries. To elaborate, suppose the Foreign economy employs Ricardian production technologies, but Home’s industries employ specific factors of production and exhibit upward-sloping supply curves. Then, a tax on import good \( f_h,k \) can affect the demand for good \( hh,g \). This effect can, in turn, alter the consumer price of Home’s exports to Foreign in industry \( g, \tilde{p}_{h_f,g} \)—i.e., \( \sigma_{h_f,g}^{f_h,k} > 0 \). Considering this, and based on Theorem 2, the non-uniform component of \( t_k \) will be non-zero (i.e., \( T_k \neq 0 \)), even though the Foreign supply curve is flat in all industries.
6 Interdependence of Trade Policies

Trade policy interdependence concerns the effect of policy choices in one area on the tradeoffs that policymakers face in other areas. The existence of these interdependencies is not a controversial idea among economists. In fact, one of the best-known results in international economics—the Lerner’s (1936) Symmetry Theorem—establishes a strong link between import and export policies. Nor is the importance of these interdependencies too hard to notice: Many disputes in the WTO are about alleged use of policy instruments that are not restricted by the WTO but have the effect of replicating trade taxes/subsidies. Nevertheless, the current literature provides very little insight about the nature of these interdependencies and their potential political and economic implications.

In the context of our model, political considerations or international trade agreements may limit the government’s freedom in choosing their trade policy from set $A$. In many instances, governments may be prohibited from conducting export policy or from setting import taxes in select industries. In the presence of general equilibrium linkages, these partial restrictions can influence the government’s choice of optimal policy with respect to unrestricted instruments.

In what follows, we outline three novel trade policy interdependencies. To streamline the presentation, we hereafter restrict attention to the Ricardian case of the model; noting that most of the trade-offs we identify are not an artifact of the Ricardian supply structure and prevail in the more general case of our framework. We present our results using a sequence of hypothetical partial liberalization episodes. In each episode, a set of previously-available policy instruments are restricted and the government sets the unrestricted instruments optimally.

6.1 Optimal Policy when Export Taxes are Restricted

Suppose Home enters a trade agreement that prohibits all export taxes, but leaves import taxes at the discretion of the government. Home’s optimal policy problem

24Two notable features of the GATT/WTO agreement resemble the partial policy restorations emphasized here. First, the GATT/WTO has only gradually introduced more constraints on the governments’ policy space over time. Earlier GATT/WTO negotiations were focused on tariff cuts in a few industries, leaving import tariffs in many other industries at the discretion of governments. Another notable feature of the GATT/WTO agreement is its rather strict stance towards export policy in comparison to import policy.
under this partial restriction can be stated as follows:

\[
\max_{(0, t, \tau; \omega) \in A} W_h (0, t, \tau; \omega)
\]

Since export taxes are restricted, the optimal tax combination is now unique and wages can no longer be normalized in both countries. Instead, due to the Ricardian supply structure, Home’s import taxes can only improve its terms-of-trade through their effect on the relative wage, \(w_h/w_f\). As we show below, given these circumstances, Home’s optimal import tax is uniform and determined by the elasticity of Foreign demand for Home’s labor.

To make this point formally, let \(L_{ji} = \mathcal{L}_{ji} (w; t, x)\) denote country \(j\)’s demand for country \(i\)’s labor. The elasticity of country \(j\)’s demand for country \(i\)’s labor demand can, thus, be defined as follows.

**D4. [Elasticity of Labor Demand]** \(\tilde{\epsilon}_{ji} \equiv \partial \ln \mathcal{L}_{ji} (w; t, x) / \partial \ln w_j = \sum_k \sum_g \left( \frac{r_{ji,k}}{r_{ji}} \tilde{\epsilon}_{ji,k} \right) \).

In the above definition, the last line follows from the Ricardian supply structure, which indicates that \(L_{ji} = \sum_k q_{ji,k}/a_{ji,k}\). One straightforward interpretation of \(\tilde{\epsilon}_{ji}\) is that it reflects country \(j\)’s collective export market power. As noted by the following proposition, Home’s optimal import tax is determined solely by \(\tilde{\epsilon}_{hf}\).

**Proposition 4.** When export taxes are restricted, the optimal policy consists of a uniform import tariff that reflects the home country’s collective export power

\[
1 + \tilde{t}^* = \frac{\tilde{\epsilon}_{hf}}{1 + \tilde{\epsilon}_{hf}}.
\]

The intuition behind the uniformity of optimal import taxes is similar to what we provided earlier. But the intuition behind the formula characterizing the optimal tax level is the following. A uniform import tax (which is isomorphic to a uniform export tax) is akin to a markup charged by Home on its labor wage rate. The optimal markup level is, thus, determined by the elasticity of Foreign demand for Home’s labor. In the widely-used multi-industry gravity model outlined in Section 4.1, the optimal import tax formula reduces to

\[
1 + \tilde{t}^* = 1 + \frac{1}{\sum_k \left( \frac{r_{hf,k}}{r_{hf}} \epsilon_k \lambda_{ff,k} \right)}.
\]
where $\epsilon_k$ denotes the trade elasticity in industry $k$. A well-know special case of the above formula is the single-industry optimal tariff formula, $\bar{t}^* = 1/e_{\lambda ff}$, popularized by Gros (1987).\(^{25}\)

Proposition 4 points to a rather surprising corollary. Given the Lerner symmetry, Proposition 4 implies that the effect of the optimal import tax can be exactly replicated with a uniform export tax. The opposite, however, is not true as the optimal export tax is typically non-uniform according to Theorem 1.

**Corollary.** *In a Ricardian economy, import taxes are only an imperfect substitute for export taxes. But export taxes can perfectly reproduce any welfare outcome that is attainable with import tariffs.*

The intuition behind the above corollary is simple. In a Ricardian economy, import taxes can only improve Home’s terms-of-trade through their effect on the economy-wide wage rate. Export taxes can do that, but they can also improve the terms-of-trade through their direct effect on consumer prices in Foreign. As a result, export taxes are more potent of a policy instrument in a Ricardian economy. These results, have an immediate implication for the design of trade agreements. That is, an incomplete agreement that restricts only import tariffs is ineffective, because the restricted import tariffs can be perfectly substituted with unrestricted export taxes. However, restricting export taxes can effectively lower the degree of protection, even without imposing restrictions on import taxes.

### 6.2 Optimal Policy when a Subset of Industries are Restricted

Now, we consider a second sequence of liberalization whereby import taxes are restricted in a *subset* of industries. To be specific, in this second sequence, Home is still obliged to set zero export taxes in all industries, i.e., $x_k = 0$ for all $k$. In addition, it is also restricted to setting zero import taxes in a subset of industries. To streamline the presentation, we let $K_R$ denote the set of import-tax-restricted industries and let $K_L$ denote the set of unrestricted industries, with $K_R \cup K_L = \ldots$ \(^{25}\)

Based on the above formula, optimal tariffs increase with the level of trade openness, $1/\lambda_{ff,k}$. This prediction, however, hinges on the convexity of the CES demand, whereby $|\epsilon_{hf,k}|$ is strictly decreasing in the level of trade in industry $k$. Conversely, if the underlying demand was sub-convex, then more trade openness would entail a lower optimal tariff.
K, by construction. We also temporarily focus on the special case where traded industries exhibit a zero income elasticity.\footnote{This assumption is consistent with a quasi-linear utility aggregator across industries, with trade costs being prohibitively high in the linear industry.}

As demonstrated in Appendix B, Home’s optimal import tax in unrestricted industries can be characterized as follows:

\[
1 + t_k^* = (1 + \bar{t}) \left( 1 + \sum_{g \neq k} \left[ \frac{1 + t_g}{1 + \bar{t}} - 1 \right] \frac{r_{f_h,g} \varepsilon_{f_h,k}}{r_{f_h,k} \varepsilon_{f_h,g}} \right), \quad k \in K_L, \tag{17}
\]

where \( \bar{t} \) is a uniform term that accounts for general equilibrium wage effects:

\[
1 + \bar{t} = \frac{\bar{\varepsilon}_{hf} + \sum_g \left( t_g - \bar{t} \right) \frac{r_{f_h,g} \varepsilon_{f_h,g}}{r_{f_h,k} \varepsilon_{f_h,k}}}{1 + \bar{\varepsilon}_{hf}}. \tag{18}
\]

Based on the above formula, the post-partial-liberalization import taxes are generally non-uniform. Furthermore, as the set of restricted industries shrinks to zero, the optimal import tax specified by the above proposition converges to the uniform import tax formula specified by Proposition 4—that is, if \( K_R = \emptyset \), then \( 1 + t_k^* = 1 + \bar{t} = \bar{\varepsilon}_{hf} / (1 + \bar{\varepsilon}_{hf}) \) for all \( k \).

Based on Equation 17, partial liberalization affects the optimal import tax in unrestricted industries through two distinct channels. The first driver of interdependence between restricted and unrestricted tariffs are cross-price elasticity effects. To elaborate, consider the case where industries are gross substitutes: \( \varepsilon_{f_h,k} > 0 \) for all \( k \) and \( g \). In that case, the second parenthesis in Equation 17 is equal to “one” without partial restrictions but smaller than “one” otherwise. Hence, partial restrictions lower the optimal import tax in unrestricted industries through cross-elasticity effects. The intuition being that restricting import taxes in a subset of industries decreases the volume of trade in unrestricted industries. The reduction in trade volume, in turn, reduces the marginal revenue from taxation and entails a lower optimal tax.

General equilibrium wage effects, which operate through \( \bar{t} \), are the second driver of tariff interdependence. Noting that \( \bar{\varepsilon}_{f_h,g} < 0 \), it follows immediately
that

\[
\frac{\varepsilon_{hf} + \sum_g \left( (t_g - \bar{t}) \frac{r_{fgh}}{r_{fgh}} \frac{\varepsilon_{fgh}}{\varepsilon_{fgh}} \right)}{1 + \varepsilon_{hf}} < \frac{\varepsilon_{hf}}{1 + \varepsilon_{hf}},
\]

where the right-hand side corresponds to the optimal import tax level without partial restrictions (Proposition 4). The above expression simply indicates that the uniform component of the optimal import tax declines in face of partial liberalization. The intuition behind this second channel can be stated as follows. When tariffs are restricted in some industries, the wage-driven component of the optimal import tax, \( \bar{t} \), introduces a relative price distortion between restricted and unrestricted industries. To partially countervail this distortion, \( \bar{t} \) assumes a lower value under the partial restriction.

The tariff interdependence arising from a reduction in \( \bar{t} \) is subject to one basic qualification. The elasticity of labor demand, \( \varepsilon_{hf} \), can itself vary with a change in the underlying trade taxes. So, to ensure that \( \bar{t} \) reduces in face of the partial restrictions, we need \( \partial | \varepsilon_{hf} | / \partial w_h \) to be sufficiently small. That is, the decline in \( w_h / w_f \) due to partial liberalization, should not lead to a too large of a decline in \( | \varepsilon_{hf} | \). This will the case if the demand for labor is sufficiently concave. The following Proposition summarizes these arguments.

**Proposition 5.** If (i) \( \partial | \varepsilon_{hf} | / \partial w_h \) is sufficiently small, and (ii) industries are gross substitutes, then tariffs are complementary across industries. That is, restricting tariffs in a subset of industries lowers the optimal tariff in unrestricted industries.

The above proposition is significant because negotiating tariff cuts can be costly, and more so for certain industries. Based on Proposition 5, it may be optimal for trade agreements to focus on tariff reductions in a subset of low-negotiation-cost industries. Once tariffs are lowered in these industries, governments will voluntarily lower their tariffs in the non-negotiated industries.

It should be noted that the assumption placed on \( \partial \ln | \varepsilon_{hf} | / \partial w_h \) by Proposition 5 is weaker than it may appear. In our multi-industry framework, two factors affect the convexity of demand for labor. On one hand, a drop in \( w_h \) alters the composition of demand in favor of high-elasticity industries. This effect always contributes to a lower \( \partial \ln | \varepsilon_{hf} | / \partial w_h \). On the other hand, a drop in \( w_h \) can also alter the demand elasticity level, \( \varepsilon_{hf,k} \), per industry, with the direction of this latter change depending on the underlying demand function. Considering this, Proposi-
tion 5 simply holds if composition effects are sufficiently large. Later in Section 7, we show that in a standard multi-industry gravity model fitted to data, the conditions outlined by Proposition 5 are satisfied, and that industry-level import taxes exhibit strong complementarity.

Importantly, the above interdependence results are derived under the assumption that export tax instruments are restricted. A considerably weaker version of tariff complementarity arises when export taxes are not restricted. Specifically, suppose Home is obliged to set zero import taxes in a subset $K_R$ of industries. Then following Theorem 1, it is optimal for Home to lower the tariff to zero in the unrestricted industries, and apply a uniform upward shift to all export taxes. By applying these changes, the import tax restriction will have essentially no effect on Home’s rate of protection. By contrast, the tariff complementarity outlined by Proposition 5 induces Home to voluntarily lower its effective rate of protection. In other words, Foreign experiences a welfare gain from the complementarity-induced reduction in import taxes.

6.3 Optimal NRTBs when Revenue-Raising Taxes are Restricted

Finally, we consider a third sequence of liberalization where all revenue-raising trade taxes are restricted. In this case, the Home government can only erect non-revenue trade barriers (NRTBs) that restrict imports without generating any revenues. It is well known that under standard partial-equilibrium or one-industry general-equilibrium trade policy models, there are no gains from erecting NRTBs. However, in our multi-industry general equilibrium framework, we find that NRTBs could improve the Home country’s welfare at the expense of the Foreign country.

We model NRTBs as wasteful iceberg transport costs that do not generate revenues for the government or utility for consumers. The government’s problem is to choose product-specific NRTBs, $\{\tau_k\}$ to maximize its welfare. We find that optimal NRTBs are

\[\begin{align*}
\tau_k & \geq 0, \\
\tau & \geq 0, \\
\tau & \in A.
\end{align*}\]

\[\text{subject to} \quad (0, 0, \tau; w) \in A.\]
from Foreign is sufficiently elastic, and (ii) zero in other industries (see Appendix C). Moreover, if $\varepsilon_{fh,k}$ is non-decreasing in $q_{fh,k}$, the optimal NRTB is prohibitively large in high-$\varepsilon$ industries.\footnote{The condition that $\partial \varepsilon_{fh,k}/\partial q_{fh,k} \geq 0$ is widely-known as Marshall’s Second Law of Demand, and is satisfied in an important class of trade models.} That is,

$$
\tau_{k}^* = \begin{cases} 
\infty & \text{if } \varepsilon_{fh,k} < 1 + \bar{\varepsilon}_h + \bar{\varepsilon}_f \\
0 & \text{if } \varepsilon_{fh,k} > 1 + \bar{\varepsilon}_h + \bar{\varepsilon}_f
\end{cases}
$$

where, as earlier, $\bar{\varepsilon}_{ji}$ denotes the elasticity of labor demand, which is weighted average of industry-level demand elasticities. The above formula indicates that in a single-industry model, the optimal NRTB is always zero since $\varepsilon_{fh,k} = \bar{\varepsilon}_f > 1 + \bar{\varepsilon}_h + \bar{\varepsilon}_h$. Similarly, the optimal NRTB will be zero in all industries if wages where assumed to be invariant to policy as in partial equilibrium models. But once we accommodate general equilibrium wage effects and allow for multiple industries, there is an incentive for setting NRTBs, which is summarizes by the following proposition.

**Proposition 6.** Absent revenue-raising taxes, it is optimal to impose a prohibitively high NRTB on imports with sufficiently high demand elasticities. The optimal NRTB on all other imports and exports will be zero.\footnote{Proof is provided in Appendix C.}

There is a simple logic behind the above result. Erecting NRTBs on a subset of products reduces the Home consumers’ welfare with respect to those products, but improves Home’s terms of trade with respect to all other imports. In high-$\varepsilon$ industries, the gains from importing Foreign varieties are relatively small, because a high $\varepsilon$ indicates strong substitutability between imported and domestic varieties. On the other hand, restricting imports in high-$\varepsilon$ industries can reduce foreign wages and, as a result, the price of imports in all other product categories. These general equilibrium wage effects can be large enough to offset the modest welfare loss due to a price increase in the NRTB-restricted industries.

In the widely-used multi-industry gravity model (outlined in Section 4.1), we show that the optimal NRTB is positive for industries that are sufficiently large (high-$\varepsilon_{h,k}$), sufficiently productive (high-$\lambda_{hh,k}$), and feature a sufficiently large trade
elasticity, $\epsilon_k$.\textsuperscript{31} Using trade elasticities estimated by Caliendo and Parro (2014), our quantitative analysis in Section 7 indicates that these conditions are typically satisfied in the ‘Mining’, ‘Petroleum’, and ‘Wood’ industries.

### 6.4 Interdependence with Multiple Countries

The multi-country version of our framework features a margin of policy interdependence that is absent in the two-country model. This margin is closely related to the Most-Favored Nation (MFN) clause of the WTO, which requires that member countries apply their import tariffs non-discriminatorily against all other WTO members. Even in the absence of the MFN clause, governments may find it administratively convenient to impose tariffs that are independent of national origin.

