Optimal Trade Policy in
Global Production Networks

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Abstract

How should countries approach trade policy when immersed in global production networks? How large are the gains from trade policy in such cases? To answer these questions, we characterize optimal trade policy in an important class of quantitative trade models with global production networks. Our theory indicates that, absent diseconomies of scale, the optimal trade policy consists of (i) uniform or zero import taxes and (ii) non-zero export taxes that are decreasing in the industry-level trade elasticity and level of upstreamness. But if export taxes are restricted, it is optimal to raise import taxes as a second-best policy to manipulate export market power through the global production network. Mapping our analytic tax formulas to data, we find that global production networks double the non-cooperative gains from optimal trade policy; but only 47% of these gains can be replicated with import taxes alone.

1 Introduction

Global production networks have transformed the world economy over the past few decades, presenting policy markers with many unprecedented challenges. Some of these challenges were manifested when the U.S. government applied tariffs on steel imports in 2018. By disrupting the global input-output network, the U.S. steel tariffs had ramifications well-beyond their local effect on the U.S. steel
industry. These unintended consequences prompted the U.S. government to respond later with additional policy measures, which may further disrupt the global input-output network.

Despite these growing challenges, our conceptual understanding of optimal trade policy in global production networks has advanced little over the past few decades.\(^1\) During this period, trade economists have developed various quantitative models that tractably accommodate global production networks. Multiple studies have used these models to quantify the welfare gains from trade or tariff liberalization.\(^2\) Yet we lack a formal theory of optimal trade policy that applies to this canonical class of models.

This paper takes a basic step towards building such a theory. We characterize optimal trade policy in a non-parametric, multi-industry neoclassical trade model with global production networks. Our theory readily applies to an important class of workhorse quantitative trade models and delivers several stark predictions:

(a) Absent diseconomies of scale, the optimal non-cooperative policy consists of

(i) uniform or zero import taxes and

(ii) non-zero export taxes that are decreasing in the industry-level trade elasticity and level of upstreamness.\(^3\)

(b) But if the use of export taxes is restricted by external agreements or institutional barriers, import taxes can be used as a second-best instrument to manipulate export market power through the global production network.

(c) Optimal second-best import taxes depend primarily on the extent to which imported goods are combined with domestic factors and used in the production of highly-differentiated exports. They are uniform across all final goods and intermediate goods that are never employed in exporting industries.

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\(^1\)The same applies to the political economy of trade taxation in global production networks. But the recent work of Blanchard et al. (2016) has bridged this gap considerably.

\(^2\)Most notably, Caliendo and Parro (2014) compute the gains from post-NFATA tariff liberalization. Costinot and Rodriguez-Clare (2014) compute the gains from a 20% uniform tariff applied by the U.S. under global input-output networks. More recently, Baqee and Farhi (2019) use a non-parametric trade model with global production networks to decompose the welfare gains from tariff liberalization into pure trade-driven gains and allocative efficiency gains.

\(^3\)The absence of diseconomies of scale is akin to the complete tariff passthrough estimated by Faigelbaum et al. (2020) and Amiti et al. (2019). That import taxes can be zero or uniform is a manifestation of the Lerner symmetry. Specifically, one way to attain the optimal policy outcome is to set import taxes to zero. Another way is for import taxes to be assigned a uniform rate and for all export taxes be lowered uniformly at the same rate.
To put these claims in perspective, we map our analytic tax formulas to data and compute the gains from optimal policy across 40 major economies. We find that a non-cooperative country can, on average, gain 2.85% from optimal export taxation, but only 47% of these gains are reproducible with second-best import taxes. These findings shed new light on the political economy of trade taxation and suggest that overlooking global production networks can lead to an underestimation of the gains from trade agreements.

Section 2 presents our theoretical model, which is a general equilibrium non-parametric neoclassical trade model that accommodates an arbitrary number of industries, an arbitrary demand structure, an arbitrary global input-output network, and diseconomies of scale due to industry-specific factors of production. Our general model nests several canonical trade models, including the basic multi-industry gravity model in Costinot et al. (2011) as well as the multi-industry gravity model with input-output linkages in Caliendo and Parro (2014).

We assume that governments have access to a full vector of industry-level import and export taxes. These taxes allow a non-cooperative government to correct two terms-of-trade-related inefficiencies:

i. Unexploited import market power, which concerns the unexploited ability of the Home country to charge a mark-down on the price of imported goods.

ii. Unexploited export market power, which concerns the unexploited ability of the Home country to charge a mark-up on the price of exported goods.

In our perfectly competitive framework, correcting these two inefficiencies allows non-cooperative countries to reach their first-best outcome. But these unilateral corrections worsen global efficiency and have beggar-thy-neighbor effects on the rest of the world.

Section 3 characterizes the optimal first-best trade taxes in our framework, delivering sufficient statistics optimal tax formulas that depend on only (i) industry-level trade elasticities, (ii) share of industry-specific capital in production, and (iii) observable expenditures, revenue, and input-output shares. A key implication of these formulas is that, unlike export taxes, first-best import taxes do not explicitly depend on the global input-output structure.

Absent industry-level diseconomies of scale, import taxes are also a redundant policy instrument. The first-best trade tax schedule consists of (a) uniform or zero
import taxes, and (b) non-zero export taxes that are decreasing in the industry-level trade elasticity and level of upstream-ness.

The redundancy of import taxes drives from the targeting principle and the Lerner symmetry. An Import tax on intermediate inputs is partially passed on to foreign consumers who buy the goods produced with these inputs. This way, import taxes can extract a markup from foreign consumers as they propagate through the production network. By the targeting principle, however, export taxes are the first-best instrument at extracting the same markup. Import taxes can also improve Home’s wage relative to the rest of the world. By the Lerner symmetry, however, the same wage effects can be replicated with a uniform export tax. So, altogether, import taxes can be discarded when constructing the first-best tax schedule.

In the presence of production networks, an export tax on intermediate inputs is also partially passed back to domestic consumers. The optimal export tax is, thus, lower on more upstream industries for which a larger fraction of the tax is re-imported back into the domestic economy. The same propagation effects also present the Home government with extra-territorial taxing power: an ability to effectively tax transactions between foreign consumers and producers. This unique ability magnifies the gains from taxing upstream exports, despite the lower optimal tax rate on these industries.

Section 4 considers a second-best scenario where the use of export taxes is restricted but governments can freely apply import taxes. We show that, absent diseconomies of scale, the optimal second-best import tax is uniform on all industries supplying final goods or intermediate inputs that are never used in exporting sectors. The optimal import tax on other industries is non-uniform and increasing in (a) the inverse of trade elasticity, and (b) the extent to which the industry supplies inputs to highly-differentiated, export-oriented sectors.

There is a simple intuition behind the above result. In a second-best scenario, import taxes can be used as an indirect tool to exploit export market power through the global production network. To that goal, governments should tax imported inputs that are used intensively in highly-differentiated, export-oriented sectors. Such import taxes are partially passed on to foreign consumers, acting as an indirect export tax. The trade-off, though, is that import taxation of this kind distorts domestic production in a way that is avoidable with first-best export taxes.

In Section 6 we use our analytic formulas to quantify the gains from trade pol-
icy. We employ data on applied tariffs, expenditure, production, and input-output shares across 16 industries and 42 major economies. Our data includes all 27 members of the European Union, Australia, Brazil, China, Indonesia, India, Japan, Korea, Mexico, Norway, Russia, Switzerland, Taiwan, Turkey, the United States, and an aggregate of the rest of the world.

With the aid of our sufficient statics tax formulas, the task of computing the gains from optimal policy reduces to solving a system of equations that requires data on only (i) industry-level trade elasticities, (ii) observable expenditure and revenue shares, as well as (iii) global input-output shares. Our quantitative analysis delivers two major results:

i. Production networks double the gains from optimal non-cooperative trade taxation, and also double to the externality of such taxes on the rest of the world. To provide numbers, the average country in our analysis can use non-cooperative export taxes to raise its real GDP by 2.85%. However, if we overlook the role of global production networks, the gains from optimal export taxation reduce to 1.40%. Furthermore, the gains are relatively larger for countries like Norway and Russia that are net exporters in upstream industries like Petroleum.

ii. Import taxes can only replicate 47% of the total gains attainable under first-best export taxes. Specifically, non-cooperative import taxes can at best raise the average country’s real GDP by 1.36%. The ineffectiveness of second-best import taxes is driven by the innate trade-off between preserving allocative efficiency in the local economy and exploiting export market power through the production network. The former discourages taxing differentiated intermediate inputs, whereas the latter requires it.

Beyond these basic results, our theory has important implications for both the political economy of trade taxation and gains from trade agreements. On the former issue, our theory can reconcile the terms-of-trade approach with evidence that consumers of intermediate inputs are more organized and that governments assign a low political weight to protecting upstream industries (Shapiro (2019)). Our optimal tariff formulas imply political weights that perfectly align with such evidence. Our theory also indicates that export taxes are underexploited, insofar as term-of-trade motives are concerned. In that regards, it speaks to an old literature that
estimates how much term-of-trade considerations matter in the governments’ objective function.

Our theory also indicates that overlooking global production networks can understate the gains from trade agreements. To make this point, suppose trade agreements break down but governments cannot raise export taxes due to institutional barriers. In that case, non-cooperative governments, that are immersed in global production networks, will apply Nash tariffs that are targeted at highly-differentiated intermediate inputs. Such targeted tariffs are unilaterally optimal in a non-cooperative equilibrium, but they are also more disruptive to global efficiency. Overlooking the efficiency loss due to targeted Nash tariffs of this sort will understate the preventative gains from trade agreements.

**Related Literature.** To the best of our knowledge, this paper is the first to analytically characterize the optimal trade policy in an important class of general equilibrium quantitative trade models that accommodate global production networks. Several studies have adopted similar models to study how production networks influence the gains from tariff liberalization (e.g., Caliendo et al. (2015); Caliendo et al. (2015); and Baqaee and Farhi (2019)). But we are not aware of any prior characterization of optimal trade policy in these canonical models.\(^4\)

The closest to our paper in this regard is Blanchard et al. (2016) who characterize the optimal final-good tariff in a partial equilibrium trade model under political economy considerations. Relative to the aforementioned study, our conceptual framework excludes political economy pressures. It, however, accommodates a wide range of general equilibrium linkages, analyzes both import and export taxation, and places no restrictions on how trade taxes discriminate between final or intermediate input varieties of the same good. These extensions allow us to identify a set of new insights that have eluded the prior literature on this topic.

Our optimal tax formulas, meanwhile, have the advantage of simplifying trade policy analysis in multi-industry general equilibrium trade models. It is well-known from the work of Costinot and Rodríguez-Clare (2014) that the analysis

\(^4\)There is an old literature that characterizes the optimal trade tax on intermediate inputs (e.g., Suzuki (1978); Das (1983)). This literature typically assumes a three-good, partial equilibrium economy with only one intermediate good that is produced in only one country. These frameworks, as a result, do not feature a global production network in its formal sense, and deliver starkly different predictions relative to our framework.
of optimal policy can become increasingly complicated in these frameworks under standard optimization techniques. Our analytic formulas allow researchers to bypass some of these complications. They reduce the task of computing the gains from optimal policy to that of solving a system of equations, which depend on observables and industry-level trade elasticities.

Our finding that optimal tariffs are uniform across industries, and therefore redundant, complements the previous assertions in Costinot et al. (2015) and Opp (2010). Both of these papers adopt a general equilibrium multi-good Ricardian trade model and show that optimal tariffs are uniform when the unit labor cost is invariant to industry-level output. We extend this finding to a non-parametric neoclassical trade model that accommodates global input-output networks.

Finally, our paper is related to recent analyses of trade agreements using quantitative trade models. Ossa (2014, 2016) highlight the importance of accounting for market imperfections when quantifying the gains from trade agreements. Bagwell et al. (2018) highlight the importance of inter-connected bilateral negotiations. Our paper contributes to this literature, by arguing that accounting for global production networks can further magnify the estimated gains from trade agreements.

2 The Economic Environment

We consider a global economy that consists of \( i = 1, ..., N \) countries (with \( C \) denoting the set of countries) and \( k = 1, ..., K \) industries (with \( K \) denoting the set of industries). Except for Section 5, our analysis focuses on a two-country case where \( h \) indexes the Home country and \( f \) indexes the Foreign country representing an aggregate of the rest of the world, i.e., \( C = \{ h, f \} \).

Each country \( i \) is populated with \( L_i \) workers that are perfectly mobile across industries but immobile across countries, with \( L_{i,k} \) denoting the number of workers employed in industry \( k \). Each industry in country \( i \) is also endowed with \( K_{i,k} \) units of an industry-specific capital that is inelastically supplied.

Industry \( k \in K \) in country \( i \in C \) produces a differentiated variety that is ultimately sold in some market \( j \in C \). We use \( i j, k \) (origin \( i \)–destination \( j \)–industry \( k \)) to index these product varieties. We use the superscript \( C \) to denote if a good is used as a final consumption good and superscript \( I \) to denote of its used as an intermediate input. Since we impose no restrictions on the size or the num-
ber of industries, we can interpret index $k$ as denoting narrowly-defined product categories rather than broadly-defined industries.