To present this particular type of interdependence, recall that unconstrained optimal import tariffs are uniform across products of the same exporter (Proposition 1). However, as shown in Appendix D, when countries are bound to MFN tariffs, it becomes optimal to vary the tariff rate across products. The intuition is that MFN tariffs do not allow Home to discriminate among various exporters based on its bilateral export market power. As a result, it is optimal for Home to vary its import tariff rate across industries based on its average bilateral market power in that industry. More specifically, optimal MFN tariffs should be higher in industries where imports are predominantly from exporters towards which Home possesses relatively more market power.

### 7 Quantitative Analysis

In this section, we fit our theoretical model to industry-level trade and production data. In doing so, we pursue two basic objectives. First, we want to quantify the gains from optimal policy and the degree of policy interdependence across industries. Second, we wish to highlight how our analytical formulas streamline the computational analysis of trade policy.

\textsuperscript{31}In theory, one can easily construct an example where industry $k$ is subject to a positive NRTB. To this end, consider the $K$-industry gravity model in Section 4.1. If $e_{ij}$’s are uniform across industries and Home and Foreign are symmetric, then $r_{ji/k} = r_{ji}/K$ for all $k$, and for the NRTB in industry $k$ to be positive it suffices that $\epsilon_k > 2\sum g \epsilon_g/K$.  

39
**Data.** Our main data source is the 2012 edition of the World Input-Output Database (WIOD, Timmer et al. 2012). The WIOD database covers 35 industries and 40 countries, which account for more than 85% of world GDP, plus an aggregate of the rest of the world. The countries in the sample include all 27 members of the European Union and 13 other major economies, namely, Australia, Brazil, Canada, China, India, Indonesia, Japan, Mexico, Russia, South Korea, Taiwan, Turkey, and the United States. The 35 industries in WIOD database include 15 tradable industries and 20 service-related industries—see Tables 2 and 3 for a thorough description of countries and industries used in the analysis.

For each country in the sample, the WIOD reports total national output, \( w_i L_i \) as well as total national expenditure, \( Y_i \), and allows for the construction of industry-level output shares, \( r_{ji,k} \), and expenditure shares, \( \lambda_{ji,k} \). Moreover, the data identifies the global input-output matrix, \( A \). We complement the WIOD with industry-level trade elasticity estimates from Caliendo and Parro (2014).

To be consistent with our analytical framework, we restructure the WIOD database in two dimensions. First, we merge all the service industries into a single aggregated non-traded sector. Second, we purge the data from trade imbalances. In this process, we closely follow the methodology in Costinot and Rodríguez-Clare (2014), who apply Dekle et al.’s (2007) hat-algebra methodology to purge the 2008 edition of the WIOD.\(^{32}\)

**Competitive Gravity Model without IO Linkages.** We first consider the two-country competitive gravity model outlined in Section 4.1. As in Costinot and Rodríguez-Clare (2014), we assume that the status-quo is free trade. That is, the WIOD data describes a global economy where countries set near-zero trade taxes. The standard Ricardian gravity model features a Cobb-Douglas utility aggregator across industries and a CES demand structure within industries. In that case, \( V (Y_i, \hat{P}_i) = Y_i / P_i \), where the aggregate price index in economy \( i \in \mathbb{C} \) is given by

\[
P_i = \prod_{k \in K} \left( \sum_{j \in \mathbb{C}} \chi_{ji,k} \hat{p}_{ji,k}^{1 - \varepsilon_i / \varepsilon_k} \right)^{-\varepsilon_i / \varepsilon_k}, \tag{19}
\]

\(^{32}\)A similar approach is also applied by Ossa (2014) to eliminate trade imbalances from the GTAP database.
with \( \chi_{ji,k} \) being a structural parameter that accounts for productivity levels and trade barriers. Accordingly, bilateral expenditure shares given by

\[
\lambda_{ji,k} = \frac{\chi_{ji,k} \hat{P}_{ji,k}^{-e_k}}{\sum_{\ell \in C} \chi_{i\ell,k} \hat{P}_{i\ell,k}^{-e_k}} e_{i,k} \quad \forall j, i \in C, \tag{20}
\]

where \( e_{i,k} = \sum_j \lambda_{ji,k} \) denotes country \( i \)'s total expenditure share on industry \( k \). The demand elasticities are also equal to \( \epsilon_{ji,k} = -1 - \epsilon_k \lambda_{ii,k}/e_{i,k} \), \( \epsilon_{ji,k}^* = \epsilon_k \lambda_{ii,k}/e_{i,k} \), and \( \epsilon_{ji,k}^g = 0 \) if \( g \neq k \). Given the above demand structure, we can use the hat-algebra methodology in combination with Theorem 1 to solve for Home’s optimal trade taxes, \( t_{fh}^* \) and \( x_{hf}^* \). Importantly, this approach eliminates the need to (i) estimate the structural parameters \( \chi_{ji,k} \), and (ii) perform a global optimization problem. Instead, \( t_{fh}^* \) and \( x_{hf}^* \) can be determined as a function of observables by merely solving a system of non-linear equations. The following proposition states this claim formally, using the hat-algebra notation that \( \hat{z} \equiv z'/z \) denotes the ratio of counterfactual-to-factual value for any variable \( z \).

**Proposition 7.** Suppose the observed data is generated by the Ricardian model outlined in Section 4 and functional forms 19 and 20. Then, the optimal trade tax can be fully characterized by solving the following system of equations

\[
\begin{cases}
(1 + x_{hf,k}) / (1 + \hat{t}) = 1 + 1/\epsilon_k \hat{\lambda}_{ff,k} \lambda_{ff,k}; & t_{fh,k} = \hat{t} \\
x_{fh,k} = 0, & t_{hf,k} = 0 \\
\hat{\lambda}_{fh,k} = \left[ \hat{R}_j (1 + t_{ji,k}) (1 + x_{ji,k}) \right]^{-e_k} \hat{P}_{i,k}^{\epsilon_k} \\
\hat{P}_{i,k}^{\epsilon_k} = \sum_j \left[ \hat{R}_j (1 + t_{ji,k}) (1 + x_{ji,k}) \right]^{-e_k} \lambda_{ji,k} \\
\hat{R}_i \hat{R}_i = \sum_k \sum_j \left[ \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{Y}_j / (1 + x_{ij,k}) (1 + t_{ij,k}) \right] \\
\hat{Y}_i Y_i = \hat{R}_i \hat{R}_i + \sum_k \sum_j \left( \frac{t_{ij,k}}{1 + t_{ij,k}} \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{Y}_j + \frac{x_{ij,k}}{1 + x_{ij,k}} \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{R}_j \hat{R}_j \right)
\end{cases}
\]

in terms of \( \{x_{hf,k}\}, \{\hat{\lambda}_{ji,k}\}, \{\hat{P}_{i,k}\}, \{\hat{R}_i\}, \) and \( \{\hat{Y}_i\} \), as a function of (i) observed expenditure shares, \( \{\lambda_{ji,k}\}; \) (ii) observed national output/income levels, \( \hat{R}_i = Y_i = w_i L_i \); and (iii) industry-level trade elasticities, \( \{\epsilon_k\} \).\(^{33}\)

To put the above proposition in perspective, it is worth comparing it to the stan-

\(^{33}\)To handle the multiplicity of optimal trade taxes, we choose a value of \( \hat{t} = 0.59 \) based on the average Smoot–Hawley tariff rates.
dard approach highlighted in Costinot and Rodríguez-Clare (2014). By appealing to the hat-algebra methodology, the standard approach eliminates the need to estimate the structural parameters \( \chi_{ji,k} \). However, it involves solving a constrained global optimization over \( 2K + 4 \) choice variables. In comparison, Proposition 7 reduces the computational task to simply solving a system of \( 4K + 4 \) equations and unknowns.

**Competitive Gravity Model with IO Linkages** Using the structure of the WIOD, we also analyze a Ricardian gravity model with IO linkages. To this end, we assume a Cobb-Douglas production function, so that the elements of the global IO matrix become invariant to trade taxes. In that case, using Theorem 2 and Lemma 2, we can produce an analog of Proposition 7 in the presence of IO linkages. Doing so, allows us to solve for the optimal trade taxes without conducting a constrained global optimization. In the interest of brevity, the details of this proposition are relegated to Appendix G.

### 7.1 The Gains from Optimal Policy

After solving for optimal trade taxes, we can compute the gains from policy as \( \hat{W}_h = \hat{Y}_h / \left( \prod_k \hat{P}_{h,k}^{\rho_{h,k}} \right) \). We do so for 14 major economies—in each case we calculate the optimal policy for that country with respect to an aggregate of the rest of the world. To put these gains in perspective, we also compare them to the gains from optimal policy when the Home government is restricted to (i) only import taxes, and (ii) only NRTBs. The computed gains from policy are reported in Table 1, both in the absence and in the presence of IO linkages.

On average, the gains from unilateral export policy dominate those of import policy by a factor of 3. The gains from unilateral NRTBs are relatively modest compared to both export and import taxes. For countries like Mexico and Taiwan, however, the gains from NRTBs are non-trivial and stand around 0.11-0.12%.

A notable observation, here, is that the welfare gains from trade policy are considerably larger in the presence of IO linkages. This outcome is perhaps related to the extraterritorial taxing power, identified in Section 5.1, whereby the Home economy generates revenue by effectively taxing transactions between Foreign suppliers and consumers.
Table 1: The gains from trade policy: % change in real GDP

<table>
<thead>
<tr>
<th>Country</th>
<th>Export tax without IO Linkages</th>
<th>Import tax without IO Linkages</th>
<th>NRTB without IO Linkages</th>
<th>Export tax with IO Linkages</th>
<th>Import tax with IO Linkages</th>
<th>NRTB with IO Linkages</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.30%</td>
<td>0.17%</td>
<td>0.03%</td>
<td>0.59%</td>
<td>0.30%</td>
<td>0.00%</td>
</tr>
<tr>
<td>E.U.</td>
<td>0.91%</td>
<td>0.47%</td>
<td>0.04%</td>
<td>1.54%</td>
<td>0.89%</td>
<td>0.05%</td>
</tr>
<tr>
<td>BRA</td>
<td>0.56%</td>
<td>0.25%</td>
<td>0.02%</td>
<td>1.00%</td>
<td>0.45%</td>
<td>0.03%</td>
</tr>
<tr>
<td>CAN</td>
<td>1.39%</td>
<td>0.37%</td>
<td>0.04%</td>
<td>2.71%</td>
<td>0.71%</td>
<td>0.03%</td>
</tr>
<tr>
<td>CHN</td>
<td>0.61%</td>
<td>0.35%</td>
<td>0.02%</td>
<td>1.71%</td>
<td>1.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td>IDN</td>
<td>0.48%</td>
<td>0.22%</td>
<td>0.02%</td>
<td>1.17%</td>
<td>0.50%</td>
<td>0.00%</td>
</tr>
<tr>
<td>IND</td>
<td>0.43%</td>
<td>0.27%</td>
<td>0.01%</td>
<td>1.20%</td>
<td>0.53%</td>
<td>0.01%</td>
</tr>
<tr>
<td>JPN</td>
<td>1.00%</td>
<td>0.41%</td>
<td>0.03%</td>
<td>1.58%</td>
<td>0.71%</td>
<td>0.05%</td>
</tr>
<tr>
<td>KOR</td>
<td>2.56%</td>
<td>0.72%</td>
<td>0.08%</td>
<td>4.01%</td>
<td>1.88%</td>
<td>0.12%</td>
</tr>
<tr>
<td>MEX</td>
<td>2.19%</td>
<td>0.51%</td>
<td>0.11%</td>
<td>1.82%</td>
<td>0.78%</td>
<td>0.05%</td>
</tr>
<tr>
<td>RUS</td>
<td>0.40%</td>
<td>0.16%</td>
<td>0.02%</td>
<td>0.67%</td>
<td>0.29%</td>
<td>0.03%</td>
</tr>
<tr>
<td>TUR</td>
<td>1.26%</td>
<td>0.34%</td>
<td>0.01%</td>
<td>1.58%</td>
<td>0.22%</td>
<td>0.01%</td>
</tr>
<tr>
<td>TWN</td>
<td>1.57%</td>
<td>0.55%</td>
<td>0.12%</td>
<td>1.97%</td>
<td>1.16%</td>
<td>0.07%</td>
</tr>
<tr>
<td>USA</td>
<td>0.56%</td>
<td>0.23%</td>
<td>0.01%</td>
<td>1.14%</td>
<td>0.39%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Average</td>
<td>0.99%</td>
<td>0.35%</td>
<td>0.04%</td>
<td>1.59%</td>
<td>0.69%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

7.2 Quantifying the Degree of Policy Interdependencies

Next, we use our quantitative model to determine how sizable trade policy interdependencies are in practice. To this end, we start from the unconstrained optimal policy equilibrium calculated in the previous step, and sequentially introduce a set of new restrictions on Home’s policy space. Namely,

i. In the first sequence, we suppose export taxes are restricted in all industries but import taxes remain unrestricted.

ii. In the second sequence, we suppose export taxes remain restricted and import taxes are also restricted in a subset of industries.

In each sequence, we compute how the partial restriction affects the Home government’s optimal choice with respect to unrestricted policies. We conduct our analysis twice: once where the United States (US) is treated as the Home country and the rest of the world is treated as Foreign; and another where the European Union (EU) is treated as the Home country.

The results displayed in Figure 1 indicate that, without any restrictions, the optimal policy involves a uniform tariff and an export tax that varies primarily
with the industry-level trade elasticity. After export tax-cum-subsidies are fully restricted, it is optimal for both the US and EU to voluntarily lower their import taxes in all industries. More importantly, this partial restriction on export policy lowers the US and EU’s effective rate of protection.

More interesting is perhaps the second sequence where import taxes are restricted in a subset of (high trade elasticity) industries. In this case, partial restriction lowers the optimal import tax in restricted industries from around 30% to 10% in both cases. From the perspective of Proposition 5, these tariff complementarity effects are driven solely by a reduction in the wage-driven term, \( \bar{t} \). While cross-substitutability between industries can magnify the degree of tariff complementarity, they are absent here due to the Cobb-Douglas assumption. So, our current analysis presents a lower bound on the degree of tariff complementarity. Finally, as illustrated in 2, introducing input-output linkages preserves and even magnifies the interdependence patterns highlighted above.

### 7.3 Optimal Policy with Multiple Countries

Finally, we analyze the case where the Home country can impose differential trade taxes on multiple Foreign countries. While we do not have an explicit formula for the optimal tax in the multi-country case, we have proven that optimal tariffs are uniform on exports from a given country. This result streamlines the computational analysis quite significantly; but we still have to conduct a constrained global optimization to determine the optimal policy.

Our multi-country analysis features 15 countries: Australia, Brazil, China, Indonesia, India, Japan, Korea, Mexico, Russia, Taiwan, Turkey, the European Union, the United States, and an aggregate of the rest of the world. We conduct two separate analyses. First, we treat the United States as the Home country and compute its optimal trade taxes with respect to the remaining 14 countries. Second, we treat the European Union as the Home country.

Figure 3 displays the multi-country optimal tax levels and compares them those produced using the two-country benchmark—each dot in the graph corresponds to an optimal tax rate for one particular import/export partner.

---

\(^{34}\)When all instruments are available, the value of \( \bar{t} \) is redundant. This, leads to an indeterminacy of optimal policy, which we handle by setting \( \bar{t} = 0 \).
Figure 1: The interdependence of polices without input-output linkages

United States

European Union

Export Taxes Restricted

Import Taxes Partially Restricted

Import Tariff

Export Tax
Figure 2: The interdependence of polices with input-output linkages

- **Import Tariff**
- **Export Tax**

**United States**

Unconstrained optimal policy

**European Union**

Unconstrained optimal policy

Export Taxes Restricted

Import Taxes Partially Restricted

Export Taxes Restricted

Import Taxes Partially Restricted
Evidently, optimal trade taxes discriminate between various partners, but only to a modest degree. There is a simple intuition behind this result. Given the factual trade levels, even the United Stated and the European Union are relatively small economies compared to the rest of the world.\textsuperscript{35} As a result, their optimal policy with respect to other countries is determined primarily by the industry-level trade elasticities that are the same across all exporters and importers. Another takeaway from this comparison is that perhaps the two-country model of optimal policy provides a reasonable approximation for optimal policy in the multi-country setup.

\textsuperscript{35}Note that that in our setup, due to presence of product differentiation, even a small open economy possess export market power. So, optimal tariffs are significant even though they do not significantly discriminate between different suppliers. See Alvarez and Lucas (2007) for a similar argument.
8 Concluding Remarks

We provide a general equilibrium analysis of optimal trade policy and trade policy interdependence. Our framework accommodates input-output linkages, specific factors of production, cross-demand effects, and multiple countries. To conclude, we highlight some of the important implications of our results.

First, we identify \( (i) \) reduced-form demand elasticities and \( (ii) \) trade tax pass-throughs (net of wage effects), as the two sufficient statistics that characterize the optimal trade tax schedule. Importantly, both of these statistics can be estimated using standard customs data and input-output matrixes. Relatedly, we find that optimal trade taxes are uniform across industries if their pass-through onto Foreign prices is zero, net of wage effects. Otherwise, optimal trade taxes vary with industry-level demand and supply characteristics.

Second, we highlight the importance of policy interdependence (i.e., complementarity or substitutability of various policy instruments) in analyzing trade agreements. These interdependencies are especially important for the optimal sequencing of trade liberalization. Our theory suggests that an optimal sequence should first restrict export taxes/subsidies, then import taxes in a subset of industries, and finally all import taxes and non-revenue import barriers.