2.1 Preferences

The representative consumer in country $i$ choose the vector of consumption quantities, $q^C_i \equiv \{q^C_{ji,k}\}$, to maximize a non-parametric utility function, $U_i(q^C_i)$, subject to a budget constraint. The superscript $C$, as noted earlier, differentiates between final consumption varieties and intermediate input varieties that are denoted by $I$. The optimal choice of the consumer yields an indirect utility function,

$$V_i(Y_i, \tilde{p}^C_i) \equiv \max_{q^C_i} U_i(q^C_i)$$

s.t. $\sum_{k \in K} \sum_{j \in C} (\tilde{p}^C_{ji,k} q^C_{ji,k}) = Y_i$. (1)

The above problem also yields a non-parametric Marshallian demand function,

$$q^C_i = D_i(Y_i, \tilde{p}^C_i),$$

which summarizes the demand-side of the economy as a function of net consumption income $Y_i$ and the vector of consumer prices $\tilde{p}^C_i \equiv \{\tilde{p}^C_{ji,k}\}$ in country $i$. The tilde notation on the price variables is used to differentiate the after-tax final prices from pre-tax producer prices. To keep track of optimal demand choices, we define the price and income elasticities of demand as follows.

.D1 [Marshallian Demand Elasticities]

(i) [own price elasticity] $\varepsilon_{ji,k} \equiv \partial \ln q^C_{ji,k} / \partial \ln \tilde{p}_{ji,k}$;

(ii) [cross-price elasticity] $\varepsilon_{ji,k}^{\mu \neq ji} \equiv \partial \ln q^C_{ji,k} / \partial \ln \tilde{p}_{ji,k}$ for $\mu, \neq ji, k$;

(iii) [income elasticity] $\eta_{ji,k} \equiv \partial \ln q^C_{ji,k} / \partial \ln Y_i$.

Throughout the paper, we restrict our attention to well-behaved demand functions that are continuous and locally non-satiated. We also assume that demand for each traded variety exhibits an elastic region where $| \varepsilon_{ji,k} | > 1$. As in monopoly problems, this condition will be necessary for obtaining a bounded solution for optimal trade taxes.
2.2 Technology

We assume that firms are competitive and operate with a non-parametric production function that employs (i) labor, (ii) intermediate inputs, and (iii) industry-specific capital. To elaborate, let $Q_{i,k} = \sum_j \tau_{ij,k} q_{ij,k}$ denote country $i$’s total output in industry $k$, where $\tau_{ij,k} \geq 1$ denotes the iceberg melt cost associated with transporting variety $ij,k$—as is standard in literature we normalize $\tau_{ii,k} = 1$. The industry-level output is produced using a non-parametric constant returns to scale production function,

$$Q_{i,k} = F_{i,k}(L_{i,k}, \bar{K}_{i,k}, q_{i,k})$$

that combines labor, $L_{i,k}$, industry-specific capital, $\bar{K}_{i,k}$, and intermediate inputs, $q_{i,k}$.

We assume that the share of labor plus industry-specific capital is constant in total production and equal to $\bar{\alpha}_{i,k}$. However, we take no parametric stance on how labor and capital are combined or how different intermediate inputs are combined in the production function.

Facing the above production structure, cost-minimizing firms charge a competitive “producer” price that is a function of (i) the wage rate in economy $i$, $w_i$, (ii) the vector of intermediate input prices employed by producers in country $i$, $\tilde{p}_{i,k} \equiv \{\tilde{p}_{ji,k}\}$, and (iii) total industry-level output, $Q_{i,k}$. Namely,

$$p_{i,j,k} = \tau_{ij,k} C_{i,k}(w_i, \tilde{p}_{i,k}^T; Q_{i,k}),$$

where $C_i(.)$ is homogeneous of degree 1 with respect to $w_i$ and $\tilde{p}_{i,k}^T$. A familiar special case of this general structure is the Ricardian case in which $C_{i,k} = a_{i,k} w_i$, where $a_{i,k}$ is a constant unit labor cost that is invariant to industry-level output.

Cost minimization by suppliers in industry $k$ of country $i$ yields an industry-level demand function for intermediate inputs, $\tilde{D}_{i,k}(Y_{i,k}, \tilde{p}_{i,k}^T)$, which depends on gross expenditure on intermediate inputs, $Y_{i,k} = \sum_n (1 - \bar{\alpha}_{i,k}) p_{in,k} q_{in,k}$, and the entire vector of intermediate input prices in country $i$. Country $i$’s overall demand for intermediate inputs, $q_{i}^T \equiv \{q_{ji,k}^T\}$, is determined as the sum of demands across all industries:

$$q_{i}^T = \sum_k \tilde{D}_{i,k}(Y_{i,k}, \tilde{p}_{i,k}^T).$$

The above production structure implicitly assumes a non-finite elasticity of transformation between varieties sold in different markets. Relaxing this assumption will lead to diseconomies of scale at the good rather than industry level (see Powell and Gruen (1968)).
For notational convenience, we assume that the after-tax price of product, \( ji,k \), is the same whether it is used as an intermediate input (indexed \( I \)) or a consumption good (indexed \( C \)):

\[
\tilde{p}^I_{ji,k} = \tilde{p}^C_{ji,k} = \tilde{p}_{ji,k}.
\]

This assumption is innocuous since we can always extend the set of goods so that \( ji,k \) indexes only the final good varieties, while \( ji,k' \) indexes only the intermediate input varieties supplied by the same industry.\(^6\) With the same rationale, we also assume that for all \( k \),

\[
\frac{\partial \ln D_i(.)}{\partial \ln \tilde{p}^I_i(.)} = \frac{\partial \ln D_i(.)}{\partial \ln \tilde{p}^C_i(.)} = \frac{\partial \ln D_i(.)}{\partial \ln \tilde{p}^T_i}.
\]

We use input-output shares to keep track of global production networks. To define these shares, let \( q^T_{i,\ell,g}(k) \) denote the optimal amount of input \( \ell_i,g \) used by industry \( k \) in country \( i \)—by construction, \( q^T_{i,\ell,g} = \sum_k q^T_{i,\ell,g}(k) \). Considering this choice of notation, we define the global input-output shares as follows.

\[ \text{D2 [Input-Output Shares]} \] The share of intermediate input goods from “country \( \ell \times \text{industry } g \)” that are used in the production of output goods in “country \( j \times \text{industry } k \)” is defined as

\[
\alpha^{\ell,g}_{i,j,k} = \frac{\tilde{p}^T_{i,\ell,g} q^T_{i,\ell,g}(k)}{p_{ji,k} q_{ji,k}}.
\]

To be clear, we do not impose that input-output shares be constant. Instead, \( \alpha \)'s can be variable, and will presumably change in response to trade taxation, e.g., taxing input \( \ell_i,g \) will lower \( \alpha^{\ell,g}_{i,j,k} \). Our input-output setup is also flexible enough to allow for the expenditure share on the intermediate input varieties of a given good to diverge from the expenditure share on its final good varieties. A well-known special case of our general input-output structure is Caliendo and Parro (2014), which is formally discussed in Section 3.1.

We use the supply elasticity, as defined below, to keep track of diseconomies of scale at the industry level.

\[ \text{D3 [Industry-Level Supply Elasticity]} \] \( \gamma_{i,k} \equiv \partial \ln C_{i,k}(\cdot)/\partial \ln Q_{i,k} \)

The above definition is motivated by the observation that \( C_{i,k}(\cdot, Q_{i,k}) \) characterizes the industry-level supply curve for country \( i \). To gain intuition about how this elasticity relates to diseconomies of scale, consider the case where the share of the industry-specific capital in production is constant and equal to \( \beta_{i,k} \). In that case,\(^6\)Following this argument, our model still accommodates cases where the government imposes differential tax rates on final good varieties \( k \) versus intermediate input varieties \( k' \).
it is straightforward to show that \( \gamma_{i,k} = \beta_{i,k} / (1 - \beta_{i,k}) \). Beyond this special case as well, \( \gamma_{i,k} \) reflects the importance of the inelastically-supplied capital in production. As we will see shortly, \( \gamma_{i,k} \) also governs the degree of national-level import market power in each industry.

### 2.3 Trade Policy Instruments

The government in country \( i \) has access to a full set of industry-level export tax-cum-subsidy instruments (denoted by \( x \)) and import tax-cum-subsidy instruments (denoted by \( t \)). Together, these policy instruments create a wedge between the after-tax price, \( \hat{p}_{ji,k} \), and the producer price, \( p_{ji,k} \), of each good \( ji,k \) as follows:

\[
\hat{p}_{ji,k} = (1 + t_{ji,k}) (1 + x_{ji,k}) p_{ji,k}.
\] (3)

In the above equation, \( t_{ji,k} \) denotes the import tax applied by country \( i \) on good \( ji,k \), while \( x_{ji,k} \) denotes the export tax applied by country \( j \) on the same good. The combination of these tax instruments raises the following tax revenue for the government in country \( i \):

\[
R_i = \sum_{k \in K} \sum_{j \in C} \left[ t_{ji,k} (1 + x_{ji,k}) p_{ji,k} q_{in,k} + x_{ij,k} p_{ij,k} q_{ij,k} \right].
\] (4)

Tax revenues are rebated to the consumers in a lump-sum fashion.\(^7\) Throughout this paper, we assume that domestic policies are unavailable, which amounts to \( t_{ii,k} = x_{ii,k} = 0 \) for all \( i \) and \( k \). As noted earlier, our product space is general enough that \( k \) can index only the final good version or the intermediate input version of each industry’s output. So, in principle, trade taxes can arbitrarily discriminate between final consumption and intermediate input varieties of the same good.

### 2.4 General Equilibrium

We assume throughout this paper that equilibrium is unique—noting that uniqueness can be formally established using the procedure in Alvarez and Lucas (2007). Below, we formally define the general equilibrium in our setup.

\(^{7}\)Since labor is inelastically supplied in our framework, lump-sum rebates are observationally equivalent to a wage subsidy or a uniform consumption subsidy.
Definition. For any given vector of taxes, $t$, and, $x$, equilibrium is a vector of wages, $w$, (pre-tax) producer and (after-tax) final prices, $p_i$ and $\tilde{p}_i$, final consumption and input demand choices, $q^C_i$ and $q^T_i$, total surplus paid to industry-specific capital, $\Pi$, net consumer expenditure, $Y$, and gross industry-level output, $\mathcal{Y}$, such that (i) the producer price for each good is characterized by Equation 2; (ii) the consumer price for each good is given by Equation 3; (iii) consumption choices, $q^C_i = D_i(Y_i, \tilde{p}_i)$, are a solution to 1; (iv) demand for intermediate inputs, $q^T_i = \sum_k \tilde{D}_i(Y_i, k, \tilde{p}^T_i)$, is chosen to minimize cost; (iv) factor markets clear

$$w_iL_i + \Pi_i = \sum_j \sum_k p_{ij,k}q_{ij,k} - \sum_j \sum_k p^T_{ji,k}q^T_{ji,k} \quad \forall i,$$

and (v) net consumption income equals factor income plus tax revenue

$$Y_i = w_iL_i + \Pi_i + R_i \quad \forall i,$$

where the tax revenue, $R_i$, is given by Equation 4.

To streamline the presentation of our theory, we henceforth express aggregate welfare in country $i$ as a function of taxes $t$, and, $x$, and wages, $w$,

$$W_i(t, x; w) \equiv V_i(Y_i(t, x; w), \tilde{p}_i(t, x; w)).$$

$w$ is an equilibrium outcome and, thus, an implicit function of trade taxes, So, we use $A$ to denote the set of all wage×policy combinations, $A = (t, x; w)$, that are feasible. The reason we express $W_i$ as a function of $(t, x; w)$ rather than just trade taxes is that an across-the-board shift in trade taxes combined with an equal-proportional adjustment to nominal wage rates preserves welfare. Expressing equilibrium outcomes in terms of the triplet $(t, x; w)$ enables us to track this kind of tax neutrality.

In equilibrium, the importance of each good for taxation purposes is determined (among other things) by its share in gross expenditure and output. So before concluding this section, it is useful to define the aforementioned shares for good $ji, k$ (origin $j$–destination $i$–industry $k$).

. D4 [Gross Expenditure and Output Shares] 
[within-industry expenditure share] $\lambda_{ji,k} \equiv \frac{\tilde{p}_{ji,k}q_{ji,k}}{\sum_{g \in K} \tilde{p}_{ji,k}q_{ji,k}}$
[overall expenditure share] $\hat{\lambda}_{ji,k} \equiv \frac{\tilde{p}_{ji,k}q_{ji,k}}{\sum_{j \in C} \sum_{g \in K} \tilde{p}_{ji,k}q_{ji,k}}$. 

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Within-industry output share \( r_{ji,k} \equiv \frac{p_{ji,k}q_{ji,k}}{\sum_{i \in C} p_{ji,k}q_{ji,k}} \)

Overall output share \( \bar{r}_{ji,k} \equiv \frac{p_{ji,k}q_{ji,k}}{\sum_{i \in C} \sum_{g \in K} p_{ji,g}q_{ji,g}} \)

Gross expenditure and output shares, as defined above, are directly observable in standard trade datasets. As we well see in the next section, optimal trade taxes can be fully characterized in terms of these observable shares, as well as input-output shares and the reduced-form demand and supply elasticities defined earlier.

3 Optimal Non-Cooperative Trade Taxes

We first characterize Home’s optimal (first-best) non-cooperative policy. This is a scenario where Home can apply both import and export taxes,

\[
\begin{cases}
  t_k \equiv t_{fh,k} \\
  x_k \equiv x_{hf,k}
\end{cases}
\]

but the rest of the world is passive and does not retaliate, \( t_{hf,k} = x_{fh,k} = 0 \). The vectors \( t \equiv \{t_k\} \) and \( x \equiv \{x_k\} \), therefore, encompass only the Home government’s trade tax instruments hereafter. With this choice of notation, Home’s optimal non-cooperative trade taxes solve the following problem:

\[
\max_{(t,x,w) \in A} W_h(t,x,w)
\]

It is needless to say that the above problem is complicated by a myriad of general equilibrium interrelations. We can nonetheless simplify the problem by appealing to several intermediate results, the first of which is the Lerner symmetry.