Third, we offer novel insights about the implications of IO linkages for optimal trade policy. Most importantly, we show that input-output linkages present the governments with a new international externality that we called “extraterritorial taxing power.” That is, by taxing re-exported intermediate goods, governments can tax transactions among foreign entities, which involve components produced and eventually consumed abroad. In line with this observation, we showed quantitatively that accounting for IO linkages increases the unilateral gains from protection substantially. The flip-side of this finding is that multilateral gains from trade agreements could be much larger in the presence of IO linkages, which justifies a particular attention to intermediate-input trade in multilateral trade negotiations.

Finally, our general equilibrium analysis of policy across multiple industries could guide future empirical studies of trade policy. In standard empirical works on trade policy, researchers usually use comparative static results from a partial-equilibrium model to explain cross-industry variation in policies. Interpreting comparative static results as cross-industry variation is less than satisfactory be-
cause it ignores cross-industry linkages, which are quantitatively important in the data. Our theory, by comparison, directly characterizes the cross-industry variation in optimal trade policies, which could be used as a guide for future empirical work in this area.

References


Appendix

A Optimal Trade Taxes in a Ricardian Economy

The optimal trade tax problem of the home country can be formulated as

$$\max_{(t, x, \tau; w) \in A} W_h (t, x, \tau; w),$$

where the set of feasible wage-policy combinations, $A$, is given by a special case of D2, where $p_{ji,k} = a_{ji,k}w_j$ and $\Pi_i = 0$ for all $i$ and $j \in C$. Since in the presence of revenue-raising taxes the optimal NRTB is zero, we drop $\tau$ when referencing a feasible wage-policy combination in our proof. That is, we express all equilibrium outcomes in terms of a revenue-raising wage-policy combination, $(t, x; w)$, with Foreign labor assigned as the numeraire (i.e., $w_f = 1$).

**Step 1: Deriving the F.O.C. for Import Taxes.** We can express the F.O.C. with respect to the tariff in sector $k$ as follows

$$\frac{dW_h (t, x; w)}{d (1 + t_k)} = \frac{\partial V_h (Y_h, \tilde{p}_h)}{\partial Y_h} \left[ \frac{\partial Y_h}{\partial (1 + t_k)} + \frac{\partial Y_h}{\partial w_h} \frac{d w_h}{d (1 + t_k)} \right] + \sum_{g} \sum_{j = f, h} \left( \frac{\partial V_h (Y_h, \tilde{p}_h)}{\partial \tilde{p}_{jh, g}} \left[ \frac{\partial \tilde{p}_{jh, g}}{\partial (1 + t_k)} + \frac{\partial \tilde{p}_{jh, g}}{\partial w_h} \frac{d w_h}{d (1 + t_k)} \right] \right) = 0,$$

Noting (i) the zero cross-passsthrough of taxes onto consumer prices, i.e., $\partial \tilde{p}_{ji, g}/\partial (1 + t_k) = 0$, if $ji, g \neq fh, k$, and (ii) the complete passthrough of taxes onto own prices, i.e., $\partial \tilde{p}_{fh, k}/\partial (1 + t_k) = 1$; we can simplify the above condition as

$$\frac{dW_h (t, x; w)}{d (1 + t_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial Y_h}{\partial (1 + t_k)} + \frac{\partial V_h}{\partial \tilde{p}_{fh, k}} \frac{\partial \tilde{p}_{fh, k}}{\partial (1 + t_k)} + \frac{\partial V_h}{\partial w_h} \frac{d w_h}{d (1 + t_k)} \right\} = 0.$$
where \( \frac{\partial V_h}{\partial w_h} = \frac{\partial Y_h}{\partial w_h} \partial Y_h + \sum g \sum_{j=h,f} \frac{\partial V_h}{\partial p_{fh,g}} \frac{\partial p_{fh,g}}{\partial w_h} \) In the above equation, \( \frac{\partial Y_h}{\partial (1+t_k)} \) can be expressed as:

\[
\frac{\partial Y_h(t, x; w)}{\partial (1+t_k)} = \frac{\partial}{\partial (1+t_k)} \left\{ \omega_i L_i + \sum g \left[ t_g p_{fh,g} q_{fh,g} + x_g p_{fh,g} q_{fh,g} \right] \right\} = p_{fh,k} q_{fh,k} + \sum g \left( t_g p_{fh,g} \frac{\partial q_{fh,g}}{\partial p_{fh,k}} \right) + \sum g \left( t_g p_{fh,g} \frac{\partial q_{fh,g}}{\partial Y_h} \right) \frac{\partial Y_h}{\partial (1+t_k)}.
\]

(21)

Note that due to the Lerner symmetry we can set \( dw_h / d (1+t_k) \) to zero to identify one of the multiple optimal policy combinations. However, to thoroughly demonstrate this point and to also be in sync with subsequent proofs, we formally derive and substitute this term. To this end, we apply the implicit function theorem to the balance trade condition, \( D_h(t, x; w) = \sum g \left[ (1 + x_g) p_{fh,g} q_{fh,g} - p_{fh,g} q_{fh,g} \right] = 0 \), which yields the following:

\[
\frac{dw_h}{d (1+t_k)} = -\frac{\partial D_h(t, x; w)}{\partial D_h(t, x; w) / \partial w_h} = -\sum g \left[ p_{fh,g} \frac{\partial q_{fh,g}}{\partial p_{fh,k}} \frac{\partial p_{fh,k}}{\partial Y_h} \right] \frac{\partial Y_h}{\partial (1+t_k)}.
\]

Plugging the expressions for \( \frac{\partial Y_h}{\partial (1+t_k)} \) and \( \frac{dw_h}{d (1+t_k)} \) back into the the F.O.C. implies the following optimality condition:

\[
\frac{dW_h(t, x; w)}{d (1+t_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ t_k p_{fh,k} \frac{\partial q_{fh,k}}{\partial p_{fh,k}} \frac{\partial p_{fh,k}}{\partial (1+t_k)} + p_{fh,k} q_{fh,k} + \sum g \left( t_g p_{fh,g} \frac{\partial q_{fh,g}}{\partial p_{fh,k}} \right) \frac{\partial p_{fh,k}}{\partial (1+t_k)} \right\} + \sum g \left( t_g p_{fh,g} \frac{\partial q_{fh,g}}{\partial Y_h} \right) \frac{\partial Y_h}{\partial (1+t_k)} \right\} = 0.
\]

Applying Roy’s identity, \( (\partial V_h / \partial \bar{p}_{fh,k}) / (\partial V_h / \partial Y_h) = -q_{fh,k} \); defining \( \bar{\tau} \equiv \left( \frac{\partial V_h}{\partial w_h} / \frac{\partial V_h}{\partial Y_h} \right) / \frac{\partial D_h}{\partial w_h} > 0 \); and noting that \( \partial \ln \bar{p}_{fh,k} / \partial \ln (1+t_k) = 1 \), we can further simplify the F.O.C. as

\[
\sum g \left[ (\bar{\tau} - t_g) p_{fh,g} q_{fh,g} \left( \frac{\partial \ln q_{fh,g}}{\partial \ln Y_h} \frac{\partial \ln Y_h}{\partial \ln (1+t_k)} + \frac{\partial \ln q_{fh,g}}{\partial \ln \bar{p}_{fh,k}} \right) \right] = 0.
\]

Recalling our Definition D1 that \( i \epsilon_{fh,k} \equiv \partial \ln q_{fh,k} / \partial \ln \bar{p}_{fh,k} \), \( i \epsilon_{fh,g} \equiv \partial \ln q_{fh,k} / \partial \ln \bar{p}_{fh,g} \), and \( \eta_{fh,k} \equiv \partial \ln q_{fh,k} / \partial \ln Y_h \), we can further simplify the F.O.C.

56
as a function of reduced-form elasticities,

\[ \sum_{g} \left[ \left(1 - \frac{1 + \tau}{1 + t_g} \right) \left( \varepsilon_{fh,k} + \eta_{fh,g} \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} \right) \lambda_{fh,g} \right] = 0, \text{ for all } k. \]

The trivial solution to the above system of K first-order conditions is \( t_k = \tau \) for all \( k \). We can also characterize conditions that ensure that the trivial solution, \( t_k = \tau_k \), is the unique solution. To this end, define the \( K \times K \) matrix \( B \equiv \left[ \lambda_{fh,g} \left( \varepsilon_{fh,k} + \eta_{fh,g} \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} \right) \right]_{k,g} \), and the \( K \times 1 \) vector \( \omega \) as \( \omega \equiv \left[ 1 - \frac{1 + \tau}{1 + t_k} \right]_{k} \). The system of F.O.C.s can, thus, be expressed as \( B \omega = 0 \). For \( \omega = 0 \) to be the unique (and trivial) solution to \( B \omega = 0 \), it suffices that \( \det B \neq 0 \).

The second-order condition for optimality is also satisfied provided that \( \varepsilon_{fh,k} < 0 \), \( \varepsilon_{fh,k} > 0 \), and \( \eta_{fh,g} > 0 \). Specifically, under the aforementioned sign of demand elasticities, one can easily verify that (i) if \( t_k < \bar{\tau} \mid t_g = \bar{\tau}, \forall g \neq k \) then,

\[ \frac{\partial W_h(t, x; \omega)}{\partial (1 + t_k)} = \sum_{g} \left( (t_g - \tau) p_{fh,g} q_{fh,g} \left[ \varepsilon_{fh,k} + \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} \eta_{fh,g} \right] \right) > 0 \]

and (ii) if \( t_k > \bar{\tau} \mid t_g = \bar{\tau}, \forall g \neq k \) then

\[ \frac{\partial W_h(t, x; \omega)}{\partial (1 + t_k)} = \sum_{g} \left( (t_g - \bar{\tau}) p_{fh,g} q_{fh,g} \left[ \varepsilon_{fh,k} + \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} \eta_{fh,g} \right] \right) < 0. \]

Hence, the solution \( t_k = \tau \) for all \( k \), is also a welfare-maximizing solution to the F.O.C.

Note that the optimal import tax is \( t_k = \bar{\tau} \) for all \( k \), irrespective of the applied export taxes. When export taxes are available and set optimally, the value of \( \bar{\tau} \) is irrelevant to the multiplicity of optimal tax schedules. However, when export taxes are unavailable or set sub-optimally, the exact value \( \bar{\tau} \) is relevant. We can derive \( \bar{\tau} \), in these circumstances, along the following lines

\[
\bar{\tau} = \frac{\partial V_h}{\partial w_h} + \frac{\partial V_h}{\partial y_h} + \sum_k \left( \frac{\partial V_h}{\partial p_{fh,k}} \frac{\partial p_{hh,k}}{\partial w_h} \right) = \frac{\partial V_h}{\partial w_h} - \sum_k \left( \frac{\partial V_h}{\partial p_{fh,k}} \frac{\partial V_h}{\partial y_h} \right) \tag{57}
\]

\[
= \frac{\partial (\sum_k p_{fh,k} q_{fh,k})}{\partial w_h} + \frac{\partial (\sum_k p_{fh,k} q_{fh,k})}{\partial y_h} - \sum_k \frac{\partial V_h}{\partial w_h} \tag{58}
\]

where the second line follows for Roy’s identity. Noting the \( p_{ji,k} q_{ji,k} = w_j \mathcal{L}_{ji,k}(w; t, x) \)
\[ \sum_{q} p_{j_i, q_{j_i, k}} = w_{j_i} L_{j_i} (w; t, x) \] (by the definition of labor demand), the above expression can be reformulated as

\[ \bar{\tau} = \frac{w_h L_{hf}}{1 + \sum_{q} \left( x_{j_i} \bar{p}_{j_i, k} (1 + \hat{\varepsilon}_{hf, k}) \right)} \]

where the last line follows from Definition D4 that

\[ \hat{\varepsilon}_{hf} \equiv \frac{\partial \ln L_{hf} (w; t, x)}{\partial \ln w_h} = \sum_{k} \frac{r_{hf, k}}{r_{hf}} \hat{\varepsilon}_{hf, k} \]

denotes the elasticity of Foreign’s demand for Home’s labor, with \( r_{hf, k} \equiv \bar{p}_{j_i, k} q_{j_i, k} / w_h L_h \) being the share of Home’s (non-tax) revenue generated from sales to Foreign in industry \( k \). Rearranging the above equation, expresses the optimal import tax for any given vector of export taxes/subsidies:

\[ 1 + \bar{\tau}^* = \frac{\hat{\varepsilon}_{hf} - \sum_{k} \left( x_{j_i} \frac{r_{hf, k}}{r_{hf}} \left( 1 + \hat{\varepsilon}_{hf, k} \right) \right)}{1 + \hat{\varepsilon}_{hf}}. \] (22)

When export taxes are restricted, the above formula reduces to that specified by Proposition 4: \( 1 + \bar{\tau}^* = \hat{\varepsilon}_{hf} / (1 + \hat{\varepsilon}_{hf}) \).

**Step 2: Deriving the F.O.C. for Export Taxes.** Noting our notation for consumer prices that \( \bar{p}_{hf, k} = (1 + x_{k}) p_{hf, k} \), the F.O.C. with respect to the export tax in sector \( k \) can be expressed as follows

\[ \frac{d W_h (t, x; w)}{d (1 + x_{k})} = \frac{\partial V_h}{\partial Y_h} \left[ \frac{\partial y_h}{\partial (1 + x_{k})} + \frac{\partial Y_h}{\partial w_h} \frac{d w_h}{d (1 + x_{k})} \right] + \sum_{g} \sum_{j=f} \left[ \frac{\partial V_h}{\partial \bar{p}_{j, g}} \frac{\partial \bar{p}_{j, g}}{\partial (1 + x_{k})} + \frac{\partial V_h}{\partial \bar{p}_{j, g}} \frac{\partial \bar{p}_{j, g}}{\partial w_h} \frac{d w_h}{d (1 + x_{k})} \right] = 0 \]

in the above expression, (i) \( \frac{\partial \bar{p}_{j, g}}{\partial (1 + x_{k})} = 0 \) for all \( g \) because the effect of export taxes on Home prices are only through their effects on wages, and (ii) \( \frac{\partial \bar{p}_{j, g}}{\partial w_h} = 0 \) for all \( g \), by normalization of foreign wage to 1. Plugging these values in to the above equation, yields the
following simplified F.O.C.,

\[
\frac{dW_h(t, x; w)}{d \ln (1 + x_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial Y_h}{\partial \ln (1 + x_k)} + \left( \frac{\partial V_h}{\partial \ln (1 + x_k)} \right) \frac{dw_h}{\partial \ln (1 + x_k)} \right\} = 0 \tag{23}
\]

where, as before, \( \frac{\partial V_h}{\partial w_h} = \frac{\partial Y_h}{\partial w_h} + \sum_g \sum_{j=1}^{\tilde{N}} \frac{\partial V_h}{\partial \tilde{p}_{h,j}} \frac{\partial \tilde{p}_{h,j}}{\partial w_h} \). The term \( \frac{\partial Y_h}{\partial \ln (1 + x_k)} \) in Equation 23 can be calculated as,

\[
\frac{\partial Y_h(t, x; w)}{\partial \ln (1 + x_k)} = \frac{\partial}{\partial (1 + x_k)} \left\{ \omega_1 + \sum_g \left( t_g p_{f,h,g} q_{f,h,g} + x_g p_{h,g} q_{h,g} \right) \right\} = \tilde{p}_{h, k} q_{h, k} + \sum_g \left( t_g p_{f,h,g} q_{f,h,g} \right) \frac{\partial \ln q_{h,g}}{\partial \ln Y_h} \frac{\partial \ln \tilde{p}_{h,k}}{\partial \ln (1 + x_k)} + \sum_g \left( t_g p_{f,h,g} q_{f,h,g} \right) \frac{\partial \ln q_{h,g}}{\partial \ln Y_h} \frac{\partial \ln \tilde{p}_{h,k}}{\partial \ln (1 + x_k)}. \tag{24}
\]

Also, as with the case of import taxes, an expression for \( \frac{dw_h}{d \ln (1 + x_k)} \) can be derived by applying the implicit function theorem to the balance trade condition:

\[
\frac{dw_h}{d \ln (1 + x_k)} = \frac{\partial D_h(t, x; w)}{\partial Y_h} \frac{\partial \ln (1 + x_k)}{\partial w_h} = \left( \frac{\partial}{\partial w_h} \right) \frac{\partial \ln \tilde{p}_{h,k} q_{h,k}}{\partial \ln (1 + x_k)} \frac{\partial \ln \tilde{p}_{h,k}}{\partial \ln (1 + x_k)}.
\]

Replacing the expressions for \( \frac{dw_h}{d \ln (1 + x_k)} \) and \( \frac{\partial Y_h}{\partial \ln (1 + x_k)} \) into Equation 23; defining \( \tau = \frac{\partial V_h}{\partial w_h} \), as before; and Noting that \( \partial \ln \tilde{p}_{h,k} / \partial \ln (1 + x_k) = 1 \), the F.O.C. reduces to

\[
\frac{dW_h(t, x; w)}{d \ln (1 + x_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \tilde{p}_{h,k} q_{h,k} + \sum_g \left( x_g p_{h,g} q_{h,g} \frac{\partial \ln q_{h,g}}{\partial \ln \tilde{p}_{h,k}} \right) + \sum_g \left( t_g p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{f,h,g}}{\partial \ln Y_h} \right) \right\} = \left( 1 + x_k \right) \left( 1 + \tau \right) + \sum_g \left( \left[ (1 + x_k) (1 + \tau) - 1 \right] p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{f,h,g}}{\partial \ln \tilde{p}_{h,k}} \right) - \sum_g \left( t_g \tilde{\tau} \right) p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{f,h,g}}{\partial \ln Y_h} \frac{\partial \ln \tilde{p}_{h,k}}{\partial \ln (1 + x_k)} p_{h,k} q_{h,k} = 0
\]

59
Step 3: Solving for the Optimal Trade Tax Combination. Noting that the optimality of import tariffs entails that $t^*_g = \tau \forall g$, the F.O.C. for optimal export policy reduces to:

$$(1 + x_k) (1 + \tau) + \sum_g \left( (1 + x_g) (1 + \bar{\tau}) - 1 \right) \frac{p_{hf,g} q_{hf,g}}{p_{hf,k} q_{hf,k}} \frac{\partial \ln q_{hf,g}}{\partial \ln p_{hf,g}} = 0$$

Recalling Definition D1 that (i) $\epsilon_{hf,k} \equiv \partial \ln q_{hf,k} / \partial \ln p_{hf,k}$, (ii) $\epsilon^h_{hf,g} \equiv \partial \ln q_{hf,g} / \partial \ln p_{hf,g}$, and noting that (iii) $p_{hf,g} q_{hf,g} / p_{hf,k} q_{hf,k} = \frac{1 + x_k \lambda_{hf,g}}{1 + x_g \lambda_{hf,k}}$, the above condition can be written in terms of trade shares and reduced-form elasticities as follows:

$$(1 + x_k) (1 + \tau) \left( 1 + \epsilon_{hf,k} \right) + \sum_{g \neq k} \left( 1 - \frac{1}{(1 + x_g) (1 + \bar{\tau})} \right) \frac{\lambda_{hf,g} \epsilon_{hf,k}}{\lambda_{hf,k} \epsilon_{hf,g}^h} = \epsilon_{hf,k}. \quad (25)$$

Defining

$$\xi_{hf,k} \equiv \sum_{g \neq k} \left[ 1 - \frac{1}{(1 + x_g) (1 + \bar{\tau})} \right] \frac{\lambda_{hf,g} \epsilon_{hf,k}}{\lambda_{hf,k} \epsilon_{hf,g}^h}$$

Equation 25 yields the following formula for the optimal import and export taxes

$$(1 + x^*_k) (1 + \tau) = \frac{\epsilon_{hf,k}}{1 + \epsilon_{hf,k} + \xi_{hf,k}}. \quad (26)$$

The term $\xi_{hf,k}$ accounts for cross-price elasticity effects. To see this, note that if cross-price elasticities are zero (i.e., $\epsilon^h_{hf,g} = 0$ for all $g \neq k$), then $\xi_{hf,k} = 0$. In order to calculate $\xi_{hf,k}$ based on cross-price elasticities, we can rewrite the equation expressing $\xi_{hf,k}$ as follows:

$$\xi_{hf,k} = -\sum_{g \neq k} \left( \xi_{hf,g} + 1 \right) \frac{\lambda_{hf,g} \epsilon_{hf,k}}{\lambda_{hf,k} \epsilon_{hf,g}}.$$

The vector $[\xi_{hf,k}]_k'$ therefore, solves $\sum_g (\xi_{hf,k} + 1) \lambda_{hf,g} \epsilon_{hf,k} / \lambda_{hf,k} \epsilon_{hf,g} = 1; namely,

$$[\xi_{hf,k}]_{k1} = \left[ \Xi^{-1} - I_K \right] 1_K,$$

where $\Xi \equiv \left[ \lambda_{hf,g} \epsilon_{hf,k} / \lambda_{hf,k} \epsilon_{hf,g} \right]_{k,g}$ is a $K \times K$ matrix and $1 \equiv [1]_k$ is a $K \times 1$ vector.

Since $\sum_g \lambda_{hf,g} \epsilon_{hf,k} = -\lambda_{hf,k} - \sum_g \lambda_{hh,g} \epsilon_{hh,g} < 0$, then $\Xi$ is strictly diagonally dominant. Therefore, $\Xi^{-1}$ exists and is monotone, which ensures that $\xi_{hf,k} + 1 > 0$ (Berman and Plemmons (1994)). That is to say, with zero import tariffs an export subsidy is never optimal.
A.1 The DFS Case

We now show how to derive the optimal tax scheme for the DFS model as a special case of our optimal tax formula. To this end, first note that cross-price elasticities are zero in the DFS model due to the assumption that the expenditure share of each is infinitesimal. Therefore, the optimal trade taxes in the DFS model are given by

$$(1 + x^*_k)(1 + t) = \frac{\epsilon_{hf,k}}{1 + \epsilon_{hf,k}}.$$ 

Assuming CES preferences with elasticity of substitution $\sigma$, the trade elasticity at an interior solution will be given by $\epsilon_{hf,k} = -\sigma$, and the optimal tariff, at an interior solution, will be given by $\frac{\sigma}{\sigma - 1}$. An interior solution is obtained if and only if

$$1 \geq \frac{a_{ff,k}w_f}{a_{hf,k}w_h} \geq \frac{\sigma}{\sigma - 1},$$

that is, if the mark-up that is induced by the tariff is not larger than the ratio of foreign to Home cost of producing goods $k$. At a corner solution, i.e., for $\frac{a_{ff,k}w_f}{a_{hf,k}w_h} < \frac{\sigma}{\sigma - 1}$, the optimal markup takes a limit-pricing form, i.e., $x^*_k = \frac{a_{ff,k}w_f}{a_{hf,k}w_h}$. We can also establish this claim more formally, by deriving the optimal monopoly markup (in the limit) as industries become homogeneous. Namely, by showing that

$$1 + x^*_k = \lim_{\epsilon_k \to \infty} \frac{\epsilon_{hf,k}}{1 + \epsilon_{hf,k}} = \lim_{\epsilon_k \to \infty} 1 + \frac{1}{\epsilon_k \lambda_{ff,k} + (\sigma - 1) \lambda_{ff,k}}.$$ 

To elaborate, our claim is that based on the above equation, if $1 \geq \frac{a_{hf,k}w_h}{a_{ff,k}w_f} \geq \frac{\sigma - 1}{\sigma}$, then

$$1 + x^*_k = \frac{a_{ff,k}w_f}{a_{hf,k}w_h}.$$ 

Noting that $\lim_{x \to 0} x \ln \frac{a}{x} = 0$, to establish the above claim, it suffices to show that $1 + x^*_k = \frac{a_{ff,k}w_f}{a_{hf,k}w_h} \left[ 1 - \frac{1}{\epsilon_k} \ln a_k \epsilon_k \right]$ is a solution to Equation 27, when $a_k \equiv \frac{a_{ff,k}w_f - 1}{a_{hf,k}w_h}$. To establish this claim, notice that conditional trade shares in industry/good $k$ are given by

$$\lambda_{ff,k} = \frac{(a_{ff,k}w_f)^{-\epsilon_k}}{(a_{ff,k}w_f)^{-\epsilon_k} + ([1 + x_k] a_{hf,k}w_h)^{-\epsilon_k}} = \frac{\left( (1 + x_k) a_{hf,k}w_h \right)^{\epsilon_k}}{1 + \left( (1 + x_k) a_{hf,k}w_h \right)^{\epsilon_k}}.$$ 

61
Plugging the above formula and our guess for the export tax, \(1 + x_k^* = \frac{a_{ff,k,wf}}{a_{hf,k,wh}} \left[ 1 - \frac{1}{a_k} \ln a_k \epsilon_k \right]\), into Equation 27 yields the following

\[
\lim_{\epsilon_k \to \infty} 1 + \frac{1}{(\epsilon_k - \sigma + 1) \lambda_{ff,k} + \sigma - 1} = 1 + \lim_{\epsilon_k \to \infty} \frac{1}{\epsilon_k \left( \frac{(1+x_k) a_{hf,k,wh}}{a_{ff,k,wf}} \right)^{\epsilon_k}} + \sigma - 1
\]

\[
= 1 + \lim_{\epsilon_k \to \infty} \frac{1}{\epsilon_k} \left[ 1 - \frac{1}{\epsilon_k} \ln a_k \epsilon_k \right]^{\epsilon_k} = 1 + \frac{1}{a_k} + (\sigma - 1) = \frac{a_{ff,k,wf}}{a_{hf,k,wh}} = \lim_{\epsilon_k \to \infty} 1 + x_k^*,
\]

where the second line uses the fact that \(\lim_{\epsilon \to \infty} \epsilon \left( 1 - \frac{\ln \epsilon}{\epsilon} \right)^{\epsilon} = \frac{1}{a}\). That is, \(1 + x_k^* = \lim_{\epsilon_k \to \infty} \frac{a_{ff,k,wf}}{a_{hf,k,wh}} \left[ 1 - \frac{1}{\epsilon_k} \ln a_k \epsilon_k \right]^{\epsilon_k}\), is the solution implied by Equation 27. Correspondingly, if \(\frac{a_{hf,k,wh}}{a_{ff,k,wf}} < \frac{\sigma - 1}{\sigma}\), then \(1 + x_k^* = \frac{\sigma}{\sigma - 1}\) is the implied solution of Equation 27, given that \(\lambda_{ff,k}(x_k^*) = 0\) and \(\lambda_{hf,k}(x_k^*) = 1\).

### B  Constrained Optimal Policy & Interdependence

In this Section we characterize the optimal policy when trade taxes are restricted in a subset of industries. Here, we analyze both export and import taxes. Our results concerning the optimal import tax is used to derive Proposition 5.

#### Optimal Import Tax when a Subset of Industries are Restricted.

Recall from Appendix A that the F.O.C. for the tariff in industry \(k\) is given by

\[
\frac{dW_h(t, x, w)}{d \ln (1 + t_k)} = \frac{\partial \ln \hat{p}_{fh,k}}{\partial \ln (1 + t_k)} + \left[ 1 - \tilde{\tau} \sum_s \left( \frac{p_{fh,g} q_{fh,g} \partial \ln q_{fh,g}}{Y_h} \right) \right] Y_h \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} - \tilde{\tau} \sum_s \left( \frac{p_{fh,g} q_{fh,g} \partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} \right) \frac{\partial \ln \hat{p}_{fh,k}}{\partial \ln (1 + t_k)} = 0.
\]

where, as before, \(\tilde{\tau} \equiv \frac{\partial V_h}{\partial \omega_h} / \frac{\partial V_h}{\partial Y_h} / \partial \omega_h / \partial Y_h\). Applying the implicit function theorem to \(Y_h = \omega_h L_h + \sum_s \left( t_g p_{fh,g} q_{fh,g} + x_g p_{hf,g} q_{hf,g} \right)\), we will have

\[
Y_h \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} = \frac{\hat{p}_{fh,g} q_{fh,g} + \sum_s \left( t_g p_{fh,g} q_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} \right) \frac{\partial \ln \hat{p}_{fh,k}}{\partial \ln (1 + t_k)} + \sum_s \left( t_g p_{fh,g} q_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln Y_h} \right)}{1 - \sum_s \left( t_g p_{fh,g} q_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln Y_h} \right)}.
\]
Considering the above equation and to account for income effects, we can define

\[ Y \equiv \frac{1 - \sum_g \left( t_g \frac{p_{fh,g} q_{fh,g}}{\bar{p}_{fh,k}} \frac{\partial \ln q_{fh,g}}{\partial \ln Y_h} \right)}{1 - \sum_g \left( \frac{t_g}{1 + t_g} \lambda_{fh,g} \bar{Y}_{fh,g} \right)} = \frac{1 - \sum_g \left( \frac{t_g}{1 + t_g} \lambda_{fh,g} \bar{Y}_{fh,g} \right)}{1 - \sum_g \left( \frac{t_g}{1 + t_g} \lambda_{fh,g} \bar{Y}_{fh,g} \right)}, \]

which allows us to further simplify the F.O.C. as

\[ (1 - Y) (1 + t_k) p_{fh,k} q_{fh,k} + \sum_g \left( t_g - Y \bar{\tau} \right) p_{fh,g} q_{fh,g} \lambda_{fh,k} = 0. \]

Rearranging the above expression implies the following formula for optimal tariff in industry \( k \) as a function of applied tariffs in other industries:

\[ 1 + t^*_k = (1 + Y \bar{\tau}) \left( 1 + \frac{1 - Y}{\lambda_{fh,k}} \right)^{-1} \left( 1 + \sum_{g \neq k} \left( \frac{1 + t_g}{1 + Y \bar{\tau}} - 1 \right) \frac{\lambda_{fh,g} \epsilon_{fh,k}}{\lambda_{fh,k} \epsilon_{fh,k}} \right). \]

Note that when traded industries exhibit a zero income elasticity, which is the case in our analysis in Section 6, then \( Y = 1 \) and the above equation reduces to 17.

**Optimal Export Tax when a Subset of Industries are Restricted.** Correspondingly, as shown in Appendix A, the F.O.C. for the export tax in industry \( k \) implies

\[ \frac{dW_h(t, x; w)}{d \ln (1 + x_k)} = \left[ 1 - \bar{\tau} \sum_g \left( \frac{p_{fh,g} q_{fh,g}}{\bar{p}_{fh,k}} \frac{\partial \ln q_{fh,g}}{\partial \ln Y_h} \right) Y_h \frac{\partial \ln Y_h}{\partial \ln (1 + x_k)} \right] - \bar{\tau} \sum_g \left( \frac{\partial \ln q_{fh,g}}{\partial \ln \bar{p}_{fh,k}} \bar{p}_{fh,k} \frac{\partial \ln \bar{p}_{fh,k}}{\partial \ln (1 + x_k)} \right) = 0. \]

Defining \( Y \) and \( \bar{\tau} \) as before, the above expression can be stated as

\[ \bar{\tau} \left( -\bar{p}_{fh,k} q_{fh,k} - \sum_g \left( \bar{p}_{fh,g} q_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln \bar{p}_{fh,k}} \frac{\partial \ln \bar{p}_{fh,k}}{\partial \ln (1 + x_k)} \right) \right) = 0. \]

Rearranging the above equation yields

\[ (1 + x_k) (1 + Y \bar{\tau}) (1 + \epsilon_{fh,k}) \left\{ 1 + \sum_{g \neq k} \left( \frac{1}{(1 + x_g) (1 + Y \bar{\tau})} \lambda_{fh,g} \epsilon_{fh,k} \right) \right\} = \epsilon_{fh,k}, \]

63
which, in turn, implies the following formula for optimal export tax in industry $k$ as a function of applied taxes in other industries:

$$(1 + x_k)(1 + Y\tau) = \frac{\varepsilon_{hf,k}}{1 + \varepsilon_{hf,k}} \left[ 1 + \sum_{g \neq k} \left( \frac{1}{1 + x_g} \right) \right] \lambda_{hf,g} \frac{h_{f,k}}{\lambda_{hf,k} 1 + \varepsilon_{hf,k}} \right]^{-1}.$$  

**Solving for $1 + \tau$.** To finalize the characterization of the restricted optimal taxes, we also need to characterize $\bar{\tau} \equiv \left( \frac{\partial V_h}{\partial w_h} / \frac{\partial V_h}{\partial w_h} \right) / \partial \ln w_h$. To this end, we can follow the same steps presented in Appendix A. That is, defining $X \equiv pq, \bar{\tau}$ can be expressed as

$$1 + \bar{\tau} = 1 + \sum_k \left( t_k p_{f,h,k} q_{f,h,k} \frac{\partial \ln (p_{f,h,k} q_{f,h,k})}{\partial \ln w_h} + x_k p_{hf,k} q_{hf,k} \frac{\partial \ln (p_{hf,k} q_{hf,k})}{\partial \ln w_h} - \frac{\partial \ln (p_{hf,k} q_{hf,k})}{\partial \ln w_h} \right) = \frac{\partial \ln \left( \sum_k \left( (1 + x_k) \frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_h} \right) - \left( 1 + \bar{x} \right)^{-1} \sum_k \left( r_{f,k} \frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_h} \right) \right)}{\partial \ln w_h} = \frac{\partial \ln \left( \sum_s \left( \frac{\partial \ln \left( \sum_g (p_{hf,g}) \right)}{\partial \ln w_h} \right) \right)}{\partial \ln w_h}.$$

where (i) $1 + \bar{x} \equiv \sum_g (p_{hf,g} q_{hf,g}) \sum_g (p_{hf,g} q_{hf,g}) = \sum_k \left( r_{f,h,k} (1 + x_k) \right)$, (ii) $1 + \bar{\tau} \equiv \sum_k \left[ (1 + t_k) r_{f,h,k} \frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_h} \right] / \partial \ln w_h$, and (vi) $\bar{\varepsilon}_{j,k} \equiv \sum_j \bar{\varepsilon}_{j,k}$ and $\bar{\varepsilon}_{j,k} \equiv \sum_j \bar{\varepsilon}_{j,k}$ denote the elasticity of labor demand per $D_4$.36

Also, note that in deriving the above expression we use the fact that $\frac{\partial \ln (p_{f,h,k} q_{f,h,k})}{\partial \ln w_h} = 1 + \frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_h}$, as well as the fact that (absent of income effects) $\frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_h} = -\frac{\partial \ln L_{f,h,k}(w)}{\partial \ln w_f} = -\bar{\varepsilon}_{f,h,k}$. Finally, note that when export taxes are zero (which is the case in our analysis in Section 6), $\bar{x} = 0$ and the formula for $\bar{\tau}$ reduces to that presented in the text—albeit in the main text we used $\bar{t}$ instead of $\tau$ to label the uniform term.