Lemma 1. [The Lerner Symmetry] For any \( a \in \mathbb{R}_+ \), combinations \( A = (1 + t_i, 1 + x_i; w_f, w_h) \) and \( A' = (a(1 + t), (1 + x)/a; w_f, aw_h) \) represent identical equilibria, i.e., \( W_i(A) = W_i(A') \) for all \( i \).

The above lemma simplifies our analysis as follows: Optimal import and export taxes both feature a uniform term that accounts for the ability of trade taxation to increase Home’s wage relative to Foreign \( (w_h/w_f) \). Following the above lemma, we need not to formally characterize this term as it is redundant. There
is another way to cast this redundancy: When both export and import taxes are available, Lemma 1 states that we can normalize wages in both economies (i.e., set \( w_h = w_f = 1 \)) and still identify one of the multiple optimal tax combinations. This result, however, follows only if a full set of industry-level export and import tax instruments are applicable. If the policy space is restricted in any way, the uniform tax component that accounts for the general equilibrium changes in \( w_h / w_f \) should be formally characterized—see Section 4 for further details.

In our general equilibrium setup, trade taxes have a non-trivial passthrough onto the vector of final prices, \( \tilde{p}_f \), in each country. A tax on one good can alter the entire vector of prices through its effect on country-level wages, input prices, and industry-wide output. To deal with this complication, we can cast our original optimal policy problem as one where the government directly chooses final prices, \( \tilde{p}_{fh}, \tilde{p}_{hf}, \) and \( \tilde{p}_{hh} \) to maximize Home’s welfare \( W_h(\tilde{p}; w) \equiv V_h(Y_h(\tilde{p}; w), \tilde{p}) \). Stated formally, the optimal policy problem can be cast as

\[
\max_{(\tilde{p}; w) \in \tilde{A}} W_h(\tilde{p}; w),
\]

where \( \tilde{A} \) denotes the set of feasible wage-price combinations that is defined analogously to \( A \). Implicit in the above formulation is the observation that (i) \( W_h(\cdot) \) does not explicitly dependent on \( \tilde{p}_f \), and (ii) the market equilibrium is efficient, so it is optimal to set \( \tilde{p}_{hh} = p_{hh} \). The above problem, as a result, corresponds to one where the government chooses the consumer-to-producer price wedges that pin down the trade taxes: \( \tilde{p}_{fh} / p_{fh} = 1 + t \) and \( \tilde{p}_{hf} / p_{hf} = 1 + x \).

Finally, we can appeal to supply- and demand-side envelop conditions to handle general equilibrium behavioral responses. On the demand side, we can appeal to Roy’s identity, whereby the direct welfare effect of a change on consumer price, \( \tilde{p}_{j,h,k}' \), is reduced to \( \partial V_h(\cdot) / \partial \tilde{p}_{j,h,k}' = -q_{j,h,k}' \). Similarly, we can appeal to Shepard’s lemma to account for input-output-driven price linkages. Specifically, a change in input price \( \tilde{p}_{li,g}' \), holding the wage and all other input prices fixed, has the following effect on the output price of a cost-minimizing supplier:

\[
\frac{\partial \ln p_{ij,k}}{\partial \ln \tilde{p}_{li,g}} = \frac{\partial \ln C_{i,k}(\cdot)}{\partial \ln \tilde{p}_{li,g}'},
\]

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where \( \alpha_{jg}^{\ell}\) denotes the share of input \( \ell i, g \) in output \( ij, k \) as defined by D3. The change in \( \bar{p}_{ijg}^T \) has an additional general equilibrium effect, which operates through changes in \( w, Y, \) and \( \mathcal{Y} \). As noted earlier, though, the handling of these general equilibrium effects can be simplified with the application of Lemma 1.

Trade taxes also affect the surplus, \( \Pi_i \), paid to industry-specific capital in country \( i \). To track these effects, we can appeal to Hotelling’s lemma, whereby the effect of a change in output prices on surplus (holding all input prices constant) can be stated as:

\[
\frac{\partial \Pi_i(.)}{\partial p_{ij,k}} = q_{ij,k}.
\]

Likewise, the effect of a change in the price of intermediate input \( ji, k \) (holding the price of all other inputs fixed) can be expressed as:

\[
\frac{\partial \Pi_i(.)}{\partial p_{ji,k}^I} = q_{ji,k}^I.
\]

By combining the aforementioned envelope conditions and appealing to Lemma 1, we can produce the following theorem that characterizes the optimal policy as a function of gross expenditure shares, \( \lambda \), gross revenue shares, \( r \), global input-output shares, \( \alpha \), reduced-form demand elasticities, \( \epsilon \), and reduced-form supply elasticities, \( \gamma \). The former three statistics are directly observable, while the latter two can be locally estimated.

**Theorem 1.** Home’s optimal non-cooperative trade taxes are unique up-to a uniform tax shifter, \( \bar{t} \), and given by

\[
1 + t_k^* = \left( 1 + \frac{\gamma_{jg}^{fH}f_{h,k}^I}{1 - \gamma_{jg}^{fH}f_{h,k}^I\epsilon_{h,f,k}} \right) (1 + \bar{t})
\]

\[
1 + x_k^* = \frac{\epsilon_{h,f,k}}{1 + \epsilon_{h,f,k} + \xi_{h,f,k} - \sum_g \alpha_{jg}^{e_{h,k}} \left( 1 + \bar{t} \right)^{-1}}
\]

where \( \xi_{h,f,k} = \left[ \Xi^{-1} - I \right] \Omega \) accounts for input-output adjusted cross-demand effects between industries, with \( \Xi \equiv \left[ \frac{\lambda_{h,g}^{e_{h,k}}}{\lambda_{h,f,k}^{e_{h,k}}} \right]_{k,g} \) and \( \Omega \equiv \left[ 1 - \sum_g \alpha_{jg}^{e_{h,k}} \right]_{k,h,f,k} \).

As discussed earlier, due to the Lerner symmetry, the optimal policy is indeterminate and unique only up to a uniform tariff, \( \bar{t} \). For a high-enough choice of \( \bar{t} \), the optimal policy will consist of import taxes paired with export subsidies. Aside from the uniform tax shifter, the optimal import tax, \( t_k^* \), is equal to the optimal industry-level mark-down on \( p_{f,h,k}^I \) (i.e., the inverse of the export supply elasticity). The optimal export tax, \( x_k^* \), is equal the optimal monopoly mark-up on \( \bar{p}_{h,f,k}^I \) that
internalizes cross-demand effects and tax propagation through the production network. In the special case with zero cross-substitutability between industries (i.e., $\Xi = I_k \iff \xi_{hf,k} = 0$) and no input-output networks (i.e., $\alpha_{hf,k} = 0$), the optimal export tax reduces to the familiar single-product optimal monopoly markup, $1 + x_k^* = \epsilon_{hf,k} / (1 + \epsilon_{hf,k})$.

Theorem 1 has an attractive feature: It characterizes the optimal policy in terms of estimable or observable sufficient statistics. In the words of Piketty and Saez (2013), such sufficient statistic formulas have two broad merits. First, they allow us “to understand the key economic mechanisms behind the formulas.” Second, they “are also often robust to changing the primitives of the model.” In the present context, the formula characterized by Theorem 1 can be empirically evaluated with readily-available trade statistics. Given these qualities and as shown later in Section 6, Theorem 1 streamlines the quantitative analysis of optimal trade policy in an important class of quantitative trade models.

There is a simple intuition for why optimal import taxes (unlike export taxes) do not explicitly depend on global input-output shares. From the perspective of the Home country, trade taxes can correct two terms-of-trade-related distortions:

i. Unexploited import market power, which concerns the unexploited ability of the Home economy to charge a mark-down on Foreign producer prices, $p_{fh,k}$.

ii. Unexploited export market power, which concerns the unexploited ability of the Home economy to charge a mark-up on Foreign consumer prices, $\tilde{p}_{hf,k}$.

By the targeting principle, industry $k$'s optimal import tax, $t_k^*$, is targeted exclusively at lowering $p_{fh,k}$ (Margin 1). For this reason, $t_k^*$ is also independent of input-output shares. To make this point more elaborately, consider the following thought experiment: Fix the price of all of Home’s export goods as these prices can be directly manipulated with export taxes. In that case, $t_k$ cannot affect either $\{p_{fh,g}\}$ or $\{\tilde{p}_{hf,g}\}$ through the input-output network. Instead, $t_k$ can only lower $p_{fh,k}$ by shrinking Foreign’s output in industry $k$. This effect, by construction, operates independent of the input-output network.

Based on the same rationale, optimal export taxes depend explicitly on global input-output shares. An export tax on intermediate input, $hf,k$, will raise the price of any Foreign-produced good employing that input, including goods that are sold back to the Home country. An export tax on intermediate inputs is, therefore,
partially passed back to Home consumers through re-importation. To mitigate these adverse feedback effects, the optimal export tax is lower on more upstream industries where re-importation of export taxes is more of an issue. The same claim is backed by the formula under Theorem 1, considering that

$$\sum_{g} \alpha_{h,k} \frac{\hat{f}_{gh}}{\hat{\lambda}_{hfk}} \in \begin{cases} 0 & \text{if } fh,k \text{ is a final good} \\ (0,1) & \text{otherwise} \end{cases}$$

That export taxes are lower on upstream industries, however, should not be confused with smaller gains from taxation. On the contrary, the unilateral gains from taxing upstream exports are actually larger. We formally document and discuss this point in Section 6.

Theorem 1 indicates that import taxes are a necessary tax instrument only if there are diseconomies of scale arising from industry-specific capital. Otherwise, if $\gamma_{fk} \approx 0$, we can discard import taxes by choice of $\tilde{t} = 0$, and attain the first-best outcome with only export taxes. The following corollary outlines this claim.

**Corollary 1.** Absent diseconomies of scale at the industry-level, i.e., $\gamma_{fk} = 0$, import taxes are a redundant trade policy instrument. That is, the first-best outcome can be reached with only export taxes that are increasing in (i) the inverse of the industry-level trade elasticity, and (ii) the industry’s level of downstreamness.

To be clear, the absence of diseconomies of scale is a sufficient but not necessary condition for the redundancy of import taxes. Import taxes can be redundant under much weaker conditions. For instance, suppose Foreign employs industry-specific capital but Home accounts for a small share of global demand for the Foreign industry. Then, $r_{fh,k} \approx 0$ and import taxes are once again redundant. The aforementioned situation provides an accurate description of any individual country’s position relative to the rest of the world. It also aligns with the complete tariff passthrough documented by Amiti et al. (2019) and Fajgelbaum et al. (2020).

---

8More generally, $\sum_{g} \alpha_{h,k} \hat{f}_{gh} / \hat{\lambda}_{hfk} = 0$ if good $fh,k$ is not used as an input in any of Foreign’s export goods. Final goods automatically satisfy this qualification.
3.1 Canonical Special Cases

To highlight the practicality of our optimal tax formula, we outline two canonical special cases: First, a basic multi-industry gravity model without input-output linkages à la Costinot et al. 2011. Second, a multi-industry gravity model with flexible input-output linkages à la Caliendo and Parro (2014).

(i) Basic Multi-Industry Gravity Model (Costinot et al. 2011). This model corresponds to a special case of our framework where labor is the only factor of production and preferences have a Cobb-Douglas-CES parameterization: 

\[ U_i = \prod_k Q_{i,k}^{\epsilon_{i,k}}, \]

where 

\[ Q_{i,k} = \left( \sum_{j=h,f} \chi_{ji} k_{j,i} \right)^{1/\rho_k}. \]

The Cobb-Douglas-CES demand structure implies that 

\[ \epsilon_{h,f,k} = -1 - \epsilon_k \lambda_{ff,k}, \]

where \( \epsilon_k \equiv \rho_k / (1 - \rho_k) \). The Cobb-Douglas assumption also eliminates cross-price elasticity effects, which amounts to \( \xi_{h,f,k} = 0 \). The absence of input-output networks implies that \( \alpha_{h,g} = 0 \) for a \( h \) and \( g \). Plugging these values into the optimal tax formula specified by Theorem 1 yields the following:

\[
1 + t_k^* = 1 + \bar{t} \\
1 + x_k^* = \left( 1 + \frac{1}{\epsilon_k \lambda_{ff,k}} \right) (1 + t)^{-1}.
\]

Stated verbally, the optimal trade tax consists of a uniform tariff, \( \bar{t} \), and an industry-specific export tax that varies primarily with the industry-level trade elasticity, \( \epsilon_k \). If the economy is modeled as a single industry, the above formula reduces to \( x^* = 1/\epsilon \lambda_{ff} \), which by the Lerner symmetry is equivalent to Gros’ (1987) optimal tariff formula, \( t^* = 1/\epsilon \lambda_{ff} \).

(ii) Multi-Industry Gravity Model with I-O Linkages (Caliendo and Parro (2014))

This model features the same CES-Cobb-Douglas utility function described above. Production, though, combines intermediate inputs and labor using a Cobb-Douglas-CES aggregator, which implies that 

\[ p_{ij,k} = w_{i}^{k} \prod_{g} P_{i,g}^{k}, \]
where $\alpha_{i,k}$ denotes the constant share of industry $k$’s inputs in industry $g$’s output, with \( \bar{\alpha}_{i,k} \equiv 1 - \sum_g \alpha_{i,g} \). \( \bar{P}_{i,g} \) denotes the (after-tax) price index of industry $g$’s composite intermediate input, which is by assumption identical to the price index of industry $g$’s composite consumption good: \( \bar{P}_{i,g} = \left( \sum_j \bar{p}_{ji,g}^{1-\epsilon_g} \right)^{1/(1-\epsilon_g)}. \) The Cobb-Douglas-CES demand for the consumption variety of good $ji,k$ is \( q^c_{ji,k} = \left( \bar{p}_{ji,k} / \bar{P}_{i,k} \right)^{-\epsilon_k} \epsilon_{i,k} Y_i. \) The demand for the intermediate input variety of good $ji,k$ is given by \( q^I_{ji,k} = \left( \bar{p}_{ji,k} / \bar{P}_{i,k} \right)^{-\epsilon_k} \sum_g \alpha_{i,g} Y_{i,g}. \)

We can apply Theorem 1 along with the following steps to determine the optimal trade tax in this setup: Since there are no industry-specific factors of production, we can set $\gamma_{f,k} = 0$ for all $k$. The Cobb-Douglas assumption meanwhile ensures that $\xi_{h,f,k} = 0$. The within-industry CES demand system implies that $\epsilon_{h,f,k} = -1 - \epsilon_k \lambda_{ff,k}$. The assumption that the demand aggregator for final and intermediate goods are identical, entails that $\alpha_{h,f,g} = \alpha_{f,g} \lambda_{hf,k}$. Plugging these values into Theorem 1 yields the following formula for optimal trade taxes in the Caliendo and Parro (2014) model:

\[
1 + t^*_k = 1 + \bar{t} \\
1 + x^*_k = \left( \frac{1 + \epsilon_k \lambda_{ff,k}}{\epsilon_k \lambda_{ff,k} - \sum_g \alpha_{f,g} x_{f,h,g}} \right) (1 + t)^{-1}. \tag{7}
\]

In-line with our earlier discussion, the optimal export tax implied by the above formula is strictly lower than that implied by the baseline gravity model (Equation 6). However, the gains from optimal trade taxation are higher relative to the baseline model. This point is quantitatively established in Section 6 where we calibrate the above formulas to actual data.

### 4 Second-Best Non-Cooperative Import Taxes

We now consider a scenario that has received considerable attention in the prior literature. In this scenario, the Home government cannot apply export taxes; due to either external trade agreements that prohibit such taxes or institutional constraints. The Home government, however, has the discretion to apply import taxes. We show that, in such circumstances, it is optimal for the Home government to use import taxes as a second-best instrument to manipulate export market
power. To make this point succinctly, we abstract from diseconomies of scale and cross-industry demand effects. These channels are, however, formally accounted for and discussed in Appendix C.

If the Home government cannot apply export taxes, its second-best import taxes solve the following problem:

$$\max_{(t,0;w) \in \mathcal{A}} W_h(t,0;w)$$

Since export taxes are set to zero, i.e., $x = 0$, the Lerner symmetry no longer implies a multiplicity of optimal tax schedules. Instead, the above problem identifies a unique vector of optimal import taxes. Since the Home economy possesses no import market power by assumption (i.e., $\gamma_{f,k}^{\ell_h, k} \approx 0$), second-best import taxes pursue one objective: to indirectly manipulate Home’s export market power.

Export market power manipulation involves two distinct channels: First, import taxes can raise Home’s wage relative to Foreign, $w_h/w_f$ (i.e., they can charge a markup on the wage rate embedded in Home’s exports). Second, import taxes can be used to charge a markup on $\tilde{p}_{h,f,k}$ through the input-output network. The former channel was previously-redundant due to the multiplicity of optimal taxes per Lerner symmetry. The latter channel was also irrelevant due to the targeting principle.

To account for the now-relevant general equilibrium wage effects, we use $L_{ij}(t,x;w)$ to denote country $j$’s demand for country $i$’s labor. The elasticity of $L_{ij}$ with respect to $w_i$ depends on the value-added content of sales. That is, the overall contribution of country $i$’s labor to its gross revenue in each industry. We can measure the value-added content of country $i$’s output in industry $k$, namely, $\delta_{i,k}$, using the input-output matrix. Specifically, let

$$\alpha_{ji} \equiv \begin{bmatrix} \alpha_{i,j}^{g} \\ \alpha_{i,k}^{g} \end{bmatrix}$$

denote the $K \times K$ input-output matrix that describes the share of country $j$’s inputs used in country $i$’s outputs. We can apply the Implicit Function Theorem to calculate $\delta_i \equiv [\delta_{i,k}]_k$ as follows:

$$\delta_i = (I_K - \alpha_{ii})^{-1} \bar{\alpha}_i,$$
where \( \bar{\alpha}_i \equiv [\bar{\alpha}_{ij}]_k \) is a \( K \times 1 \) vector denoting industry-level labor shares in country \( i \). Using the above notation, we can calculate the elasticity of labor demand in terms of gross reduced-form demand elasticities, \( \varepsilon_{ij,k} \), and value-added shares, \( \delta_{i,k} \).

**D4 [Elasticity of Labor Demand]**

\[
\bar{\varepsilon}_{ij} \equiv \frac{\partial \ln L_{ij}}{\partial \ln w_i} = \sum_k \omega_{ij,k} \varepsilon_{ij,k}, \text{ where } \omega_{ij,k} \equiv \frac{\delta_{i,k} \lambda_{ij,k}}{\sum_g \delta_{i,k} r_{ij,k}}
\]

is a weight that reflects the value-added contribution of good \( ij,k \) to country \( i \)'s exports.

In principle, \( \bar{\varepsilon}_{ij} \) measures country \( i \)'s economy-wide export market power net of input-output linkages. As noted by the following Theorem, Home’s optimal import tax in each industry is determined by \( \bar{\varepsilon}_{hf} \) and an industry-specific term that accounts for the ability of import taxes to manipulate industry-specific export market power through the production network.

**Theorem 2.** In a second-best scenario where the use of export taxes is restricted, optimal import taxes feature of a uniform component and an industry-specific competent that captures the extent to which import taxes are passed on to Foreign consumers. Namely,

\[
1 + t_k^* = \bar{\varepsilon}_{hf} \left( \frac{1 + \tau_k}{\varepsilon_{fh,k}} \right),
\]

where \( \tau \equiv [\tau_k] \) is given by

\[
\tau = \left[ \begin{array}{c}
1 - S + \frac{\varepsilon_{hf,S} \bar{\lambda}_{fh,k}}{\varepsilon_{fh,k}} \bar{\varepsilon}_{kh,g}
\end{array} \right]^{-1} \sum_{g \in K} \left( \bar{\lambda}_{fh,k} - \left[ 1 + \frac{\varepsilon_{hf,S} \bar{\lambda}_{h,f,g}}{\bar{\varepsilon}_{fh,k} \varepsilon_{fh,k}} \right] \bar{\lambda}_{fh,k} \bar{\alpha}_{fh} \right),
\]

with \( \bar{\alpha}_{fh,k} = (I_K - \alpha_{ff})^{-1} \alpha_{fh} \alpha_{hf} \), and \( \bar{\lambda}_{fh,k} = (I_K - \alpha_{fh})^{-1} \alpha_{fh} \). So, \( \tau_k = 0 \) for import goods that are not used as intermediate inputs in export goods and \( \tau_k \neq 0 \) otherwise.

In the above formula, the uniform term, \( \bar{\varepsilon}_{hf} / (1 + \bar{\varepsilon}_{hf}) \), corresponds to an optimal export mark-up on the wage rate, \( w_h \). To elaborate, a uniform import tax is equivalent to a uniform export tax, which is itself equivalent to a uniform markup applied to the wage rate, \( w_h \), in Foreign markets. The optimal level of this markup is proportional to the elasticity of Foreign demand for Home’s labor, \( \bar{\varepsilon}_{hf} \). To give further perspective, consider the basic gravity model without input-output linkages as presented in Section 3.1. In this case, \( \bar{\lambda}_{fh,k} = 0 \) and \( \delta_{i,k} = 1 \), implying a
uniform optimal import tax across all industries:

$$
\bar{t}_k^* = 1/ \sum_k \left( \frac{\hat{r}_{hf,k}}{\hat{r}_{hf}} \epsilon_k \lambda_{ff,k} \right),
$$

where $\epsilon_k$ denotes the trade elasticity in industry $k$. The above formula itself strictly generalizes the optimal single-industry tariff formula in Gros (1987) to many asymmetric industries.

The industry-specific component, $1 + \tau_k / \epsilon_{fh,k}$, accounts for the fact that an import tax on intermediate inputs is partially passed on to Foreign consumers. So, such a tax can be used to imperfectly mimic (the restricted first-best) export taxes. To connect this point to Theorem 2, note that the optimal import tax on good $fh,k$ depends primarily on $\tilde{\alpha}_{fh,fg}$, which measures the degree to which the tax is passed on to Foreign consumers. It is straightforward to show that

$$
\left[ \tilde{\alpha}_{fh,fg} \right] = a_{fh} + a_{fh}^2 + a_{fh}^3 + ...$

The above expression implies that if (i) industry $k$ is a strictly downstream industry in a vertical economy, or (ii) the inputs produced by industry $k$ are never used in exporting industries, then $\tilde{\alpha}_{fh,fg} = \tilde{\alpha}_{fh} = 0$ and, as a result, $\tau_k = 0$. Plugging this value back into Theorem 2, the optimal import tax on such industries is uniform and equal to $t_k^* = \bar{\epsilon}_{hf} / (1 + \bar{\epsilon}_{hf})$. The intuition is that an import tax on the aforementioned industries is never passed on to Foreign consumers beyond general equilibrium wage effects. So, there is no rationale for taxing such imports beyond what is justified by the flat wage markup.

By contrast, if industry $k$ supplies inputs to export-oriented industries, then $\tilde{\alpha}_{fh,fg} > 0$, which implies that $\tau_k / \epsilon_{fh,k} > 0$. Plugging this value into Theorem 2 implies that industry $k$ imports should be taxed above the uniform rate. Intuitively, an import tax on industry $k$ is partly borne by Foreign consumers. So, with the unavailability of export taxes, $t_k$ can be used to extract an additional markup, $\tau_k / \epsilon_{fh,k}$, from Foreign consumers besides the flat wage markup, $\bar{\epsilon}_{hf} / (1 + \bar{\epsilon}_{hf})$.

Aside from $\tilde{\alpha}_{fh,fg}$, the effectiveness of $t_k$ at mimicking first-best export taxes is governed by two additional factors: (a) the degree to which intermediate input

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$^9$This is true provided that Home is sufficiently small relative to the rest of the world, i.e., $\tilde{\alpha}_{fh,fg} \approx 0$. 

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22
variety \( fh,k \) is substitutable with other inputs (i.e., \( \epsilon_{fh,k} \) in the formula specified by Theorem 1); and (b) the intensity at which input \( fh,k \) is used in highly-differentiated export-oriented industries (i.e., \( \tilde{\alpha}_{fh,g}^{f_k} \lambda_{hf,g}^{h \epsilon_{hf,g}} \) in the formula specified by Theorem 1). Specifically, \( t^*_k \) is higher if intermediate input \( fh,k \) is low-demand elastic and is used more-intensively in highly-differentiated, export-oriented industries. The following corollary summarizes these arguments.

**Corollary 2.** If export taxes are restricted, it is optimal to use import taxes as a second-best instrument to manipulate export market power through the global production network. Optimal second-best import taxes are uniform across all industries that supply final goods or intermediate goods that are never used in export sectors. They are otherwise non-uniform and increasing in the extent to which the imported goods are used as inputs in highly-differentiated, export-oriented sectors.

The above corollary raises a basic question: How effective are import taxes at mimicking export taxes? Answering this question is ultimately a quantitative matter, which we explore in Section 6. But we can gain valuable insights by approaching the question theoretically. If imported goods are re-exported without being processed, then an import tax on these goods is identical to an export tax. Beyond this extreme case, import taxes are less efficient than export taxes for two reasons: First, they distort production-related choices in the Home economy. Second, they may be partially passed on to domestic consumers, even when applied to intermediate inputs. Considering this, the effectiveness of second-best trade taxes is constrained by a basic trade-off. To ensure that import taxes are passed on minimally to domestic consumers, they should be applied on highly-differentiated intermediate inputs. However, taxes targeted at differentiated inputs create significant misallocation in the local economy. This basic trade-off means that, even in theory, second-best import taxes as largely ineffective.

Corollary 2 has several other implications for the existing literature. First, it sheds fresh light on the political economy of trade taxation in upstream versus downstream industries. Second, it suggests that the gains from trade agreements will be understated if we overlook global production networks. The following section outlines these other implications.
4.1 Implication for the Existing Literature

Political Economy of Trade Taxation: The dominant view in the trade policy literature is that governments (that are not constrained by external agreements) apply tariffs that are the sum of political economy and terms-of-trade considerations. Under this interpretation, the political weight of industry-level protection can be inferred from the wedge between the socially optimal tariff and the applied tariff rate in each industry. A high political weight simply indicates more lobbying for protective tariffs a given industry.