36 The above equation implicitly assumes that $\partial (1 + \bar{x}) / \partial w_h \approx 0$.  

64
C Optimal NRTBs

The optimal NRTB problem of the home country can be formulated as

\[
\max_{(0,0,\tau, w) \in A} W_h (0, 0, \tau; w).
\]

To tighten the notation, we henceforth use \( W_h (\tau; w) \equiv W_h (0, 0, \tau; w) \) to denote welfare arising from the imposition of NRTBs when revenue-raising taxes are restricted. The F.O.C. with respect to the NRTB in industry \( k \) can be stated as

\[
\frac{dW_h (\tau; w)}{d (1 + \tau_k)} = \frac{\partial V_h}{\partial Y_h} \left[ \frac{\partial Y_h}{\partial Y_h} \frac{dY_h}{d w_h} \frac{d w_h}{d (1 + \tau_k)} + \sum_{g} \sum_{j=f,h} \left[ \frac{\partial V_h}{\partial p_{jh,g}} \frac{\partial p_{jh,g}}{\partial (1 + \tau_k)} + \frac{\partial V_h}{\partial \tilde{p}_{jh,g}} \frac{\partial \tilde{p}_{jh,g}}{\partial w_h} \frac{d w_h}{d (1 + \tau_k)} \right] \right] = 0.
\]

Noting that (i) \( \partial Y_h / \partial (1 + \tau_k) = 0 \), (ii) \( \partial \tilde{p}_{jh,g} / \partial (1 + \tau_k) = 0 \) for all \( g \), (iii) \( \partial \tilde{p}_{jh,g} / \partial (1 + \tau_k) = 0 \) if \( g \neq k \) or, and (iv) applying the implicit function theorem to derive \( d w_h / d (1 + \tau_k) \), the F.O.C. can be written as:

\[
\frac{dW_h (\tau; w)}{d \ln (1 + \tau_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial V_h}{\partial p_{fh,k}} \frac{p_{fh,k}}{\partial \ln (1 + \tau_k)} + \frac{\partial V_h}{\partial \tilde{p}_{fh,k}} \left( p_{fh,\tilde{p}_{fh,k}} \frac{\partial q_{fh,k}}{\partial \tilde{p}_{fh,k}} \frac{\partial \tilde{p}_{fh,k}}{\partial \ln (1 + \tau_k)} \right) \right\} = 0,
\]

where, as before, \( \frac{\partial V_h}{\partial w_h} = \frac{\partial V_h}{\partial Y_h} \frac{\partial Y_h}{\partial w_h} + \sum_{g} \sum_{j=h,f} \left( \frac{\partial V_h}{\partial p_{jh,g}} \frac{\partial p_{jh,g}}{\partial w_h} \right) \). The above condition can be simplified as follows:

\[
\frac{dW_h (\tau; w)}{d \ln (1 + \tau_k)} = -\frac{\partial V_h}{\partial Y_h} p_{fh,k} q_{fh,k} (1 + \tilde{e}_{fh,k}) = 0.
\]
Given its definition, $\tau$ can be calculated as:

$$
\tau = \frac{\partial V_h}{\partial w_h} + \frac{\partial V_h}{\partial \tau} = \frac{\partial Y_h}{\partial w_h} + \sum_k \left( \frac{\partial p_{fkh}q_{fhk}}{\partial w_h} \right) = \frac{\partial Y_h}{\partial w_h} - \sum_k \left( \frac{p_{fkh}q_{fhk}}{\partial w_h} \right)
$$

where

$$
\frac{\partial L_{fh}(w; \tau)}{\partial w_h} = -1 + \tilde{\epsilon}_f h + \tilde{\epsilon}_h f
$$

with the last line following from our earlier derivation that (per D4) $\partial L_{fh}(w; \tau) / \partial w_h = -\tilde{\epsilon}_f h$ and $\partial \ln L_{fh}(w; \tau) / \partial \ln w_h = 1 + \tilde{\epsilon}_h f$. Plugging $\tau$ from above expression back into the F.O.C., implies the following:

$$
\left\{ \begin{array}{ll}
\frac{\partial W_h}{\partial \ln (1 + \tau_k)} > 0 & \text{if } \epsilon_{fkh} < 1 + \tilde{\epsilon}_h f + \tilde{\epsilon}_f h \\
\frac{\partial W_h}{\partial \ln (1 + \tau_k)} < 0 & \text{if } \epsilon_{fkh} > 1 + \tilde{\epsilon}_h f + \tilde{\epsilon}_f h
\end{array} \right.
$$

Note the imposition of $\tau_k$ reduces $q_{fhk}$. So if demand is super-convex, i.e., $\partial \epsilon_{fkh}/\partial q_{fhk} > 0$, then the above conditions imply following formula for optimal NRTBs:

$$
\tau_k = \left\{ \begin{array}{ll}
\infty & \text{if } \epsilon_{fkh} < 1 + \tilde{\epsilon}_h f + \tilde{\epsilon}_f h \\
0 & \text{if } \epsilon_{fkh} > 1 + \tilde{\epsilon}_h f + \tilde{\epsilon}_f h
\end{array} \right.
$$

The condition that $\partial \epsilon_{fkh}/\partial q_{fhk} > 0$ is widely-known as Marshall’s Second Law of Demand, and is satisfied in an important class of trade models.
D Optimal Trade Taxes with Multiple Foreign Countries

Country $i$’s optimal trade tax solves the following problem

$$\max_{t_i, x_i} V_i(Y_i, \tilde{p}_i)$$

s.t.

$$\begin{align*}
 w_j L_j &= \sum_{k} \sum_{C} \left(p_{j, k} q_{j, k}\right) \\
 Y_j &= w_j L_j + \sum_{k} \sum_{C} \left( t_{j, k} p_{j, k} q_{j, k} + x_{j, k} p_{j, k} q_{j, k}\right) \\
 \tilde{p}_{j, k} &= (1 + x_{j, k}) (1 + t_{j, k}) p_{j, k} \\
 p_{j, k} &= a_{j, k} w_j \\
 t_{j, k} &= x_{j, k} = 0 \quad \forall j \neq i
\end{align*}$$

Below, we show that the solution to the above problem features a uniform import tax on each supplier. Then, we solve a more restrictive case of the problem to show that MFN tariffs are non-uniform.

Uniformity of Import Taxes [Proposition 1]. Note that we want to solve for country $i$’s optimal import tax schedule. As in the baseline model, the welfare in country $i$, can be expressed as $W_i = \partial V_i (Y_i, \tilde{p}_i)$, where $Y_i = w_i L_i + \sum_{k} \left( t_{j, k} p_{j, k} q_{j, k} + x_{j, k} p_{j, k} q_{j, k}\right)$. Correspondingly, $W_i$ is uniquely determined by the vector of import and export taxes, $t_i = \{t_{j, k}\}$ and $x_i = \{x_{j, k}\}$, plus the vector of country-level wages, $w = \{w_j\}$:

$$W_i (t_i, x_i; w) = V_i (Y_i (t_i, x_i; w), \tilde{p}_i (t_i, x_i; w))$$

Defining $D_i (t_i, x_i; w) = w_i L_i - \sum_{k} \sum_{C} \left(p_{i, k} q_{i, k}\right)$, the equilibrium vector of aggregate wages, $w$, solves the following system of equations:

$$\begin{cases}
 D_1 (t_1, x_1; w) = 0 \\
 \vdots \\
 D_N (t_N, x_N; w) = 0
\end{cases}$$

(28)
Plugging the above equation back into the F.O.C. and defining
\[
\eta_{\ell i, g} \equiv \frac{\partial V_i(Y_i, \tilde{p}_i)}{\partial Y_i} \left( \frac{\partial Y_i}{\partial \ln (1 + t_{ji,k})} + \frac{d w_i}{\partial \ln (1 + t_{ji,k})} \right)
\]

where

\[
\frac{\partial Y_i}{\partial \ln (1 + t_{ji,k})} = \tilde{p}_{ji,k} q_{ji,k} + \sum_{\ell \in C} \sum_{g \in K} \left( t_{\ell i, k} p_{\ell i, k} q_{\ell i, k} (\xi_{\ell i, g} + \eta_{\ell i, g} \frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})}) \right)
\]

Invoking Roy’s identity, \( \frac{\partial V_i(Y_i, \tilde{p}_i)}{\partial \ln Y_i} = q_{ji,k} \), and noting that

\[
\frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})} = \eta_{\ell i, g} \frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})}
\]

Plugging the above equation back into the F.O.C. and defining \( \Delta_{\ell i, g} \equiv \xi_{\ell i, g} + \eta_{\ell i, g} \frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})} \), will yield the following optimality condition

\[
\frac{d W_i(t_i, x_i; \omega)}{d \ln (1 + t_{ji,k})} = \frac{\partial V_i(Y_i, \tilde{p}_i)}{\partial Y_i} \left[ \tilde{p}_{ji,k} q_{ji,k} - \tilde{p}_{ji,k} q_{ji,k} + \sum_{\ell \in C} \sum_{g \in K} \left( t_{\ell i, k} p_{\ell i, k} q_{\ell i, k} \Delta_{\ell i, g} \right) - \sum_{\ell \in C} \left( v_{\ell i} \frac{d \ln w_{\ell}}{d \ln (1 + t_{ji,k})} \right) \right]
\]

where \( v_{\ell i} \equiv \partial W_i / \partial \ln w_{\ell} \). Applying the implicit function theorem to the System of Equations 28, we can solve for \( d \ln w / d \ln (1 + t_i) \) as follows:

\[
\begin{bmatrix}
\frac{d \ln w_1}{d \ln (1 + t_{ji,k})} & \cdots & \frac{d \ln w_1}{d \ln (1 + t_{N, ji,k})} \\
\vdots & \ddots & \vdots \\
\frac{d \ln w_N}{d \ln (1 + t_{ji,k})} & \cdots & \frac{d \ln w_N}{d \ln (1 + t_{N, ji,k})}
\end{bmatrix}
= \left( \frac{\partial \ln D}{\partial \ln w} \right)^{-1}
\begin{bmatrix}
\frac{\partial \ln D_1}{\partial \ln (1 + t_{ji,k})} & \cdots & \frac{\partial \ln D_1}{\partial \ln (1 + t_{N, ji,k})} \\
\vdots & \ddots & \vdots \\
\frac{\partial \ln D_N}{\partial \ln (1 + t_{ji,k})} & \cdots & \frac{\partial \ln D_N}{\partial \ln (1 + t_{N, ji,k})}
\end{bmatrix},
\]

Letting \( \tau_{\ell i} \) denotes element \( \ell i \) of matrix \( (\partial \ln D / \partial \ln w)^{-1} \), the above system implies that for every \( \ell \in C \)

\[
\frac{d \ln w_{\ell}}{d \ln (1 + t_{ji,k})} = \sum_{\ell \in C} \left( \tau_{\ell n} \sum_{g \in K} \left( p_{n i, g} q_{n i, g} \Delta_{n i, g} \right) \right).
\]

Plugging the above expression back into the F.O.C. implies the following

\[
\sum_{n \in C} \sum_{g \in K} \left( t_{n i, g} p_{n i, g} q_{n i, g} \Delta_{n i, g} \right) - \sum_{\ell \in C} \left( v_{\ell} \sum_{n \in C} \left( \tau_{\ell n} \sum_{g \in K} p_{n i, g} q_{n i, g} \Delta_{n i, g} \right) \right) = 0.
\]
The above expression can in turn be rearranged as

\[
\sum_{n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left[ \left( t_{ni,g} - \sum_{\ell \in \mathcal{C}} \tau_{\ell n} v_{i\ell} \right) p_{ni,g} q_{ni,g} \Delta_{ni,g}^{ji,k} \right] = \sum_{n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left[ \left( t_{ni,g} - \tau_{ni} \right) p_{ni,g} q_{ni,g} \Delta_{ni,g}^{ji,k} \right] = 0,
\]

where \( \tau_{ni} \equiv \sum_{\ell \in \mathcal{C}} \tau_{\ell n} v_{i\ell} \), or, equivalently

\[
T_i \Delta_i = 0,
\]

where \( T_i = \left[ t_{ni,g} - \bar{\tau}_{ni} \right]_{n \in \mathcal{C} \times \mathcal{K}} \) and \( \Delta_i = \left[ p_{ni,g} q_{ni,g} \Delta_{ni,g}^{ji,k} \right]_{n \in \mathcal{C}, j \in \mathcal{C}, g \in \mathcal{K}} \) are respectively \( 1 \times N \cdot K \) and \( N \cdot K \times N \cdot K \) matrices. If \( \det \Delta_i \neq 0 \), then \( T_i = 0 \) is the unique solution to the above system, which implies that the optimal import tax is uniform across products originating from the same exporting country:

\[
t_{ji,k} = \bar{\tau}_{ji}, \quad \forall j, k
\]

**MFN Tariffs.** No suppose the country \( i \) is bound by the MFN clause, whereby it has to impose the same tariff \( t_{i,k} \) on industry \( k \) irrespective of origin country. In that case, following the same steps as above the F.O.C. can be stated as

\[
\sum_{n \in \mathcal{C}} \left( \sum_{g \in \mathcal{K}} t_{i,g} \sum_{j} \left( p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right) \right) - \sum_{n \in \mathcal{C}} \left( \tau_{ni} \sum_{g \in \mathcal{K}} \sum_{j} \left( p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right) \right) = 0.
\]

The above equation that unless for all \( k \) and \( k' \in \mathcal{K} \),

\[
\frac{\sum_{j,n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left( \tau_{mi} p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right)}{\sum_{j,n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left( \tau_{mi} p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right)} = \frac{\sum_{j,n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left( \bar{\tau}_{mi} p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right)}{\sum_{j,n \in \mathcal{C}} \sum_{g \in \mathcal{K}} \left( \bar{\tau}_{mi} p_{ji,g} q_{ji,g} \Delta_{ji,g}^{ni,k} \right)},
\]

The optimal MFN tariff is non-uniform across industries. That is, unless industries are symmetric the optimal tariffs is non-uniform.
E Proof of Theorem 2

The optimal trade tax problem of the home country can be formulated as

\[
\max_{(t, x, \tau; w) \in A} W_h(t, x, \tau; w),
\]

where the set of feasible wage-policy combinations, \( A \), is given by D2. Trivially, when revenue-raising taxes are available, the optimal NRTB is zero, i.e., \( \tau = 0 \). So, hereafter we can restrict our attention to characterizing the optimal revenue-raising taxes, \( t^* \) and \( x^* \).

To handle the complex nature of the above problem, we proceed in several steps. First, we characterize the optimal policy as a function of the general equilibrium trade tax pass-throughs and reduced-form demand elasticities. Note that the pass-through of taxes onto consumers were always equal to “one” in the Ricardian mode, so this first step was previously irrelevant. To fix minds, we define the pass-through of taxes (net of wage effects) as follows:

\[
\sigma_{ji,k} \equiv \frac{\partial \ln \hat{p}_{ji,k}(t, x; w)}{\partial \ln (1 + t_{ji,k})} = \frac{\partial \ln \hat{p}_{ji,k}(t, x; w)}{\partial \ln (1 + x_{ji,k})}.
\]

To elaborate, \( \sigma_{ji,k} \) denotes the pass-through of a tax on good \( ji,g \) to the consumer price of good \( ji,k \), net of wage-driven effects.

Second, to handle the effects of policy on “aggregate” variables (e.g., wage and income), we invoke the Lerner symmetry. Specifically, we first identify a solution to the above problem for which the “aggregate” term corresponding to general equilibrium income effects drops out of the FOCs. Once this particular solution is identified, we can construct the remaining solutions by an across-the-board multiplication of the export and import tax vectors.