A prominent example undertaking this approach is Ossa (2014). He infers political weights using optimal tariffs implied by the multi-industry Krugman (1980) model. Optimal tariffs in the aforementioned model are determined primarily by the industry-level trade elasticities. Accordingly, Ossa (2014) estimates political weights that are strongly increasing in an industry’s trade elasticity but are less dependent on an industry’s level of upstreamness.

On face value, these estimates may defy the conventional wisdom that upstream versus downstream status is a primary determinant of political pressures. It is well-established that producers collectively lobby for a lower tariff on inputs, whereas final good consumers are poorly organized (Shapiro (2019)).

Our theory indicates that accounting for input-output networks can help resolve this tension. That is, applying Ossa’s (2014) strategy to a model with input-output networks will imply political weights that align with existing evidence on lobbying. To elaborate, Theorem 2 indicates that (socially) optimal tariffs are decreasing in both an industry’s trade elasticity and level of downstreamness. Given this prediction, estimating political weights using Ossa’s (2014) approach but with our optimal tariff formula will imply a systematic and negative relationship between the political weight of protection and the level of downstreamness, which perfectly aligns with the evidence noted earlier.

Our theory also speaks to the importance of terms-of-trade versus political motives in the governments’ objective function. A prominent study on this topic is Goldberg and Maggi (1999). They analyze the cross-industry variation in applied tariffs.

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10 In the Krugman model with restricted entry, optimal import taxes are increasing in the industry-level markup, which is by assumption equal to the inverse of the trade elasticity. This result, however, only holds in a second-best scenario where domestic taxes are inapplicable. Otherwise, the optimal import tax is uniform (see Lashkaripour and Lugovskyy (2019)).
and optimal tariffs and estimate that governments assign a higher weight to terms-of-trade motives than to political motives. Our theory, however, suggests that accounting for under-exploited optimal export taxes can revise this finding along certain dimensions—we discuss this implication more in Section 6.

**The Gains from Trade Agreements:** Trade agreements help prevent non-cooperative equilibria in which countries erect retaliatory Nash tariffs against each other. The gains from trade agreements can, accordingly, be measured as the welfare loss from a multilateral adoption of Nash tariffs. Theorem 2 not only characterizes the optimal tariff when Foreign is passive, but also the Nash tariff when Foreign retaliates. It can, therefore, shed new light on gains from trade agreements in global production networks.11

Theorem 2 suggests that, absent trade agreements, countries have an incentive to target Nash tariffs at highly-differentiated intermediate inputs. As argued earlier, targeted Nash tariffs of this sort are optimal for a non-cooperative government: They extract monopoly markups from foreign consumers through the production network. But they also inflict a greater deadweight loss on the global economy compared to the uniform Nash tariffs that will prevail without global production networks.

Considering the above, accounting for global production networks increases the estimated gains from trade agreements. This increase is, on one hand, driven by the well-known cascading effects that also apply to standard gains from trade calculations. On the other hand, it is driven by trade agreements preventing Nash tariffs that are targeted at differentiated intermediate inputs. This latter channel is typically overlooked in the existing estimates of the gains from trade agreements.

## 5 Optimal Multilateral Policy

We now discuss how our theory extends to a multilateral setup with \( N > 2 \) countries. The optimal policy problem facing country \( i \) in this setup is similar to 5; but it now involves \( (N - 1) K \) import tax instruments, \( t = \{t_{ji,k}\} \), and \( (N - 1) K \) export

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11The implicit assumption here is that, in the event of a trade war, governments will resort to only import taxation. This assumption applies to many countries like the U.S. where export taxes are prohibited by the constitution.
tax instruments, \( x = \{ x_{ij,k} \} \). An extra complication here is that a tax on good \( ji,k \) can alter the entire vector of national-level wages \( \{ w_1, \ldots, w_N \} \). This detail aside, we can follow the same steps outlined earlier to derive sufficient statistics formulas for optimal multilateral trade taxes (see Appendix D):

\[
1 + t^*_{ji,k} = \left( 1 + \frac{\gamma_{j,k} r_{ji,k}}{1 - \sum_{l \neq i} \gamma_{j,k} r_{ji,k} \epsilon_{ji,k}} \right) (1 + \bar{t}) \\
1 + x^*_{ij,k} = \frac{\epsilon_{ij,k}}{1 + \epsilon_{ij,k} + \xi_{ij,k} - \sum_{g} \sum_{j} \bar{\lambda}_{ij,k} \bar{a}_{ji,k}^g (1 + \bar{t})^{-1}}.
\]

In the above formula, \( \xi_{ij,k} \) accounts for cross-industry demand effects and is defined analogously to the cross-demand term specified under Theorem 1, \( \Lambda_{ji,g} \equiv p_{ji,g} q_{ji,g} / \sum_{l \neq i} \sum_{k} p_{ji,k} q_{ji,k} \) and \( \bar{\Lambda}_{ji,g} \equiv \bar{p}_{ji,g} q_{ji,g} / \sum_{l \neq i} \sum_{k} p_{ji,k} q_{ji,k} \) denote import and export shares; and \( \bar{a}_{ji,k}^g \) = \( \bar{a}_{ji,k}^i \) if \( j \neq j \), with \( \bar{a}_{ji,k}^j = a_{ji,k}^j \).

The optimal tax formula specified above differs from the baseline formula (presented under Theorem 1) in one important aspect. The optimal export tax accounts for the fact that a tax on any individual export variety may be passed back to domestic consumers through multiple partners. The term \( \bar{a}_{ji,k}^g \) in the denominator, accounts for the extent to which \( x_{ij,k} \) is passed back to domestic consumers through country \( j \)'s export of industry \( g \) goods. The higher the \( \bar{a}_{ji,k}^g \)'s from various \( j \)'s, the lower the optimal export tax. The same formula also indicates that the burden of an export tax on one partner may be borne primarily by a third country that is not directly involved in the export transaction.

These subtle differences notwithstanding, the claims presented under Corollaries 1 and 2 readily extend to the multi-country case. As before, the optimal import taxes do not explicitly depend on the global input-output shares, which is a manifestation of the targeting principle. Absent diseconomies of scale, import taxes are also uniform and, therefore, redundant under the first-best scenario.\(^{14}\)

\(^{12}\)We can handle this extra complication by appealing to the first-order approximation, \( \frac{\partial \ln \bar{w}_n}{\partial \ln w_n} \propto \lambda_{ni,k} \lambda_{ji,k} r_{ni,k} \approx 0 \) iff \( n \neq i,j \). In theory, this approximation is implied by, but is strictly weaker, than the small open economy assumption.

\(^{13}\)Stated formally, \( \xi_{ij,k} \) is given by \( \left[ \xi_{ij,k} \right]_k = \left( \left[ \frac{\bar{\lambda}_{ij,k} a_{ji,k}^i}{\bar{\lambda}_{ij,k} a_{ji,k}^i} \right]^{-1} - I_K \right) \left[ 1 - \sum_{g} \sum_{j} \frac{\bar{\lambda}_{ij,k} a_{ji,k}^g}{\bar{\lambda}_{ij,k} a_{ji,k}^g} \right] \).

\(^{14}\)The uniformity of optimal tariffs across industries is not driven by the first-order approximation discussed in Footnote 12. As shown in Appendix D, this prediction holds irrespectively.
6 Quantitative Analysis

In this section, we map our theoretical model to industry-level trade and production data. Our analysis here pursues two distinct objectives. First, we want to quantify the gains from optimal trade policy in the presence of global production networks. Second, we wish to highlight how our analytic tax formulas streamline the computational analysis of optimal trade policy.

6.1 Mapping Optimal Tax Formulas to Data

With the aid of our optimal tax formulas, the gains from optimal policy can be computed by solving a system of equations that depend on observables and trade elasticities. We demonstrate this point first with a baseline model that overlooks global input-output networks. We then move on to our main model that formally accounts for global input-output networks. Our goal, ultimately, is to compare the predictions of the two models to isolate the role of production networks.

Baseline Model without Input-Output Networks. We first consider the basic multi-industry gravity model without input-output networks as outlined in Section 3.1. Recall that this baseline setup features a Cobb-Douglas utility aggregator across industries and a CES demand structure within industries. This demand structure implies that 

\[ V(Y_i, \bar{P}_i) = \frac{Y_i}{\bar{P}_i}, \]

where the aggregate price index in economy \( i \in C \) is given by

\[ \bar{P}_i = \prod_{k \in K} \left( \sum_{j \in C} \chi_{ji,k} \bar{p}_{ji,k}^{-\epsilon_k} \right)^{-\epsilon_i,k,\epsilon_k}. \]  

In the above formulation, \( \chi_{ji,k} \) accounts for policy invariant taste shifters and \( \bar{p}_{ij,k} = (1 + t_{ij,k})(1 + x_{ij,k}) \tau_{ij,k} a_{i,k} w_i \). Consider a counterfactual policy change, whereby the vector of trade taxes changes from its applied rate \( \{t_{ij,k}\} \), and \( \{x_{ji,k}\} \) to a counterfactual rate, \( \{t'_{ij,k}\} \), and \( \{x'_{ji,k}\} \). In response to this policy change, let \( z' \) denote the counterfactual value of any variable \( z \) and let \( \bar{z} = z'/z \) denote the
corresponding change in that variable. The CES demand structure implies that

\[
\hat{P}_{i,k} = \sum_{j} \left( \hat{\omega}_j (1 + t_{ji,k}) (1 + x_{ji,k}) \right)^{-\epsilon_k} \lambda_{ji,k}^{-1/\epsilon_k}
\]

where \(1 + t_{ji,k} \equiv (1 + t'_{ji,k}) / (1 + t_{ji,k})\) and \(1 + x_{ij,k} = (1 + x'_{ij,k}) / (1 + x_{ij,k})\) denote the change in import and export taxes. The change in trade shares is then given by

\[
\hat{\lambda}_{ji,k} = \left[ (1 + t_{ji,k}) (1 + x_{ji,k}) \hat{\omega}_j \right]^{-\epsilon_k} \sum_{n} \lambda_{ni,k} \left[ (1 + t_{ni,k}) (1 + x_{ni,k}) \hat{\omega}_n \right]^{-\epsilon_k}.
\]

The new vector of wages and income levels in the counterfactual equilibrium are determined by the labor market clearing condition:

\[
w'_{i}L_{i} = \sum_{k} \sum_{j} \left( \lambda'_{ij,k} e_{i,k} Y'_{j} \right)
\]

and the condition that total income equals wage income plus tax revenue:

\[
Y'_{i} = w'_{i}L_{i} + \sum_{k} \sum_{j} \left( \frac{t'_{ji,k}}{1 + t'_{ji,k}} \lambda'_{ij,k} e_{i,k} Y'_{j} + \frac{x'_{ij,k}}{1 + x'_{ij,k}} \lambda'_{ij,k} e_{j,k} Y'_{j} \right).
\]

Both of the above equations can be written in terms of changes, by noting that \(Y'_{i} = \hat{Y}_i Y_i, w'_{i} = \hat{\omega}_i w_i, \) and \(\lambda'_{ij,k} = \hat{\lambda}_{ij,k} \lambda_{ij,k}\).\(^{15}\)

With the above background in mind, consider our counterfactual policy experiment of interest: One in which Home applies its optimal trade tax rates, as characterized by Equation 6:

\[
\begin{cases}
t'_{fh,k} = t^*_f, k = 0 \\
x'_{hf,k} = x^*_f, k = 1 / \epsilon_k \lambda'_{ff,k}
\end{cases}
\]

while the Foreign country is passive and retains their applied tax rates, i.e., \(t'_{hf,k} = t_{hf,k}\) and \(x'_{hf,k} = x_{hf,k}\). Inserting these tax changes into the formulas outlined earlier, we can compute the \(\hat{\lambda}_{ij,k}, \hat{\omega}_{i}, \) and \(\hat{Y}_i\) that result from this policy change.

\(^{15}\)Likewise, the post-change tax rates in the above equations can be expresses as \(t'_{ji,k} = (1 + t_{ji,k}) (1 + t_{ji,k}) - 1\) and \(x'_{ij,k} = (1 + x_{ij,k}) (1 + x_{ij,k}) - 1\).
With this information, we can then calculate the welfare consequences of the policy change. The following proposition outlines this procedure.

**Proposition 1.** Suppose the observed data is generated by the baseline gravity model without input-output networks, as outlined in Section 3. Then, Home’s optimal trade tax and its effect on equilibrium outcomes can be fully characterized by solving the following system of equations

\[
\begin{align*}
1 + x_{hf,k}^* &= 1 + \frac{1}{\epsilon_k \lambda_{ij,k} \lambda_{jj,k}}; \quad t_{hf,k}^* = 0 \\
x_{hf,k}^* &= x_{hf,k}; \quad t_{hf,k}^* = t_{hf,k} \\
\hat{\lambda}_{ji,k} &= \left( \frac{(1 + t_{ji,k}^*)(1 + x_{ji,k}^*)}{(1 + t_{ji,k}^*)(1 + x_{ji,k}^*)} \hat{w}_j \right)^{-\epsilon_k} \hat{\lambda}_{ji,k} \\
\hat{P}_{ij,k} &= \sum_j \left( \left( \frac{(1 + t_{ij,k}^*)(1 + x_{ij,k}^*)}{(1 + t_{ij,k}^*)(1 + x_{ij,k}^*)} \hat{w}_j \right)^{-\epsilon_k} \hat{\lambda}_{ij,k} \right) \\
\hat{\omega}_i w_i L_i &= \sum_k \sum_j \left( \frac{\hat{\lambda}_{ij,k} \lambda_{ij,k} e_{ij,k} \hat{Y}_i}{1 + t_{ij,k}^*} \hat{\lambda}_{ij,k} \lambda_{ij,k} e_{ij,k} \hat{Y}_i ight) + \frac{x_{ij,k}^*}{1 + x_{ij,k}^*} \hat{\lambda}_{ij,k} \lambda_{ij,k} e_{ij,k} \hat{Y}_i \hat{Y}_j \\
\hat{Y}_i &= \hat{\omega}_i w_i L_i + \sum_k \sum_j \left( \frac{t_{ij,k}^*}{1 + t_{ij,k}^*} \hat{\lambda}_{ij,k} \lambda_{ij,k} e_{ij,k} \hat{Y}_i + \frac{x_{ij,k}^*}{1 + x_{ij,k}^*} \hat{\lambda}_{ij,k} \lambda_{ij,k} e_{ij,k} \hat{Y}_i \hat{Y}_j \right)
\end{align*}
\]

The above system solves \( K + 2N \) independent unknowns, namely, \( \{ x_{hf,k}^* \}, \{ \hat{w}_i \}, \) and \( \{ \hat{Y}_i \} \), as a function of industry-level trade elasticities, \( \epsilon_k \), and the following set of observables: (i) applied trade taxes; (ii) within- and across-industry expenditure shares \( \lambda_{ji,k} \), and \( e_{ij,k} \); (iii) wage income, \( w_i L_i \); and (iv) total income, \( Y_i = w_i L_i + R_i \).

Solving the system specified by Proposition 1, determines the welfare consequences of Home’s optimal trade taxes as

\[
\hat{W}_i = \hat{Y}_i / \prod_k \hat{P}_{ij,k}.
\]

Importantly, all of this is implementable without appealing to any global optimization routine or without knowledge of structural parameters like \( \tau_{ij,k}, a_{ij,k}, \) or \( \chi_{ij,k} \). To put our approach in perspective, compare it to the standard approach that involves a constrained global optimization subject to equilibrium constraints (Costinot and Rodríguez-Clare (2014); Ossa (2014)). The standard approach can be implemented using a nested fixed-point procedure, which is impractical with many industries (i.e., a high \( K \)). Or, alternatively, it can be implemented using the MPEC procedure in Su and Judd (2012), which requires specialized commercial solvers like SNOPT.
or Knitro to attain credible results. The procedure outlined by Proposition 1 bypasses these challenges by reducing the global optimization problem to a system of equations that is straightforward to solve.

**Main Model with Input-Output Networks.** Now, consider the multi-industry gravity model with input-output networks as presented in Section 3.1. In this extended model, preferences are still characterized by Equation 8; but the price of good \( ij, k \) depends on the entire vector of industry-level price indexes in country \( i \):

\[
\hat{p}_{ij,k} = (1 + t_{ij,k})(1 + x_{ij,k})\tau_{ij,k}\alpha_{i,k}w_i \prod_{g} \hat{\bar{P}}_{i,g}^{a_{i,gk}}.
\]

The change in expenditure shares in response to a change in trade taxes can be, accordingly, specified as

\[
\hat{\lambda}_{ij,k} = \frac{\left[ (1 + \hat{t}_{ij,k})(1 + \hat{x}_{ij,k})\hat{w}_i \prod_{g} \hat{\bar{P}}_{i,g}^{a_{i,gk}} \right]^{-\epsilon_k}}{\sum_n \lambda_{nj,k} \left[ (1 + \hat{t}_{nj,k})(1 + \hat{x}_{nj,k})\hat{w}_n \prod_{g} \hat{\bar{P}}_{n,g}^{a_{n,gk}} \right]^{-\epsilon_k}}
\]

where the change in price indexes are given by

\[
\hat{\bar{P}}_{i,k}^{-\epsilon_k} = \sum_{j} \left[ 1 + \hat{t}_{ni,k}(1 + \hat{x}_{ni,k})\hat{w}_n \prod_{g} \hat{\bar{P}}_{n,g}^{a_{n,gk}} \right]^{-\epsilon_k} \lambda_{ji,k}.
\]

In the counterfactual equilibrium that arises after the tax change, gross industry-level revenues can be calculated as the sum of sales net of taxes:

\[
\hat{Y}_{i,k} = \sum_{j} \left( \frac{\lambda_{ij,g}^i}{(1 + x_{ij,g}^i)(1 + t_{ij,g}^i)} E_{j,k}^i \right),
\]

where \( E_{i,k} = e_{i,k} Y_{i,k} + \sum_g a_{i,gk} Y_{i,k}^g \) denotes gross expenditure on industry \( k \) in country \( i \). Net consumption income is equal to wage income plus tax revenues:

\[
\hat{Y}_{i} = w_i^L L_i + \sum_k \sum_j \left( \frac{t_{ji,k}}{1 + t_{ji,k}} \lambda_{ji,k}^i E_{i,k} + \frac{x_{ij,k}}{1 + x_{ij,k}} \lambda_{ij,k}^i E_{j,k} \right).
\]
Wages income in the above expression is itself equal to the sum of labor compensation across all industries: \( w'_i L_i = \sum_k \hat{\alpha}_{i,k} \check{Y}'_{i,k} \). As before we can write the above equilibrium conditions in changes by noting that \( \lambda'_{ji,k} = \hat{\lambda}_{ji,k} \lambda_{ji,k}, w'_i = \hat{w}_i w_i, \check{Y}'_{i,k} = \check{Y}_{i,k} \check{Y}_{i,k}, \) and \( Y'_i = \hat{Y}_i Y_i \).

To compute the gains from optimal policy, we need to apply the above procedure to a tax change where Home applies the optimal trade tax rates specified by Equation 7, while Foreign retains its applied tax rates. The following proposition summarizes this procedure.

**Proposition 2.** Suppose the observed data is generated by a multi-industry gravity model with global input-output networks, as outlined in Section 3.1. Then, Home's optimal trade taxes and their effect on equilibrium outcomes can be fully characterized by solving the following system of equations

\[
\begin{cases}
1 + x_{hf,k}^* = \frac{(1 + \epsilon_k \hat{\lambda}_{hf,k} \lambda_{hf,k})}{\epsilon_k \hat{\lambda}_{hf,k} \lambda_{hf,k} - \sum_g \alpha_{fg,k} \hat{r}_{fg,k} r_{fg,k}}; & 1 + t_{hf,k}^* = 1 \\
1 + x_{fh,k}^* = 1 + x_{fh,k}; & 1 + t_{hf,k}^* = 1 + t_{hf,k} \\
\hat{\lambda}_{ji,k} = \left[ \frac{(1 + t_{ij,k}^*)(1 + x_{ij,k}^*)}{(1 + t_{ij,k})(1 + x_{ij,k})} \right] \hat{w}_{ij,k} \prod_g \hat{P}_{ig,k}^{g_{ig,k}} \left[ \frac{1}{\lambda_{ij,k}} \right]; & 1 + t_{fh,k} = 1 \\
\hat{P}_{i,k}^{-\epsilon_k} = \sum_j \left[ \left( \frac{(1 + t_{ij,k})}{(1 + t_{ij,k})(1 + x_{ij,k})} \right) \hat{w}_{ij,k} \prod_g \hat{P}_{ig,k}^{g_{ig,k}} \left[ \frac{1}{\lambda_{ij,k}} \right] \right] \lambda_{ji,k} \\
\hat{r}_{ji,k} r_{ji,k} = \frac{\hat{\lambda}_{ji,k} \lambda_{ji,k} \check{E}_{i,k} E_{i,k}}{(1 + x_{ij,k})(1 + t_{ij,k})} / \sum_n \left( \frac{\hat{\lambda}_{jn,k} \lambda_{jn,k} \check{E}_{n,g} E_{n,g}}{(1 + x_{jn,k})(1 + t_{jn,k})} \right) \\
\hat{E}_{i,k} E_{i,k} = e_{ik} \hat{Y}_i Y_i + \sum_g \sum_j \alpha_{ig,k} \lambda_{ij,k} \check{Y}_{i,k} \check{Y}_{i,k} \\
\hat{\omega}_i w_i L_i = \sum_k \sum_j \hat{\alpha}_{i,k} \lambda_{ji,k} \hat{Y}_{i,k} Y_{i,k} \\
\hat{Y}_i Y_i = \hat{\omega}_i w_i L_i + \sum_k \sum_j \left( \frac{t_{ij,k}^*}{1 + x_{ij,k}^*} \right) \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{E}_{i,k} E_{i,k} + \frac{x_{ij,k}}{1 + x_{ij,k}} \hat{\lambda}_{ij,k} \lambda_{ij,k} \hat{E}_{i,k} E_{i,k}
\end{cases}
\]

The above system solves \( K + 2N(K + 1) \) independent unknowns, namely, \( \{x_{hf,k}^*\}, \{\hat{P}_{i,k}\}, \{\hat{\omega}_i\}, \{\hat{Y}_i\}, \{\check{Y}_{i,k}\}, \) as a function of industry-level trade elasticities, \( \epsilon_k, \) and the following set of observables: (i) gross expenditure and revenue shares, \( \lambda_{ji,k}, \) and \( r_{ij,k}; \) (ii) industry-level consumption shares, \( e_{ij,k}; \) (iii) wage income, \( w_i L_i; \) (iv) final good expenditure, \( Y_i = w_i L_i + R_i; \) (v) gross industry-level revenue and expenditure levels, \( \check{Y}_{i,k} \) and \( E_{i,k}; \) and (vi) input-output shares, \( \alpha_{i,k}. \)

As with the baseline model, the system specified by Proposition 2 also deter-
mines welfare consequences, which are given by \( \hat{W}_i = \hat{Y}_i / \prod_k \hat{P}_{i,k} \).

### 6.2 Data Description

Our main data source is the 2014 edition of the World Input-Output Database (WIOD, Timmer et al. 2012). The WIOD database covers 56 industries and 42 countries that account for more than 85% of world GDP, plus an aggregate of the rest of the world. The countries in the sample include all 27 members of the European Union (EU) and 15 other major economies: Australia, Brazil, Canada, China, India, Indonesia, Japan, Mexico, Norway, Russia, South Korea, Switzerland, Taiwan, Turkey, and the United States—see Table 3 in the appendix for a full list of countries. Following Costinot and Rodríguez-Clare (2014), we aggregate the original 56 industries in the WIOD into 16 industrial categories, which are listed in Table 1. Finally, we make the WIOD database consistent with our theoretical model by purging it from trade imbalances. In this process, we closely follow the methodology in Costinot and Rodríguez-Clare (2014) who apply Dekle et al.’s (2007) hat-algebra methodology to purge the 2008 edition of the WIOD.\(^{16}\)

We take data on applied tariffs, \( t_{ij,k} \), from the United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAINS). The 2014 version of the UNCTAD-TRAINS reports the simple tariff line average of the effectively applied tariff (AHS) across 31 two-digit (in ISIC rev.3) sectors, 185 importers, and 243 export partners. When tariff data are missing in a given year, we use tariff data for the nearest available year, giving priority to earlier years. We aggregate the UNCTAD-TRAINS data into individual WIOD industries, following the methodology in Kucheryavyy et al. (2016). Since individual EU member countries are not represented in the UNCTAD-TRAINS data during the 2000-2014 period, our analysis treats the 27 EU members as one taxing authority.

To map our model to the WIOD and UNCTAD-TRAINS datasets, we treat good \( ij, k \) as a good pertaining to WIOD industry \( k \) that is supplied by country \( i \) to market \( j \). Under this interpretation, our data reports the following information:

i. The gross expenditure on good \( ij, k \): \( \hat{p}_{ij,k} q_{ij,k} \)

ii. The applied import tax on good \( ij, k \): \( t_{ij,k} \)

\(^{16}\)A similar approach is also applied by Ossa (2014) to eliminate trade imbalances from the GTAP database.
iii. The share of industry $k$ inputs used in industry $g$’s output in country $i$: $\alpha_{i,kg}$.

Using the above data points, we can construct the within-industry gross expenditure shares as

$$\lambda_{ij,k} = \frac{\hat{p}_{ij,k}q_{ij,k}}{\sum_n \hat{p}_{in,k}q_{in,k}}.$$  

Similarly and assuming $x_{ij,k} = 0$, the within-industry gross output shares can be constructed as

$$\tau_{ij,k} = \frac{\hat{p}_{ij,k}q_{ij,k} / (1 + t_{ij,k})}{\sum_n \hat{p}_{in,k}q_{in,k} / (1 + t_{ij,k})}.$$  

The total gross output of industry $k$ in country $i$ can be calculated as the sum of gross sales net of taxes:

$$Y_{i,k} = \sum_n \left[ \hat{p}_{in,k}q_{in,k} / (1 + t_{ij,k}) \right].$$  

With information on gross output, country $i$’s net spending on final goods can be calculated as the gross minus intermediate input expenditure:

$$Y_i = \sum_j \sum_k \left( \hat{p}_{ji,k}q_{ji,k} \right) - \sum_k \sum_g \left( \alpha_{i,kg}Y_{i,g} \right).$$  

The industry-level consumption shares are, accordingly, given by

$$e_{i,k} = \frac{\left[ \sum_j \left( \hat{p}_{ji,k}q_{ji,k} \right) - \sum_g \left( \alpha_{i,kg}Y_{i,g} \right) \right]}{Y_i}.$$  

Finally, wage income can be calculated as the sum of labor compensation across all industries: $w_iL_i = \sum \bar{\alpha}_{i,k}Y_{i,k}$, where $\bar{\alpha}_{i,k} = 1 - \sum_g \alpha_{i,kg}$.