Finally, as with the Ricardian case, we invoke a set of supply-side and demand-side envelop conditions to account for general equilibrium behavioral responses. It is this latter steps that allows us to characterize the optimal tax schedule without imposing strong parametric assumptions on the supply or demand-sides of the economy.
Step 1: Deriving the F.O.C. for Import Taxes. The F.O.C. with respect to sector $k$'s tariff can be expressed as

$$\frac{dW_h(t, x, w)}{d(1 + t_k)} = \frac{\partial V_h}{\partial Y_h} \left[ \frac{\partial Y_h}{\partial (1 + t_k)} + \frac{\partial Y_h}{\partial w_h} \frac{dw_h}{d(1 + t_k)} \right] + \sum_g \sum_{j=h} \left( \frac{\partial V_h}{\partial \tilde{p}_{j, h, g}} \left[ \frac{\partial \tilde{p}_{j, h, g}}{\partial (1 + t_k)} + \frac{\partial \tilde{p}_{j, h, g}}{\partial w_h} \frac{dw_h}{d(1 + t_k)} \right] \right) = 0,$$

It should be noted upfront that the difference between the present setup and the pure Ricardian case, is that the (conditional) tariff pass-through $\sigma_{j, h, g}^{f, k} = \partial \ln \tilde{p}_{j, h, g}/\partial \ln (1 + t_k)$ can be non-zero (even if $g \neq k$) due to (i) the upward sloping supply curve in industry $g$, plus (ii) the cross-substitutability between industry $k$ and industry $g$ goods. Plugging $Y_h = w_h L_h + \Pi_h + R^X_h + R^M_h$, into the F.O.C. yields the following:

$$\frac{dW_h(t, x, w)}{d(1 + t_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial \Pi_h}{\partial (1 + t_k)} + \frac{\partial R^X_h}{\partial (1 + t_k)} + \frac{\partial R^M_h}{\partial (1 + t_k)} \right\} + \sum_g \sum_{j=h} \left( \frac{\partial V_h / \partial \tilde{p}_{j, h, g}}{\partial V_h / \partial Y_h} \frac{\partial \tilde{p}_{j, h, g}}{\partial (1 + t_k)} \right) + \frac{\partial V_h / \partial w_h}{\partial V_h / \partial Y_h} \frac{dw_h}{d(1 + t_k)} \right\} = 0,$$

where $\partial V_h / \partial w_h = \partial Y_h / \partial w_h + \sum_g (\partial V_h / \partial \tilde{p}_{j, h, g}) (\partial \tilde{p}_{j, h, g} / \partial w_h)$. The above F.O.C. is characterized by five different elements that can be characterized as follows. First, The effect of tariffs on producer surplus, $\partial \Pi_h / \partial 1 + t_k$, can be expressed as

$$\frac{\partial \Pi_h(t, x, w)}{\partial \ln (1 + t_k)} = \sum_g \sum_{i=h} \left( \frac{\partial \Pi_{hi, g}}{\partial \tilde{p}_{hi, g}} \frac{\partial \tilde{p}_{hi, g}}{\partial \ln (1 + t_k)} \right) + \sum_g \sum_{j=h} \left( \frac{\partial \Pi_{hi, g}}{\partial \tilde{p}_{j, h, s}} \frac{\partial \tilde{p}_{j, h, s}}{\partial \ln (1 + t_k)} \right)$$

$$= \sum_g \sum_{i=h} \left( \tilde{p}_{hi, g} \frac{\partial \tilde{p}_{hi, g}}{\partial \ln (1 + t_k)} \right) - \sum_g \sum_{j=h} \left( \tilde{p}_{j, h, s} \frac{\partial \tilde{p}_{j, h, s}}{\partial \ln (1 + t_k)} \right),$$

where the last line follows from Hotelling's lemma that $\partial \Pi_{ji, g} / \partial \tilde{p}_{ji, g} = q_{ji, k}$ and $\sum_g \partial \Pi_{hi, g} / \partial \tilde{p}_{j, h, g} = q^T_{j, h, g}$. Second, the effect of import taxes on import tax revenues,
\[ \frac{\partial R^M}{\partial (1 + t_k)} \text{, can be expressed as} \]
\[
\frac{\partial R^M (t, x, w)}{\partial \ln (1 + t_k)} = \sum_g \frac{\partial t_g p_{fh,g} q_{fh,g}}{\partial \ln (1 + t_k)}
\]
\[
= \sum_g \left( t_g p_{fh,g} q_{fh,g} \right) \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} + p_{fh,k} q_{fh,k}
\]
\[
+ \sum_g \left( t_g p_{fh,g} q_{fh,g} \left[ \frac{\partial \ln p_{fh,g}}{\partial \ln (1 + t_k)} + \sum_j \left( \frac{\partial \ln q_{fh,g}}{\partial \ln p_{jh,s}} \frac{\partial \ln \bar{q}_{jh,s}}{\partial \ln (1 + t_k)} \right) \right] \right)
\]

Third, the effect of import taxes on export tax revenues, \( \partial R^X / \partial (1 + t_k) \), can be expressed as
\[
\frac{\partial R^X (t, x, w)}{\partial \ln (1 + t_k)} = \sum_g \frac{\partial x_g p_{hf,g} q_{hf,g}}{\partial \ln (1 + t_k)}
\]
\[
= \sum_g \left( x_g p_{hf,g} q_{hf,g} \left[ \frac{\partial \ln p_{hf,g}}{\partial \ln (1 + t_k)} + \sum_j \left( \frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{jf,s}} \frac{\partial \ln \bar{p}_{jf,s}}{\partial \ln (1 + t_k)} \right) \right] \right)
\]

Fourth, the effect of taxes on the consumer prices can be simplified using Roy’s identity, \( \frac{\partial V_i}{\partial \bar{p}_{jgh}} = q_{ji,g} \), as follows
\[
\sum_g \sum_{j=f,h} \left( \frac{\partial V_h}{\partial \bar{p}_{jgh}} \frac{\partial \bar{p}_{jgh}}{\partial \ln (1 + t_k)} \right) = \sum_g \sum_{j=f,h} \left( \bar{p}_{jgh} q_{jgh} \frac{\partial \ln \bar{p}_{jgh}}{\partial \ln (1 + t_k)} \right)
\]

Finally, the effect of tariffs on wages can be determined by applying the implicit function theorem to the balanced trade condition, \( D_h (t, x, w) = \sum_g (p_{fh,g} q_{fh,g} - (1 + x_g) p_{hf,g} q_{hf,g}) \). Doing so, implies \( \frac{dw_h}{d(1+t_k)} = -\frac{\partial D_h (t, x, w)}{\partial (1+t_k)} / \frac{\partial D_h (t, x, w)}{\partial w_h} \).
Hence, defining \( \tilde{\tau} \equiv \left( \frac{\partial V_h}{\partial w_h} / \frac{\partial V_h}{\partial Y_h} \right) / \frac{\partial D_h (t, x, w)}{\partial w_h} \),
\[
\frac{\partial V_h / \partial w_h}{\partial V_h / \partial Y_h \, d \ln (1 + t_k)} = -\tau \left\{ \sum_g \left( p_{fh,g} q_{fh,g} \eta_{fh,g} \right) \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} \right. \]
\[
+ \sum_g \left( p_{fh,g} q_{fh,g} \left[ \frac{\partial \ln p_{fh,g}}{\partial \ln (1 + t_k)} + \sum_j \left( \frac{\partial \ln q_{fh,g}}{\partial \ln p_{jh,s}} \frac{\partial \ln \bar{q}_{jh,s}}{\partial \ln (1 + t_k)} \right) \right] \right) \]
\[
- \sum_g \left( \bar{p}_{hf,g} q_{hf,g} \left[ \frac{\partial \ln p_{hf,g}}{\partial \ln (1 + t_k)} + \sum_j \left( \frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{jf,s}} \frac{\partial \ln \bar{p}_{jf,s}}{\partial \ln (1 + t_k)} \right) \right] \right) \right\}
\]

To simplify the above expressions, we can employ our notation for the tax pass-through, \( \sigma_{ji,g}^{fh,k} \equiv \partial \ln \bar{p}_{ji,g} / \partial \ln 1 + t_k \), and the Marshallian demand elasticity, \( e_{ji,k}^{gh} \equiv \)
\[ \frac{\partial \ln q_{ji,k}}{\partial \ln \tilde{p}_{\mu,g}}. \] We can also use the fact that the pass-through of taxes onto “producer” prices \((p_{\mu,g} = \tilde{p}_{\mu,g} / (1 + t_{\mu,g}) (1 + x_{\mu,g}))\) are, by construction, described by the following:

\[
\frac{\partial \ln p_{\mu,g}}{\partial \ln (1 + t_k)} = \begin{cases} 
\sigma_{fh,k}^{\mu,g} & \mu, g \neq fh, k \\
\sigma_{fh,k}^{\mu,g} - 1 & \mu, g = fh, k.
\end{cases}
\]

Doing as stated above and noting that \(\partial V_h / \partial Y_h > 0\) and \(q_{ji,k} = q_{ji,k}^T + q_{ji,k}'\), the F.O.C. can be expressed as follows:

\[
\sum_g \sum_{i=h,f} \left( p_{hi,g} q_{hi,g} \sigma_{hi,g}^{fh,k} \right) - \sum_g \sum_j \left( \tilde{p}_{jh,g} q_{jh,g} \sigma_{jh,g}^{fh,k} \right) + \sum_g \left[ x_g p_{hf,g} q_{hf,g} \left( \sigma_{hf,g}^{fh,k} + \sum_j \epsilon_{hf,g}^{ij,f,s} \sigma_{jh,s}^{fh,k} \right) \right] + p_{fh,k} q_{fh,k} + \sum_g \left[ t_g p_{fh,g} q_{fh,g} \left( \sigma_{fh,g}^{fh,k} + \sum_j \epsilon_{fh,g}^{jh,s} \sigma_{jh,s}^{fh,k} \right) \right] - \bar{\tau} \left( -p_{fh,k} q_{fh,k} + \sum_g \left[ p_{fh,g} q_{fh,g} \left( \sigma_{fh,g}^{fh,k} + \sum_j \epsilon_{fh,g}^{jh,s} \sigma_{jh,s}^{fh,k} \right) \right] \right) + \bar{\tau} \sum_g \left( \tilde{p}_{hf,g} q_{hf,g} \left[ \sigma_{hf,g}^{fh,k} + \sum_j \epsilon_{hf,g}^{jh,s} \sigma_{jh,s}^{fh,k} \right] \right) + \sum_g \left[ (t_g - \bar{\tau}) p_{fh,g} q_{fh,g} \right] \frac{\partial \ln Y_h}{\partial \ln (1 + t_k)} = 0
\]

Given the Lerner symmetry and the multiplicity of the optimal trade tax, there always exists a solution to the above problem where \(\sum_g \left( [t_g - \bar{\tau}] p_{fh,g} q_{fh,g} \right) = 0\). Henceforth, we restrict our attention to solving for this particular solution. Once we do that, the remaining solutions can be identified with a basic multiplicative transformation of the import and export tax vectors. Considering this and rearranging the above expression, yields the following F.O.C.

\[
\sum_g \left( p_{hf,g} q_{hf,g} \sigma_{hf,g}^{fh,k} - \tilde{p}_{hf,g} q_{hf,g} \sigma_{hf,g}^{fh,k} \right) + (1 + \bar{\tau}) p_{fh,k} q_{fh,k} + \sum_g \left( t_g - \bar{\tau} \right) p_{fh,g} q_{fh,g} \left[ \sigma_{fh,g}^{fh,k} + \sum_j \epsilon_{fh,g}^{jh,s} \sigma_{jh,s}^{fh,k} \right] + \sum_g \left[ (1 + \bar{\tau}) (1 + x_g - 1) \left( \sigma_{hf,g}^{fh,k} + \sum_j \epsilon_{hf,g}^{jij,s} \sigma_{jij,s}^{fh,k} \right) \right] p_{hf,g} q_{hf,g} = 0.
\]
Finally, dividing the above expressions by 1 + τ and \(Y_f = w_f L_f\) (noting that \(p_{fh,k} q_{fh,k} = r_{fh,k} Y_f\) and \(\tilde{p}_{hf,g} q_{hf,g} = \lambda_{hf,g} Y_f\)) leads us the following optimality condition:

\[
\sum_g \left[ \left( \frac{1 + t_g}{1 + \tau} - 1 \right) \left( \sum_j \sum_s \epsilon_{fh,g}^j s^j \sigma_{j}^{f,h,k} \right) r_{fh,g} \right. \\
+ \left. \left( 1 - \frac{1}{(1 + \tau)(1 + x_g)} \right) \left( \sum_j \sum_s \epsilon_{hf,g}^s s^j \sigma_{j}^{f,h,k} \right) \lambda_{hf,g} \right] = r_{fh,k} - \sum_g \left( \lambda_{hf,g} \sigma_{j}^{f,h,k} - r_{fh,g} \sigma_{j}^{f,h,k} \right)
\]

(29)

**Step 2. Deriving the F.O.C. for Export Taxes.** The F.O.C. with respect to sector \(k\)’s export tax can be stated as

\[
\frac{dW_h(t, x, w)}{d (1 + x_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial \Pi_h}{\partial (1 + x_k)} + \frac{\partial R_h}{\partial (1 + x_k)} \right\} + \sum_{j=f,h} \left( \frac{\partial V_h}{\partial \tilde{p}_{j,h,g}} \frac{\partial \tilde{p}_{j,h,g}}{\partial (1 + x_k)} \right) + \frac{\partial V_h}{\partial w_h} \frac{dw_h}{d (1 + x_k)} = 0,
\]

Adopting our earlier definition for \(\partial V_h / \partial w_h\) and noting that \(Y_h = w_h L_h + \Pi_h + \mathcal{R}_h^X + \mathcal{R}_h^M\), the above condition can be reformulated as

\[
\frac{dW_h(t, x, w)}{d (1 + x_k)} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial \Pi_h}{\partial (1 + x_k)} + \frac{\partial R_h}{\partial (1 + x_k)} \right\} + \sum_{j=f,h} \left( \frac{\partial V_h}{\partial \tilde{p}_{j,h,g}} \frac{\partial \tilde{p}_{j,h,g}}{\partial (1 + x_k)} \right) + \frac{\partial V_h}{\partial w_h} \frac{dw_h}{d (1 + x_k)} = 0,
\]

As before, the above condition is composed of five elements that can be expressed as follows. First, The effect of export taxes on producer surplus, \(\partial \Pi_h / \partial 1 + x_k\), can be expressed as

\[
\frac{\partial \Pi_h(t, x, w)}{\partial \ln (1 + x_k)} = \sum_{g} \sum_{i=h,f} \left( \frac{\partial \Pi_{hi,g}}{\partial p_{hi,g}} \frac{\partial p_{hi,g}}{\partial \ln (1 + x_k)} \right) + \sum_{s} \sum_{j=h,f} \left( \frac{\partial \Pi_{hi,s}}{\partial \tilde{p}_{j,h,s}} \frac{\partial \tilde{p}_{j,h,s}}{\partial \ln (1 + x_k)} \right)
\]

\[
= \sum_{g} \sum_{i=h,f} \left( p_{hi,g} q_{hi,g} \frac{\partial p_{hi,g}}{\partial \ln (1 + x_k)} \right) - \sum_{s} \sum_{j=h,f} \left( \tilde{p}_{j,h,s} q_{j,h,s} \frac{\partial \tilde{p}_{j,h,s}}{\partial \ln (1 + x_k)} \right),
\]

where the last line follows from Hotelling’s lemma that \(\partial \Pi_{ji,g} / \partial p_{ji,g} = q_{ji,k}\) and \(\sum_g \partial \Pi_{hi,g} / \partial p_{j,h,g} = q_{j,h,g}^T\). Second, the effect of export taxes on import tax revenues,
$\partial R^M / \partial 1 + x_k$, can be expressed as

$$\frac{\partial R^M (t, x, w)}{\partial \ln (1 + x_k)} = \sum_g \frac{\partial t_g p_{fh,g} \eta_{fh,g}}{\partial \ln (1 + x_k)}$$

$$= \sum_g \left( t_g p_{fh,g} \eta_{fh,g} \right) \frac{\partial \ln Y_h}{\partial \ln (1 + x_k)}$$

$$+ \sum_g \left( t_g p_{fh,g} \eta_{fh,g} \right) \left[ \frac{\partial \ln p_{fh,g}}{\partial \ln (1 + x_k)} + \sum_s \left( \frac{\partial \ln q_{fh,g}}{\partial \ln p_{fh,g}} \frac{\partial \ln \tilde{p}_{j,h,s}}{\partial \ln (1 + x_k)} \right) \right]$$

Third, the effect of export taxes on export tax revenues, $\partial R^X / \partial (1 + x_k)$, can be expressed as

$$\frac{\partial R^X (t, x, w)}{\partial \ln (1 + x_k)} = \sum_g \frac{\partial x_g p_{hf,g} \eta_{hf,g}}{\partial \ln (1 + x_k)}$$

$$- \tilde{p}_{hf,k} \eta_{hf,k} + \sum_g \left( x_g p_{hf,g} \eta_{hf,g} \right) \left[ \frac{\partial \ln p_{hf,g}}{\partial \ln (1 + x_k)} + \sum_s \sum_j \left( \frac{\partial \ln q_{hf,g}}{\partial \ln p_{hf,g}} \frac{\partial \ln \tilde{p}_{j,f,s}}{\partial \ln (1 + x_k)} \right) \right]$$

Fourth, the effect of on the consumer prices can be simplified using Roy’s identity, $\frac{\partial V_i}{\partial \eta_{i,g}} = q_{i,g}$, as follows

$$\sum_g \sum_j \left( \frac{\partial V_h / \partial \tilde{p}_{j,h,g}}{\partial V_h / \partial Y_h} \frac{\partial \tilde{p}_{j,h,g}}{\partial \ln (1 + x_k)} \right) = \sum_g \sum_j \left( \tilde{p}_{j,h,g} \eta_{j,h,g} \frac{\partial \tilde{p}_{j,h,g}}{\partial \ln (1 + x_k)} \right)$$

Finally, the effect of export taxes on wages can be determined by applying the implicit function theorem to the balanced trade condition, $D_h = \sum_g p_{fh,g} \eta_{fh,g} - \tilde{p}_{hf,g} \eta_{hf,g}$. Doing so, implies $\frac{dw_h}{d(1+x_k)} = -\frac{\partial D_h}{\partial (1+x_k) / \partial w_h}$. Hence, adopting our earlier definition, $\bar{\tau} \equiv \left( \frac{\partial V_h}{\partial w_h} / \frac{\partial V_h}{\partial Y_h} \right) / \frac{\partial D_h}{\partial w_h}$,
As before, we can simplify the above expressions by using our notation that \( \partial_{j^i_k} \equiv \partial \ln p_{j^i_k} / \partial \ln (1 + x_k) \) and \( \epsilon_{j^i_k} \equiv \partial \ln q_{j^i_k} / \partial \ln p_{j^i_k} \); plus the fact that

\[
\frac{\partial \ln p_{j^i_k}}{\partial \ln (1 + x_k)} = \begin{cases} 
\frac{\partial_{h^f_k}}{\partial_{j^i_k}}, & \mu, g \neq h^f, k \\
\frac{\partial_{h^f_k}}{\partial_{j^i_k}}, & \mu, g = h^f, k 
\end{cases}
\]