**Estimating the Industry-Level Trade Elasticities**  We estimate the industry-level trade elasticities using the triple-difference estimator developed by Caliendo and Parro (2014). To present this procedure, note that the multi-industry gravity model (with or without input-output networks) predicts the following formulation for trade shares:

$$\lambda_{ij,k} = \Phi_{i,k}\Omega_{j,k}\tau_{ij,k}^{-\epsilon_k}(1 + t_{ij,k})^{-\epsilon_k},$$

---

17To be clear, when applying the above data points to Propositions 1 and 2, we need to assign one country as Home and aggregate the remaining countries into the rest of world.
where \( \Phi_{ik} \equiv a_{ik}^{-\epsilon_k} w_{ik}^{-\tilde{\epsilon}_k} \prod_g \tilde{P}_{ig}^{-\tilde{\alpha}_g \epsilon_k} \) and \( \Omega_{jk} \equiv \sum_n \left[ \tau_{nj,k} (1 + t_{nj,k})^{-\epsilon_k} \Phi_{nk} \right] \) can be viewed as exporter\times industry and importer\times industry fixed effects. Suppose the iceberg trade cost, \( \ln \tau_{ij,k} = \ln d_{ij,k} + \epsilon_{ij,k} \), is composed of two components: (i) a systematic and symmetric component, \( d_{ij,k} = d_{ji,k} \), that accounts for the effect of distance, common language, and common border, and (ii) a random disturbance term, \( \epsilon_{ji,k} \), that represents deviation from symmetry. Using this decomposition, we can produce the following estimating equation for any triplet \((j, i, n)\):

\[
\ln \frac{\lambda_{ji,k} \lambda_{in,k} \lambda_{nj,k}}{\lambda_{ij,k} \lambda_{ni,k} \lambda_{jn,k}} = -\epsilon_k \ln \frac{(1 + t_{ji,k})(1 + t_{in,k})(1 + t_{nj,k})}{(1 + t_{ij,k})(1 + t_{ni,k})(1 + t_{jn,k})} + \tilde{\epsilon}_{jn,k},
\]

where \( \tilde{\epsilon}_{jn,k} \equiv \epsilon_k (\epsilon_{ij,k} - \epsilon_{ji,k} + \epsilon_{in,k} - \epsilon_{ni,k} + \epsilon_{nj,k} - \epsilon_{jn,k}) \). Using the above estimating equation, we can attain unbiased and consistent estimates for \( \epsilon_k \) under the identifying assumption that \( \text{Corr}(t_{ji,k}, \epsilon_{ji,k}) = 0 \). We estimate the above equation separately for each industry, using data on \( \{\lambda_{ji,k}\} \) from the 2014 version of the WIOD and data on \( \{t_{ji,k}\} \) from the UNCTAD-TRAiNS database. The estimation results are reported in Table 1 and broadly align with those produced by Caliendo and Parro (2014), who use data on a smaller sample of countries from 1993.

6.3 Quantitative Results

For each country, the computed gains from optimal policy are reported in Table 2. Each iteration of our analysis treats one of the countries listed in Table 2 as the Home country and aggregates the remaining countries into one Foreign economy. Alternatively, we can use the multilateral formulas presented in Section 5 to avoid aggregating other countries into one Foreign economy. Doing so delivers welfare gains that are practically indistinguishable from our benchmark results.
Table 1: List of industries and estimated trade elasticities.

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>$\epsilon_k$</th>
<th>std. err.</th>
<th>Obsv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Crop and animal production, hunting</td>
<td>0.93</td>
<td>0.19</td>
<td>12,341</td>
</tr>
<tr>
<td></td>
<td>Forestry and logging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fishing and aquaculture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mining and Quarrying</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Forestry and logging</td>
<td>0.53</td>
<td>0.13</td>
<td>12,300</td>
</tr>
<tr>
<td>3</td>
<td>Fishing and aquaculture</td>
<td>2.71</td>
<td>0.51</td>
<td>12,341</td>
</tr>
<tr>
<td>4</td>
<td>Wood and Products of Wood and Cork</td>
<td>5.64</td>
<td>0.87</td>
<td>12,183</td>
</tr>
<tr>
<td>5</td>
<td>Textiles, Wearing Apparel and Leather</td>
<td>2.71</td>
<td>0.51</td>
<td>12,341</td>
</tr>
<tr>
<td>6</td>
<td>Paper and Paper Products Printing and Reproduction of Recorded Media</td>
<td>4.65</td>
<td>1.49</td>
<td>12,300</td>
</tr>
<tr>
<td>7</td>
<td>Other Non-Metallic Mineral</td>
<td>1.51</td>
<td>0.89</td>
<td>12,341</td>
</tr>
<tr>
<td>8</td>
<td>Basic Pharmaceuticals Products</td>
<td>2.36</td>
<td>0.91</td>
<td>12,300</td>
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<tr>
<td>9</td>
<td>Rubber and Plastics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Other Non-Metallic Mineral</td>
<td>1.51</td>
<td>0.89</td>
<td>12,341</td>
</tr>
<tr>
<td>11</td>
<td>Basic Metals</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Fabricated Metal Products</td>
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</tr>
<tr>
<td>12</td>
<td>Computer, Electronic and Optical Products</td>
<td>4.07</td>
<td>1.02</td>
<td>12,341</td>
</tr>
<tr>
<td>13</td>
<td>Electrical Equipment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Motor Vehicles, Trailers and Semi-Trailers</td>
<td>2.70</td>
<td>0.45</td>
<td>12,341</td>
</tr>
<tr>
<td>15</td>
<td>Other Transport Equipment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>All Service-Related Industries</td>
<td>3.80</td>
<td>0.84</td>
<td>12,341</td>
</tr>
<tr>
<td></td>
<td>(WIOD Industry No. 23-56)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: This table estimates the industry-level trade elasticities using the Caliendo and Parro (2014) methodology. The original WIOD industry classification features 56 industries, which we aggregate into 16 industrial categories.

This finding has basic implications for the weight of *terms-of-trade* versus *political* consideration in the governments’ objective function. Looking at import taxes in isolation implies a significant role for terms-of-trade considerations (see Goldberg and Maggi (1999)). But if that were true, govern-

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19To elaborate, Goldberg and Maggi (1999) study the cross-industry variation in import taxes
ments should be applying export taxes more vigorously, as they are not prohibited by the WTO. The fact that export taxes are underexploited implies that export-oriented industries have a greater-than-previously-estimated political weight in the government's objective function.

ii. Interestingly, second-best import taxes are only slightly more effective after we account for global production networks. As argued earlier this outcome is an artifact of an innate tension between two welfare considerations: (a) to maintain allocative efficiency in the domestic economy governments should not tax highly-differentiated intermediate inputs, but (ii) to exploit export market power with second-best import taxes, governments have to levy a tax on highly-differentiated intermediate imports. This tension renders import taxes as an ineffective second-best substitute for export taxes.

iii. The welfare gains from trade taxation are 2-times larger once we account for global production networks. Intuitively, in the presence of global production networks, the excess burden per dollar of tax revenue falls relatively more on the rest of the world. Consider, for instance, an export tax on rare earth minerals that are used as an input in smartphone production abroad. A fraction of this export tax is borne on transactions involving foreign producers and foreign consumers of smartphones. So, through the global production network, the tax-imposing country can effectively raise revenue on transactions occurring outside its jurisdiction—i.e., global production networks present governments with extra-territorial taxing power.

Figure 1 illustrates the optimal trade tax schedule for the U.S. under the baseline model that does not accommodate input-output networks and the main model that does. As implied by Theorem 1, the optimal export tax is lower once we account for input-output networks. The intuition is that export taxes can propagate through the input-output network, in which case a portion of the tax is ultimately borne by domestic consumers. The optimal export tax is set lower to mitigate these adverse propagation effects, and especially so in more upstream industries like Chemicals, Mining, and Services.

and find that “the weight of welfare in the government’s objective function is many times larger than the weight of contributions.”

36
Table 2: The gains from trade policy: % change in real GDP

<table>
<thead>
<tr>
<th>Country</th>
<th>Baseline Model</th>
<th>Model with IO Networks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Export tax</td>
<td>Import tax</td>
</tr>
<tr>
<td>AUS</td>
<td>1.49%</td>
<td>0.88%</td>
</tr>
<tr>
<td>EU</td>
<td>0.70%</td>
<td>0.32%</td>
</tr>
<tr>
<td>BRA</td>
<td>1.00%</td>
<td>0.66%</td>
</tr>
<tr>
<td>CAN</td>
<td>1.79%</td>
<td>0.71%</td>
</tr>
<tr>
<td>CHE</td>
<td>2.05%</td>
<td>0.71%</td>
</tr>
<tr>
<td>CHN</td>
<td>0.48%</td>
<td>0.32%</td>
</tr>
<tr>
<td>IDN</td>
<td>1.61%</td>
<td>1.01%</td>
</tr>
<tr>
<td>IND</td>
<td>0.72%</td>
<td>0.21%</td>
</tr>
<tr>
<td>JPN</td>
<td>0.72%</td>
<td>0.37%</td>
</tr>
<tr>
<td>KOR</td>
<td>1.44%</td>
<td>0.74%</td>
</tr>
<tr>
<td>MEX</td>
<td>1.65%</td>
<td>0.96%</td>
</tr>
<tr>
<td>NOR</td>
<td>3.00%</td>
<td>2.30%</td>
</tr>
<tr>
<td>RUS</td>
<td>1.54%</td>
<td>0.51%</td>
</tr>
<tr>
<td>TUR</td>
<td>1.33%</td>
<td>0.75%</td>
</tr>
<tr>
<td>TWN</td>
<td>1.76%</td>
<td>0.85%</td>
</tr>
<tr>
<td>USA</td>
<td>0.50%</td>
<td>0.16%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.40%</td>
<td>0.73%</td>
</tr>
</tbody>
</table>

Note: This paper reports the change in a country’s welfare from applying its optimal trade taxes. The baseline model is the perfectly competitive multi-industry gravity model ‘a la Costinot et al. (2011). The model with IO networks is the perfectly competitive multi-industry gravity model with input-output networks ‘a la Caliendo and Parro (2014). In each iteration one country is treated as Home and the other countries are aggregated into one Foreign economy. The first-best taxes are solved using the system specified by Propositions 1 and 2. Second best import taxes correspond to a scenario where export taxes are prohibited.

The second-best import taxes are uniform if we do not account for input-output networks. Otherwise, they vary across industries in a way that resembles the optimal first-best export taxes. In particular, the optimal import tax is higher on low trade elasticity industries. But, unlike first-best export taxes, it is also relatively higher on upstream industries. The optimal import tax is, for example, higher on Mining and Chemical industries that are both relatively more upstream and have a relatively low trade elasticity. The Petroleum industry, in comparison, is relatively more upstream but exhibits the highest trade elasticity of all industries. As a result, the second-best import tax on the Petroleum industry is the lowest across all industries.
Figure 1: The United States’ Optimal Non-Cooperative Trade Taxes

Baseline Model  Model with I-O Netwroks

First-Best Export Taxes

Second-Best Import Taxes

Note: This graph plots the optimal trade tax for the U.S. economy against the rest of the world. Industries are ordered by the trade elasticity estimates reported in Table 1. The left panel reported first-best export taxes, normalizing the redundant import taxes to zero. The hollow squares correspond to the optimal export taxes predicted by the multi-industry competitive gravity model without input-output networks. Second best import taxes correspond to a scenario where export taxes are prohibited.

The fact that second-best import taxes vary modestly across industries reflects a tension between misallocation-worsening and terms-of-trade-improving effects. To explain this tension again, consider the extreme case where all imported intermediates are re-exported without processing in the domestic economy. In that case, the second-best import tax schedule converge to the first-best export tax schedule. Because, in this extreme case, neither tax instrument distorts input choices in the domestic economy. This is far from what happens in reality, though—as is clearly manifested in Figure 1. In practice, industries like Chemicals and Mining that should be targeted with second-best import taxes are both high-value-added and more upstream. So, disrupting them with taxes will be quite detrimental to allocative efficiency.
7 Concluding Remarks

Our understanding of global trade has improved drastically over the past two decades; thanks in part to the emergence of tractable quantitative trade models that admit a rich geography of trade costs and global production networks. Surprisingly, these canonical models have had much less impact on how we think about trade policy. To address this gap, we presented a full analytical characterization of optimal trade policy in the aforementioned class of quantitative trade models. We demonstrated that export taxes are necessary and sufficient for attaining the first-best policy outcome. But when the use of export taxes is restricted, it is optimal to adopt import taxes as a second-best instrument and manipulate export market power through the global production network. We estimated, however, that import taxes are an ineffective second-best instrument, as they tax differentiated inputs that are crucial to allocative efficiency.

Our theory presents researchers with a new benchmark to study the political economy of trade taxation in the age of global production networks. The current practice in the literature is to infer political economy motives by contrasting the applied trade taxes with their socially optimal level. Under this approach, our general theory of optimal trade policy will enable researchers to credibly estimate the political weight of protecting upstream versus downstream industries. Likewise, our theory can shed fresh light on the relative importance of terms-of-trade versus political economy considerations in the conduct of trade policy. Previous estimates that focus only on import taxation indicate that political considerations are dwarfed by terms-of-trade considerations. Our theory suggests that accounting for export taxes may revise this prediction.