Doing so and noting that \( \partial V_h / \partial Y_h > 0 \), the F.O.C. can be stated as follows:

\[
- \sum_g \sum_{j=h,f} \left( \tilde{p}_{j^g_h} q_{j^g_h} \sigma_{j^g_h}^{h^f_k} \right) + \left( \sum_g \sum_{j=h,f} p_{j^g_h} q_{j^g_h} \sigma_{j^g_h}^{h^f_k} \right) - p_{h^f_k} q_{h^f_k} + \tilde{p}_{h^f_k} q_{h^f_k} - x_k p_{h^f_k} q_{h^f_k} + \sum_g \left( x_g p_{h^g_s} q_{h^g_s} \left[ \sigma_{h^g_s}^{h^f_k} + \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right] \right) + \sum_g \left[ t_g p_{h^g_s} q_{h^g_s} \left[ \sigma_{h^g_s}^{h^f_k} + \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right] \right] - \tau \sum_g \left( p_{h^g_s} q_{h^g_s} \left[ \sigma_{h^g_s}^{h^f_k} + \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right] \right) + \tau \sum_g \left( \tilde{p}_{h^g_s} q_{h^g_s} \left[ \sigma_{h^g_s}^{h^f_k} + \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right] \right) + \sum_g \left( \left[ t_g - \tau \right] p_{h^g_s} q_{h^g_s} q_{h^g_s} \right) \frac{\partial \ln Y_h}{\partial \ln (1 + x_k)} = 0.
\]

Recall that our aim is to initially identify the solution where \( \sum_g \left( \left[ t_g - \tau \right] p_{h^g_s} q_{h^g_s} q_{h^g_s} \right) = 0 \). Considering this, the last term drops out and the above expression reduces to

\[
\sum_g \left( p_{h^g_s} q_{h^g_s} \sigma_{h^g_s}^{h^f_k} - \tilde{p}_{h^g_s} q_{h^g_s} \sigma_{h^g_s}^{h^f_k} \right) + \sum_g \left[ \left( t_g - \tau \right) \left( \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right) p_{h^g_s} q_{h^g_s} \right] + \sum_g \left( \left[ (1 + x_g) (1 + \tau) - 1 \right] \left( \sum_s \sum_{j=h,f} \epsilon_{h^g_s}^{j^h_s} \sigma_{j^h_s}^{h^f_k} \right) p_{h^g_s} q_{h^g_s} \right) = 0.
\]

76
Rearranging the above equation and dividing by \( Y_f = w_f L_f \) and \( 1 + \bar{\tau} \) yields the following optimality condition:

\[
- \sum_g \left[ \left( \frac{1 + t_g}{1 + \bar{\tau}} - 1 \right) \left( \sum_s \sum_f \epsilon_{fh,g}^s \sigma_{jh,s}^{h_f,k} \right) r_{fh,g} \right. \\
+ \left. \left( \frac{1}{(1 + \bar{\tau}) (1 + x_g)} - 1 \right) \left( \sum_s \sum_f \epsilon_{fh,g}^s \sigma_{jh,s}^{h_f,k} \right) \right] \lambda_{fh,g} = \sum_g \left( \sigma_{fh,g}^{h_f,k} \lambda_{fh,g} - \sigma_{fh,g}^{h_f,k} r_{fh,g} \right)
\]

(30)

**Step 3: Simultaneously Solving the System of F.O.C.**  As a final step, we simultaneously solve the system of F.O.C.s for all tax instruments. To simplify the process, we solve the system of F.O.C.s in terms of \( 1 + \mathcal{T}_k \equiv (1 + t_k) / (1 + \bar{\tau}) \) and \( 1 + \mathcal{X}_k \equiv 1 / (1 + x_k) (1 + \bar{\tau}) \). Noting that, by solving solving for \( \mathcal{T}_k \) and \( \mathcal{X}_k \), we also automatically pin down the optimal tax schedule:

\[
\begin{align*}
1 + t_k &= (1 + \bar{\tau}) (1 + \mathcal{T}_k) \\
1 + x_k &= 1 / (1 + \bar{\tau}) (1 + \mathcal{X}_k)
\end{align*}
\]

Given the definition for \( \mathcal{T}_k \), and \( \mathcal{X}_k \), Equation 29, which describes the set of FOCs w.r.t. import taxes, can be simplified as follows

\[
\sum_g \left( -\mathcal{T}_g r_{fh,g} \sum_{s \in \mathcal{K}} \sum_{\ell \in \mathcal{C}} \bar{\epsilon}_{fh,g}^s \sigma_{j\ell,s}^{h_f,k} + \mathcal{X}_g \lambda_{fh,g} \sum_{s \in \mathcal{K}} \sum_{\ell \in \mathcal{C}} \epsilon_{fh,g}^s \sigma_{j\ell,s}^{h_f,k} \right) = r_{fh,k} + \sum_g \left( \sigma_{fh,g}^{h_f,k} \lambda_{fh,g} - \sigma_{fh,g}^{h_f,k} r_{fh,g} \right)
\]

(31)

Similarly, Equation 30, which describes the set of FOCs w.r.t. export taxes, adopts the following simplified expression

\[
\sum_g \left( \mathcal{T}_g r_{fh,g} \sum_{s \in \mathcal{K}} \sum_{\ell \in \mathcal{C}} \epsilon_{fh,g}^s \sigma_{j\ell,s}^{h_f,k} - \mathcal{X}_g \lambda_{fh,g} \sum_{s \in \mathcal{K}} \sum_{\ell \in \mathcal{C}} \epsilon_{fh,g}^s \sigma_{j\ell,s}^{h_f,k} \right) = - \sum_g \left( \sigma_{fh,g}^{h_f,k} \lambda_{fh,g} - \sigma_{fh,g}^{h_f,k} r_{fh,g} \right)
\]

(32)

The gains intuition about the above equations, note that

\[
\sum_{s \in \mathcal{K}} \sum_{\ell \in \mathcal{C}} \epsilon_{j\ell,s}^{i,k} \sigma_{j\ell,s}^{i,k} = \frac{\partial \ln q_{i,j,k} (t, x; w, y)}{\partial \ln (1 + t_{i,j,k})} = \frac{\partial \ln q_{i,j,k} (t, x; w, y)}{\partial \ln (1 + x_{j,i,k})}.
\]

So, the term on the left-hand side of both the above equations can be viewed as the trade volume loss (at current prices) from taxes. The right-hand sides of both equations, meanwhile, correspond to the terms-of-trade gains from policy net of wage effects. More specifically, the right-hand side expression in Equation 31, corresponds to a trade-weighted
(\(t_k\)-induced) change in the relative price of Home’s exports:

\[
\frac{\partial TOT_h}{\partial \ln (1 + t_k)} = \sum_g \left( \lambda_{hf,g} \frac{\partial \ln \hat{p}_{hf}}{\partial \ln (1 + t_k)} - r_{fh,g} \frac{\partial \ln p_{fh}}{\partial \ln (1 + t_k)} \right) = r_{hf,k} + \sum_g \left( \sigma_{hf,g}^{fh,k} \lambda_{hf,g} - \sigma_{fh,g}^{fh,k} r_{fh,g} \right)
\]

Likewise, the right-hand side expression in Equation 32, corresponds to a trade-weighted (\(x_k\)-induced) change in the relative price of Home’s exports:

\[
\frac{\partial TOT_h}{\partial \ln (1 + x_k)} = \sum_g \left( \lambda_{hf,g} \frac{\partial \ln \hat{p}_{hf}}{\partial \ln (1 + x_k)} - r_{fh,g} \frac{\partial \ln p_{fh}}{\partial \ln (1 + x_k)} \right) = \lambda_{hf,k} - \sum_g \left( \sigma_{hf,g}^{fh,k} \lambda_{hf,g} - \sigma_{fh,g}^{fh,k} r_{fh,g} \right)
\]

Given the definitions for \(\partial TOT_h / \partial \ln (1 + t_k)\) and \(\partial TOT_h / \partial \ln (1 + x_k)\), we can write the system of F.O.C.s described by Equations 29 and 30 in matrix-form as follows

\[
\begin{bmatrix}
-r_{fh} \circ \epsilon_{fh} \sigma_{fh} & \lambda_{hf} \circ \epsilon_{hf} \sigma_{fh} \\
r_{fh} \circ \epsilon_{fh} \sigma_{fh} & -\lambda_{hf} \circ \epsilon_{hf} \sigma_{fh}
\end{bmatrix}
\begin{bmatrix}
\mathcal{T} \\
\mathcal{X}
\end{bmatrix} =
\begin{bmatrix}
\nabla_{\ln (1 + t)} TOT_h \\
-\nabla_{\ln (1 + x)} TOT_h
\end{bmatrix},
\]

where \(\epsilon_{ji} = [\epsilon_{ij,g}]_{k,j,g}\) is a \(K \times 4K\) matrix of demand elasticities; \(\sigma_{ji} = [\sigma_{ji,k}]_{j,k}\) is a \(4K \times K\) matrix of tax pass-throughs; while \(\mathcal{T} \equiv [T_k]_k\), \(\mathcal{X} \equiv [X_k]_k\), \(\nabla_{\ln (1 + t)} TOT_h \equiv [\partial \ln TOT_h / \partial \ln (1 + t_k)]_k\), \(\nabla_{\ln (1 + x)} TOT_h \equiv [\partial \ln TOT_h / \partial \ln (1 + x_k)]_k\), and \(\lambda_{hf} \equiv [\lambda_{hf,k}]_k\) are \(K \times 1\) vectors.

**Characterizing \(\tau\) when Export Taxes are Restricted**

To characterize the uniform component of the optimal import tax, \(\tau\), we follow same steps conducted in Appendix A. Specifically, along the same line of arguments presented there, \(\tau\) can be expressed

\[
\tau \equiv \frac{\partial V_h}{\partial \ln w_h} / \frac{\partial V_h}{\partial \ln y_h} = \frac{\partial Y_h}{\partial \ln w_h} - \sum_k \left( q_{hh,k} \frac{\partial \phi_{hh,k}}{\partial \ln w_h} \right) / \frac{\partial \ln \mathcal{L}_{h}(t; w)}{\partial \ln w_h} - \frac{\partial \ln \mathcal{L}_{h}(t; w)}{\partial \ln w_h},
\]

where \(\mathcal{L}_{ji}(t; w)\) denotes the demand for country \(j\)’s labor in market \(i\) (i.e., \(w_i L_{ji} = \sum_k p_{ji,k} q_{ji,k}\)), and the last equality is derived using Roy’s identity that \(\partial V_h / \partial \ln y_h = -q_{hh,k}\).

To simplify the above expression, we note that (i) \(Y_h = w_h L_h + \Pi_h + \sum_k t_k p_{fh,k} q_{fh,k}\), (ii)
To determine $\frac{\partial \ln p_{hh,k}}{\partial \ln w_h} = \frac{\partial \ln p_{hi,k}}{\partial \ln w_h} = 1$, as well as (iii) by Hotelling’s lemma,

$$\frac{\partial \Pi_h}{\partial \ln w_h} = \sum_k \sum_i \left( p_{hi,k} \frac{\partial \Pi_{hi,k}}{\partial \ln w_h} \right) \frac{\partial \ln p_{hi,k}}{\partial \ln w_h} + \frac{\partial \Pi_{hi,k}}{\partial \ln w_h} = \sum_k \sum_i \left( p_{hi,k}q_{hi,k} \right) - w_h L_h$$

Plugging these expressions back into the initial formula for $\tau$, yields the following:

$$\tau = \frac{\sum_k \sum_i \left( p_{hi,k}q_{hi,k} \right) + \sum_k \left( t_k \frac{\partial p_{fh,k}q_{fh,k}}{\partial \ln w_f} \right) - \sum_k \left( p_{hh,k}q_{hh,k} \right)}{w_f \frac{\partial \ln w_f L_{fh}}{\partial \ln w_f} - w_h \frac{\partial \ln w_h L_{hf}}{\partial \ln w_h}} = \frac{\sum_k \left( (t_k - \tau) r_{fh,k} \tilde{e}_{fh,k} \right)}{-\tilde{e}_{hf} - 1}$$

where the last line follows from that fact that (i) $w_h L_{hf} = \sum_k \sum_i \left( p_{hi,k}q_{hi,k} \right) - \sum_k \left( p_{hh,k}q_{hh,k} \right)$; (ii) $w_f L_{fh} = w_h L_{hf}$ by the balanced trade condition; and (iii) $\frac{\partial \ln L_{fh,k}(\cdot)}{\partial \ln w_f} = -\tilde{e}_{fh,k}$, and $\tilde{e}_{fh} \equiv L_{fh}(\cdot)/\partial \ln w_f = \sum_k r_{fh,k} \tilde{e}_{fh,k}$ per D4. Given the above equation, we can thus express $1 + \tau$ as

$$1 + \tau = \frac{\tilde{e}_{hf} + \sum_k \left( (t_k - \tau) r_{fh,k} \tilde{e}_{fh,k} \right)}{1 + \tilde{e}_{hf}}$$

which is the expression presented as Equation 18 in Section 6.2.

**F Characterizing the Pass-Throughs**

Below, we characterize the general equilibrium pass-through of a trade tax $1 + t_{ji,k}$ (or $1 + x_{ji,k}$) on to consumer prices, $\bar{p}_{ij}$, in both countries. To clarify the notation, $1 + t_{ji,k}$ (or $1 + x_{ji,k}$) corresponds to an import (or export) tax on variety $ji,k$. In our analysis we are interested in home’s import tax $1 + t_k \equiv 1 + t_{fh,k}$ and home’s export tax $1 + x_k = 1 + x_{hf,k}$.

**F1 Ricardian Model with IO Linkages**

To determine $\frac{\partial \ln \bar{p}_{ij,g}}{\partial \ln (1 + t_{ji,k})} = \frac{\partial \ln \bar{p}_{ij,g}}{\partial \ln (1 + x_{ji,k})}$ for any two varieties $ji,g$ and $ji,k$, we can applying the implicit function theorem to the price function, $F(t,x,\bar{p}) \equiv$
\{F_{jig}(t,x,\bar{p})\}_{jig}, \text{ which (in the presence of IO linkages) is defined as follows:}\n
\[ F_{jig}(t,x,\bar{p}) \equiv \ln \tilde{p}_{jig} - \ln (1 + t_{jig}) (1 + x_{jig}) p_{jig} \left( \frac{\tilde{p}^T_j}{w_j} \right) = 0, \quad \forall j,i \in \mathbb{C}; \quad g \in \mathbb{K} \]

where \( \tilde{p}_i \equiv \{ p_{jik} \}_{j \in \mathbb{C}, k \in \mathbb{K}} \) is the 2K \times 1 vector of all “consumer” prices in country \( i \), while \( t = [t_{jik}]_{jik} \), \( x = [x_{jik}]_{jik} \), and \( \bar{p} = [p_{jik}]_{jik} \) are 4K \times 1 vectors that contain all tax and price combinations in both the Home and Foreign economies \( (i = h \text{ and } f) \). Considering the above, the pass-through elasticity, \( \sigma_{jig} \equiv \partial \ln \tilde{p}_{jig} / \partial \ln (1 + t_{jik}) \) (which is a partial elasticity conditional on the wage rate, \( w_j \)), can be expressed as follows

\[
\sigma \equiv \left[ \frac{\partial \ln \tilde{p}_{jig}}{\partial \ln (1 + t_{jik})} \right]_{jig,ik} = \left[ \frac{\partial \ln \tilde{p}_{jig}}{\partial \ln (1 + x_{jik})} \right]_{jig,ik}
\]

\[
= (\nabla \ln \bar{p} F)^{-1} \nabla 1 + t \bar{F} = (\nabla \ln \bar{p} F)^{-1},
\]

where \( \sigma \), \( \nabla \ln \bar{p} F \), and \( \nabla 1 + t \bar{F} \) are 4K \times 4K matrixes, and the last line follows from the fact that \( \nabla 1 + t \bar{F} = I \). To characterize \( (\nabla \ln \bar{p} F)^{-1} \), we can invoke Shepherd’s Lemma. That is,

\[
\frac{\partial \ln p_{jig}(\cdot)}{\partial \ln \tilde{p}_{jik}} = \frac{\tilde{p}^T_{jik} a_{jig}^{\mu g}}{p_{jig} q_{jig}}, \quad \forall j,i \in \mathbb{C}; \quad g,k \in \mathbb{K}
\]

where \( a_{jig}^{\mu g} \equiv \tilde{p}^T_{jik} q_{jig} / p_{jig} q_{jig} \) denotes the share of intermediate input \( ji,k \) in the total production cost of output variety \( ji,g \) (with \( q_{jig}^{\mu g} \) denoting the amount of \( ji,k \)-type inputs used in the production of \( ji,g \)). Note that, by construction, \( a_{jig}^{\mu g} = 0 \) if \( i \neq j \). Considering the above equation, it is also straightforward to verify that

\[
\sigma = (\nabla \ln \bar{p} F)^{-1} = (I - A)^{-1},
\]

where \( A \equiv \left[ a_{jig}^{\mu g} \right]_{jig,ik} \) is the full 4K \times 4K input-output matrix of world production. So, altogether, the pass-through of tax \( 1 + t_{jik} \) (or \( 1 + x_{jik} \)) on to consumer prices, is fully characterized by the \( jik' \)th column of \( \sigma \), which is itself a function of the elements of the input-output matrix:

\[
\sigma_{ji,k} = (I - A)^{-1} \mathbf{1}_{ji,k},
\]

where \( \mathbf{1}_{ji,k} \) is a 4K \times 1 vector where the \( jik' \)th element is “one” and the remaining elements are “zero.” Correspondingly, \( \sigma_{ji} = [\sigma_{ji,k}]_k \) is a 4K \times K matrix comprised of all \( K \sigma_{ji,k} \)

\[37\text{Note that } F(\cdot) \text{ is a } 4K \times 1 \text{ vector.} \]
vectors. To apply Theorem 2, we specifically need (i) \( \sigma_{fh} \) that describes the pass-through of home’s import taxes, as well as (ii) \( \sigma_{hf} \) that describes the pass-through of home’s export taxes.

Note that we, hereafter, assume that \( I - A \) satisfies the Hawkins–Simon conditions and is non-singular. Moreover, note that note that since all \( a^\mu_g \) < 1 for all \( \mu, g \) and \( ji, k \), we can write \( I - A \) as:

\[
(I - A)^{-1} = I + A^2 + A^3 + A^4 + ...
\]

Given the above expression it is straightforward to show that if good \( ji, k \), is a “final” good (so that the \( ji \)'th column of \( A \) is zero), then \( \sigma_{ji,k} \) is the identity vector. That is, all elements of \( \sigma_{ji,k} \) are zero except for the \( ji \)'th element which is equal to one.

### F.2 Generalized Specific-Factors Model

S done above, to determine \( \partial \ln \tilde{p}_{\mu,g} / \partial \ln (1 + t_{ji,k}) = \partial \ln \tilde{p}_{\mu,g} / \partial \ln (1 + x_{ji,k}) \) for any two varieties \( \mu, g \) and \( ji, k \), we can applying the implicit function theorem to the price function, \( F(t, x, \tilde{p}) \equiv \{ F_{\mu,g} (t, x, \tilde{p}) \}_{\mu,g} \) which is defined as follows:

\[
F_{\mu,g} (t, x, \tilde{p}) \equiv \ln \tilde{p}_{\mu,g} - \ln (1 + t_{\mu,g}) (1 + x_{\mu,g}) a_{\mu,g} (q_{\mu,h} (\tilde{p}_h), q_{\mu,f} (\tilde{p}_f)) w_i = 0, \ \forall j, t \in C; g \in K
\]

where \( \tilde{p}_i \equiv \{ p_{ji,k} \}_{j \in C, k \in K} \) is the \( 2K \times 1 \) vector of all “consumer” prices in country \( i \), while \( t = [t_{ji,k}]_{ji,k} \), \( x = [x_{ji,k}]_{ji,k} \) and \( \tilde{p} = [p_{ji,k}]_{ji,k} \) are \( 4K \times 1 \) vectors that contain all tax and price combinations in both the Home and Foreign economies (\( i = h \) and \( f \)). Considering the above, the pass-through elasticity, \( \sigma_{ji,g} \equiv \partial \ln p_{\mu,g} / \partial \ln 1 + t_{ji,k} \) (which is a partial elasticity conditional on the wage rate, \( w_j \)), can be expressed as follows

\[
\sigma \equiv \left[ \frac{\partial \ln \tilde{p}_{\mu,g}}{\partial \ln (1 + t_{ji,k})} \right]_{\mu,g,ji,k} = \left[ \frac{\partial \ln \tilde{p}_{\mu,g}}{\partial \ln (1 + x_{ji,k})} \right]_{\mu,g,ji,k}
\]

\[
= (\nabla_{\ln \tilde{p}} F)^{-1} \nabla_{1+t} F = (\nabla_{\ln \tilde{p}} F)^{-1},
\]

where \( \sigma, \nabla_{\ln \tilde{p}} F \), and \( \nabla_{1+t} F \) are \( 4K \times 4K \) matrixes, and the last line follows from the fact that \( \nabla_{1+t} F = I \). It is also straightforward to verify that

\[
\sigma = (\nabla_{\ln \tilde{p}} F)^{-1} = (I - \Sigma)^{-1},
\]

---

38Note that \( F(.) \) is a \( 4K \times 1 \) vector.
where $\Sigma = \begin{bmatrix} \gamma_{j\mu, k} & \epsilon_{j\mu, k} \\ \mu, g, n, t, s \end{bmatrix}$. So, altogether, the pass-through of tax $1 + t_{ji,k}$ (or $1 + x_{ji,k}$) on to consumer prices, is fully characterized by the $ji,k$'th column of $\sigma$ as as function of supply and demand elasticities. In particular,

$$\sigma_{ji,k} = (I - \Sigma)^{-1} 1_{ji,k},$$

Recall that the two cases we are interested in are (i) vector $\sigma_{fh,k}$ describing the pass-through of home’s import tax, $1 + t_{fh,k}$, and (ii) vector $\sigma_{hf,k}$ describing the pass-through of home’s export tax, $1 + x_{hf,k}$, for every industry $k$.

### F.2.1 Special Cases: Ricardo-Viner model with Zero Cross-Demand Effects

The Ricardo-Viner model is a special case where $\gamma_{ji,k} \equiv \gamma_{ji,k} = \gamma_{ji,k}$ for all $j, i$, and $k$. But, in line with traditional trade policy literature, let us also assume that cross-price elasticities are zero (i.e., preference are additively separable and feature a non-traded quasi-linear sector à la Broda et al. (2008)). In this special case, the pass-through is characterized by the following expression:

$$\sigma_{ji,k} = \begin{cases} 0 & j, g \neq j, k \\ \frac{\epsilon_{ji,k} \gamma_{ji,k}}{1 - \epsilon_{ji,k} \gamma_{ji,k}} & j, g = j, k; \ i \neq t \\ \frac{1}{1 - \epsilon_{ji,k} \gamma_{ji,k}} & ji, g = ji, k \end{cases}$$

Given the expression for $\sigma_{ji,k}$, the terms-of-trade effects can be immediately calculated as follows:

$$\begin{cases} TOT_{t,k} = r_{fh,k} \left( 1 - \sigma_{fh,k} \right) = \frac{\epsilon_{fh,k} \gamma_{fh,k}}{1 - \gamma_{fh,k} \epsilon_{fh,k}} r_{fh,k} \\ \lambda_{hf,k} - TOT_{x,k} = \lambda_{hf,k} \sigma_{hf,k} = \frac{1}{1 - \gamma_{hf,k} \epsilon_{hf,k}} \lambda_{hf,k} \end{cases}$$

Finally, since $\epsilon_{ji,k} = 0$ if $i, k \neq ji, g$, plugging the above expressions into Equation 33, yields the following formulas for $T_k$ and $X_k$

$$\begin{cases} T_k = \left[ r_{fh,k} \left( 1 - \sigma_{fh,k} \right) \right] / \left( r_{fh,k} \epsilon_{fh,k} \sigma_{fh,k} \right) = \gamma_{fh,k} \\ X_k = \left( \lambda_{hf,k} \sigma_{hf,k} / \left( \lambda_{hf,k} \epsilon_{hf,k} \sigma_{hf,k} \right) \right) = 1 / \epsilon_{hf,k} \end{cases}$$

82
The above equation, in turn, leads us to the following familiar-looking optimal tax formula:

\[
\begin{align*}
1 + t_k &= (1 + \tilde{\tau}) \left( 1 + \gamma_{fh,k} \right) \\
1 + x_k &= 1 / (1 + \tilde{\tau}) \left( 1 + 1 / \varepsilon_{hf,k} \right)
\end{align*}
\]

In other words, Home’s optimal import tax is equal to the inverse of Foreign’s export supply elasticity, $\gamma_{fh,k}$. The above formula clearly outlines the symmetry between export and import taxes. In fact, in the special case where $\varepsilon_{hf,k} \to \infty$ (which is analog to the Ricardian assumption of $\gamma_{fh,k} = 0$, but imposed on the demand side), the export tax is uniform while the import tax is not.

### G Quantitative Methodology with IO Linkages

In this appendix, we present an analog to Proposition 7, but in the presence of IO linkages. To be able to conduct the quantitative analysis, we impose the additional restrictions that the global IO matrix, $A = [a_{ji,k}]$, is invariant to policy. Considering this, the first step is to use the global IO matrix from the WIOD to compute the pass-through matrix. This, can be simply done as follows:

\[
\sigma = (I - A)^{-1}
\]

Then, we can appeal to the following proposition to solve for Home’s optimal import and export tax levels as a solution to system of non-linear equations.

**Proposition 8.** Suppose the observed data is generated by a Ricardian model with input-output linkages and functional forms 19 and 20. Then, the optimal trade tax can be fully characterized by
solving the following system of equations

\[
\begin{bmatrix}
1 + t_{fh,k} \\
1 + x_{hf,k}
\end{bmatrix}
= 
\begin{bmatrix}
1 + T_k \\
1 / (1 + \lambda'_k)
\end{bmatrix},
\begin{bmatrix}
t_{hf,k} \\
x_{fh,k}
\end{bmatrix}
= 0
\forall k \in K
\]

\[
\begin{bmatrix}
\mathcal{T} \\
\mathcal{X}
\end{bmatrix}
= 
\begin{bmatrix}
(\hat{r}_{fh} \circ r_{fh}) \circ a_{fh}^{fr} \\
(\hat{r}_{fh} \circ r_{fh}) \circ a_{fh}^{fr}
\end{bmatrix}^{-1}
\begin{bmatrix}
b_{fh} \\
\lambda_{hf} - b_{hf}
\end{bmatrix}
\]

\[
a_{ji} = \epsilon_k \lambda_{ij,k} \lambda_{ij,k} \sigma_{ij,k}^{i,j} - (1 + \epsilon_k \hat{\lambda}_{ij,k} \lambda_{ij,k}) \sigma_{ij,k}^{i,j} \forall j, i, t \in C
\]

\[
b_{ji} = \left[ \sum_k \sigma_{ij,k}^{i,j} \lambda_{ij,k} \lambda_{ij,k} \sigma_{ij,k}^{i,j} - \sigma_{ij,k}^{i,j} \hat{r}_{fh} \circ r_{fh} \right] \kappa
\forall j, i \in C; \forall k \in K
\]

\[
\hat{\lambda}_{ij,k} = \left[ \hat{R}_j (1 + t_{ij,k}) (1 + x_{ij,k}) \right]^{-\epsilon_k} \hat{p}_{ij,k}
\forall j, i \in C; \forall k \in K
\]

\[
\hat{p}_{ij,k} = \left[ \hat{R}_j \sum_j \left[ \left( 1 - \sum_{ij,k} \sigma_{ij,k}^{i,j} \right) \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{Y}_i / (1 + x_{ij,k}) (1 + t_{ij,k}) \right] \right]^{-\epsilon_k} \lambda_{ij,k}
\forall j, i \in C
\]

in terms of \{t_{fh,k}\}, \{x_{hf,k}\}, \{\hat{r}_{ij,k}\}, \{\hat{p}_{ij,k}\}, \{\hat{R}_i\}, and \{\hat{Y}_i\}, as a function of (i) tax pass-through, \{\sigma_{ij,k}^{i,j}\} that are fully determined by the global IO matrix, \(A = [a_{ij,k}^{i,j}]_{ij,k}\), (ii) observed expenditure shares, \{\lambda_{ij,k}\}; (iii) observed output shares, \{\lambda_{ij,k}\};(iv) observed national output and income levels, \(R_i = w_i L_i \) and \(Y_i\); as well as (v) industry-level trade elasticities, \[\epsilon_k\].

Note that one we solve for the optimal policy, \(e\) can immediately calculate the gains from policy as \(\hat{Y}_i / \Pi_k \hat{p}_{ij,k}^{\epsilon_k}\).

**H Additional Tables**
Table 2: List of countries in quantitative analysis

<table>
<thead>
<tr>
<th>Country name</th>
<th>WIOD code</th>
<th>Basic aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>AUS</td>
<td>Australia</td>
</tr>
<tr>
<td>Brazil</td>
<td>BRA</td>
<td>Brazil</td>
</tr>
<tr>
<td>Canada</td>
<td>CAN</td>
<td>Canada</td>
</tr>
<tr>
<td>China</td>
<td>CHN</td>
<td>China</td>
</tr>
<tr>
<td>Indonesia</td>
<td>IDN</td>
<td>Indonesia</td>
</tr>
<tr>
<td>India</td>
<td>IND</td>
<td>India</td>
</tr>
<tr>
<td>Japan</td>
<td>JPN</td>
<td>Japan</td>
</tr>
<tr>
<td>Korea</td>
<td>KOR</td>
<td>Korea</td>
</tr>
<tr>
<td>Mexico</td>
<td>MEX</td>
<td>Mexico</td>
</tr>
<tr>
<td>Russia</td>
<td>RUS</td>
<td>Russia</td>
</tr>
<tr>
<td>Turkey</td>
<td>TUR</td>
<td>Turkey</td>
</tr>
<tr>
<td>Taiwan</td>
<td>TWN</td>
<td>Taiwan</td>
</tr>
<tr>
<td>United States</td>
<td>USA</td>
<td>United States</td>
</tr>
<tr>
<td>Austria</td>
<td>AUT</td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>BEL</td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>BGR</td>
<td></td>
</tr>
<tr>
<td>Cyprus</td>
<td>CYP</td>
<td></td>
</tr>
<tr>
<td>Czech Republic</td>
<td>CZE</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>DEU</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>DNK</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>ESP</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>FIN</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>FRA</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>GBR</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>GRC</td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>HUN</td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>IRL</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>ITA</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>NLD</td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>POL</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>PRT</td>
<td></td>
</tr>
<tr>
<td>Romania</td>
<td>ROM</td>
<td></td>
</tr>
<tr>
<td>Slovakia</td>
<td>SVK</td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>SVN</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>SWE</td>
<td></td>
</tr>
<tr>
<td>Estonia</td>
<td>EST</td>
<td></td>
</tr>
<tr>
<td>Latvia</td>
<td>LVA</td>
<td></td>
</tr>
<tr>
<td>Lithuania</td>
<td>LTU</td>
<td></td>
</tr>
<tr>
<td>Luxemburg</td>
<td>LUX</td>
<td></td>
</tr>
<tr>
<td>Malta</td>
<td>MLT</td>
<td></td>
</tr>
<tr>
<td>Rest of the World</td>
<td>RoW</td>
<td>Rest of the World</td>
</tr>
</tbody>
</table>

European Union
Table 3: List of industries in quantitative analysis

<table>
<thead>
<tr>
<th>WIOD Sector</th>
<th>Sector’s Description</th>
<th>Trade Elasticity (Caliendo-Parro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture, Hunting, Forestry and Fishing</td>
<td>8.11</td>
</tr>
<tr>
<td>2</td>
<td>Mining and Quarrying</td>
<td>15.72</td>
</tr>
<tr>
<td>3</td>
<td>Food, Beverages and Tobacco</td>
<td>2.55</td>
</tr>
<tr>
<td>4</td>
<td>Textiles and Textile Products, Leather and Footwear</td>
<td>5.56</td>
</tr>
<tr>
<td>5</td>
<td>Wood and Products of Wood and Cork</td>
<td>10.83</td>
</tr>
<tr>
<td>6</td>
<td>Pulp, Paper, Paper, Printing and Publishing</td>
<td>9.07</td>
</tr>
<tr>
<td>7</td>
<td>Coke, Refined Petroleum and Nuclear Fuel</td>
<td>51.08</td>
</tr>
<tr>
<td>8</td>
<td>Chemicals and Chemical Products</td>
<td>4.75</td>
</tr>
<tr>
<td>9</td>
<td>Rubber and Plastics</td>
<td>1.66</td>
</tr>
<tr>
<td>10</td>
<td>Other Non-Metallic Mineral</td>
<td>2.76</td>
</tr>
<tr>
<td>11</td>
<td>Basic Metals and Fabricated Metal</td>
<td>7.99</td>
</tr>
<tr>
<td>12</td>
<td>Machinery, Nec</td>
<td>1.52</td>
</tr>
<tr>
<td>13</td>
<td>Electrical and Optical Equipment</td>
<td>10.60</td>
</tr>
<tr>
<td>14</td>
<td>Transport Equipment</td>
<td>0.37</td>
</tr>
<tr>
<td>15</td>
<td>Manufacturing, Nec; Recycling</td>
<td>5.00</td>
</tr>
<tr>
<td>16</td>
<td>Electricity, Gas and Water Supply, Construction, Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel, Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles, Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods, Hotels and Restaurants, Inland Transport, Water Transport, Air Transport, Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies, Post and Telecommunications, Financial Intermediation, Real Estate Activities, Renting of M&amp;Eq and Other Business Activities, Education, Health and Social Work, Public Admin and Defence; Compulsory Social Security, Other Community, Social and Personal Services, Private Households with Employed Persons</td>
<td>100</td>
</tr>
</tbody>
</table>