Finally, our optimal trade tax formulas can be used to attain improved estimates for the gains from trade agreements. These gains can be calculated as the welfare loss that would otherwise occur if non-cooperative governments adopt retaliatory Nash tariffs. On a conceptual level, our theory indicates that accounting for global production networks implies systematically higher Nash tariffs on differentiated inputs. So, in theory, global production networks will magnify the gains from trade agreements beyond what is already known in the literature. On a technical level, our formulas simplify the task of computing Nash tariffs in the presence of global production networks—a task that is known to be computationally cumbersome.


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Appendix

A Proof of Lemma 1

The proof of Lemma 1 follows from two intermediate claims, labeled C1 and C2. To present these claims, note that $A = (1 + t, 1 + x, w_h, w_f)$ and $A' = (a(1 + t), (1 + x)/a, aw_h, w_f)$. Accordingly, all equilibrium variables corresponding to the wage-tax combination $A'$ will be denoted by the prime notation. Claim C1 is based on the demand function $D_{ji,k}(\bar{p}_i, Y_i)$ being homogeneous of degree zero. Specifically,

$$\begin{align*}
\begin{cases}
\bar{p}_i' = a\bar{p}_h', & Y_h' = aY_h \\
\bar{p}_j' = \bar{p}_f', & Y_f' = aY_f
\end{cases} \implies q_{ji,k}' = q_{ji,k} \quad \forall ji, k. \quad (C1)
\end{align*}$$

where recall that $\bar{p}_i \equiv \{\bar{p}_{ji,k}\}_{j,k}$ denotes the vector of final prices in country $i$. Claim C2 can be stated as follows

$$q_{ji,k}' = q_{ji,k} \quad \forall ji, k \implies \begin{cases}
\bar{p}_h' = a\bar{p}_h, & Y_h' = aY_h \\
\bar{p}_f' = \bar{p}_f, & Y_f' = aY_f
\end{cases}. \quad (C2)$$

The claim that $\bar{p}_h' = a\bar{p}_h$ and $\bar{p}_f' = \bar{p}_f$ follow trivially from the price equation:

$$\bar{p}_{ji,k} = (1 + x_{ji,k})(1 + t_{ji,k})C_{i,k}(w_j, \bar{p}_j; \sum_n \tau_{jn,k}q_{jn,k}),$$

given that $C_{i,k}$ is homogeneous of degree 1 w.r.t. $w_j$ and $\bar{p}_j$. That $Y_f' = w_fL_f = w_fL_f = Y_f$, holds by construction. The claim that $Y_h' = aY_h$, meanwhile, can be shown along the following steps:

$$Y_h' = w_h'L_h + \sum_k \left[ \frac{t_k'}{1 + t_k'} \bar{p}_{fh,k}q_{fh,k} + \frac{x_k'}{1 + x_k'} \bar{p}_{hf,k}q_{hf,k} \right]$$

$$= a\omega_h' + \sum_k \left[ \left( 1 - \frac{a}{1 + t_k'} \right) \bar{p}_{fh,k}q_{fh,k} + \left[ a - \frac{1}{1 + x_k'} \right] \bar{p}_{hf,k}q_{hf,k} \right]$$

$$aw_hL_h + \sum_k \left[ \left( 1 - \frac{a}{a(1 + t_k)} \right) a\bar{p}_{fh,k}q_{fh,k} + \left[ a - \frac{1}{(1 + x_k)/a} \right] \bar{p}_{hf,k}q_{hf,k} \right]$$

$$= aw_iL_i + a \sum_k \left[ \frac{t_k}{1 + t_k} \bar{p}_{fh,k}q_{fh,k} + \frac{x_k}{1 + x_k} \bar{p}_{hf,k}q_{hf,k} \right] = a(w_hL_h + \mathcal{R}_h),$$

43
where the second line follows from the balanced trade condition, i.e.,
\[ \sum_k \left[ \frac{\tilde{p}_{fh,k} q_{fh,k}}{1 + t_k} - \tilde{p}_{hf,k} q_{hf,k} \right] = 0. \] Together, Claims C1 and C2 establish Lemma 1.

B Proof of Theorem 1

Note that by definition \(1 + t_k \equiv \tilde{p}_{fh,k} / p_{fh,k}\) and \(1 + x_k \equiv \tilde{p}_{hf,k} / p_{hf,k}\). Hence, the optimal policy problem can be reformulated as a problem of directly choosing consumer prices, \(\tilde{p}_{fh,k}\) and \(\tilde{p}_{hf,k}\), rather than taxes, \(t_k\) and, \(x_k\). In that case, the optimal policy problem can alternatively be expressed as
\[
\max_{(\tilde{p}; w) \in \tilde{A}} W_h (\tilde{p}; w),
\]
where \(\tilde{A}\) denotes the set of feasible wage-price combinations that is defined analogous to D2. Note that in the presence of feasible wage-price linkages, equilibrium is conventionally defined in terms three sets of equilibrium outcomes: (a) wages, \(w\), (b) net income \(Y\), and (c) gross revenue, \(\mathbf{Y}\). However, combining equilibrium conditions \((iv)\) and \((v)\) in D2, we can uniquely solve \(Y \equiv Y(\tilde{p}; w)\) and \(\mathbf{Y} \equiv \mathbf{Y}(\tilde{p}; w)\) in terms of prices and wages. Considering this we can uniquely solve all equilibrium variables, including welfare, in terms of prices and wages.

To handle the complex nature of the above problem, we use two technical tricks. First, we assume that the government can choose \(\tilde{p}_{hh}\) in addition to \(\tilde{p}_{fh}\) and \(\tilde{p}_{hf}\). In that case, Home’s tax revenues include revenue’s from domestic taxation, which we denote by
\[
\mathcal{R}^{D}_{hh} \equiv \sum_k (\tilde{p}_{hh,k} - p_{hh,k}) q_{hh,k}.
\]
Including the aforementioned instrument is innocuous since (i) markets are perfectly competitive, and (ii) governments have not political economy motives for intra-national redistribution. As a result, there is no rationale for domestic taxation (i.e., \(\tilde{p}_{hh,k} = p_{hh,k}\)) and the problem with extended instruments becomes isomorphic to our main problem of interest. The second trick that greatly simplifies our analysis is the observation that
\[
\frac{dW_h(\tilde{p}, w)}{d\tilde{p}_{ff,k}} = 0.
\]
In light of the above result, we are simply solving a problem where the government can directly choose all price variables that matter for Home’s welfare. So, we should not track how a change in, say price variable \(\tilde{p}_{fh,k}\), affects any other consumer price variables.
Step 1: Deriving the F.O.C. for Import Taxes. The F.O.C. with respect to sector $k$’s tariff can be expressed as

$$\frac{dW_h(p; w)}{d\ln \bar{p}_{fh,k}} = \frac{\partial V_h(.)}{\partial Y_h} \left[ \frac{\partial Y_h}{\partial \ln \bar{p}_{fh,k}} + \frac{\partial Y_h}{\partial w_h} \frac{dw_h}{d\ln \bar{p}_{fh,k}} \right] + \frac{\partial V_h(.)}{\partial \bar{p}_{fh,k}}$$

It should be noted upfront that the difference between the present setup and the pure Ricardian case, is that the (conditional) tariff pass-through $\sigma_{fh,k}^{\partial \ln \bar{p}_{fh,k}} = \partial \ln \bar{p}_{fh,k}/\partial \ln (1 + t_k)$ can be non-zero (even if $g \neq k$) due to (i) the upward sloping supply curve in industry $g$, plus (ii) the cross-substitutability between industry $k$ and industry $g$ goods. Plugging $Y_h = w_hL_h + \Pi_h + \mathcal{R}_h^X + \mathcal{R}_h^M + \mathcal{R}_h^D$, into the F.O.C. yields the following:

$$\frac{dW_h(p; w)}{d\ln \bar{p}_{fh,k}} = \frac{\partial \Pi_h}{\partial Y_h} \left\{ \frac{\partial \Pi_h}{\partial \ln \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^X}{\partial \ln \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^M}{\partial \ln \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^D}{\partial \ln \bar{p}_{fh,k}} \right\} + \frac{\partial \Pi_h}{\partial \bar{p}_{fh,k}} \left( \frac{\partial \Pi_h}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^X}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^M}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^D}{\partial \bar{p}_{fh,k}} \right) = 0,$$

where $\partial V_h/\partial w_h = \partial Y_h/\partial w_h + \sum_g (\partial V_h/\partial \bar{p}_{fh,k}^g) (\partial \bar{p}_{fh,k}^g/\partial w_h)$. The above F.O.C. is characterized by six different elements that can be characterized as follows. First, The effect of tariffs on producer surplus, $\partial \Pi_h/\partial \ln \bar{p}_{fh,k}$, can be expressed as

$$\frac{\partial \Pi_h(p; w)}{\partial \ln \bar{p}_{fh,k}} = \sum_g \sum_{i=h,f} \left( \frac{\partial \Pi_h}{\partial \ln \bar{p}_{hi,g}} \left[ \frac{\partial \ln \bar{p}_{hi,g}}{\partial \ln \bar{p}_{fh,k}} + \frac{\partial p_{hi,g}}{\partial \ln \bar{p}_{fh,k}} \frac{\partial \ln q_{hh,g}}{\partial \ln \bar{p}_{fh,k}} \right] \right) + \frac{\partial \Pi_h}{\partial \bar{p}_{fh,k}} \left( \frac{\partial \Pi_h}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^X}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^M}{\partial \bar{p}_{fh,k}} + \frac{\partial \mathcal{R}_h^D}{\partial \bar{p}_{fh,k}} \right)$$

where the second line follows from Hotelling’s lemma that $\partial \Pi_{hi,g}/\partial p_{hi,g} = q_{hi,g}$ and $\partial \Pi_h/\partial \bar{p}_{fh,k}^T = q_{fh,k}^T$. Second, noting that $dW_h/\partial \bar{p}_{fh,g} = 0$, the effect of import taxes on import tax revenues, $\mathcal{R}_h^M = \sum_g (\bar{p}_{fh,g} - p_{fh,g})q_{fh,g}$, can be expressed as

$$\frac{\partial \mathcal{R}_h^M(p; w)}{\partial \ln \bar{p}_{fh,k}} = \frac{\partial}{\partial \ln \bar{p}_{fh,k}} \left\{ \sum_g (\bar{p}_{fh,g} - p_{fh,g})q_{fh,g} \right\}$$

$$= \bar{p}_{fh,k}q_{fh,k} + \sum_g \left[ (\bar{p}_{fh,g} - p_{fh,g})q_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln \bar{p}_{fh,k}} - p_{fh,g}q_{fh,g} \frac{\partial \ln p_{fh,g}}{\partial \ln \bar{p}_{fh,k}} \right],$$

where $\partial \ln q_{fh,g}/\partial \ln \bar{p}_{fh,k}$ accounts for the overall effect $\bar{p}_{fh,k}$ on final and intermediate input demand. More specifically, letting $\mathcal{D}_{fh,gs}(.)$ denote the demand for intermediate
input variety $fh_g$ from industry $s$,

$$\frac{\partial \ln q_{fh_g}}{\partial \ln p_{fh_k}} = \frac{q_{fh_g}^c}{q_{fh_g}} \left( \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln p_{fh_k}} + \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln Y_h} \frac{d \ln Y_h}{d \ln p_{fh_k}} \right)$$

$$+ \frac{q_{fh_g}^T}{q_{fh_g}} \left( \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln p_{fh_k}} + \sum_s \alpha_{h,gs} \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln Y_{hs}} \frac{d \ln Y_{hs}}{d \ln p_{fh_k}} \right) = \varepsilon_{fh_g} + \eta_{fh_g} \frac{d \ln E_{hg}}{d \ln p_{fh_k}}$$

where, $\alpha_{h,gs} \equiv q_{fh_g}^T/q_{fh_g}^T$ and $E_{ig} \equiv \omega_{i,g} Y_i + \sum_s \omega_{i,gs} Y_{is}$ denotes demand-weighted gross expenditure on final plus intermediate inputs fro industry $g$, with the last line following for the implicit assumption that $\frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln Y_{hk}} = \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln Y_h}$ and $\frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln p_{fh_k}} = \frac{\partial \ln D_{fh_g}(\cdot)}{\partial \ln p_{fh_k}}$ for all $s$. Third, the effect of import taxes on export tax revenues, $R_h^X \equiv \sum_g (\tilde{p}_{fh,g} - p_{fh,g}) q_{fh,g}$, can be expressed as

$$\frac{\partial R_h^X (\tilde{p}; w)}{\partial \ln \tilde{p}_{fh_k}} = \frac{\partial}{\partial \ln \tilde{p}_{fh_k}} \left\{ \sum_g (\tilde{p}_{fh,g} - p_{fh,g}) q_{fh,g} \right\}$$

$$= \sum_g \left[ (\tilde{p}_{fh,g} - p_{fh,g}) q_{fh,g} \frac{\partial \ln E_{fg}}{\partial \ln \tilde{p}_{fh_k}} - p_{fh,g} q_{fh,g} \left( \frac{\partial \ln p_{fh,g}}{\partial \ln \tilde{p}_{fh_k}} + \frac{\partial \ln p_{fh,g}}{\partial \ln q_{fh,g}} \right) \right].$$

Fourth, the effect of import taxes on domestic tax revenues, $R_h^D \equiv \sum_g (\tilde{p}_{hh,g} - p_{hh,g}) q_{hh,g}$, can be expressed as

$$\frac{\partial R_h^D (\tilde{p}; w)}{\partial \ln \tilde{p}_{fh_k}} = \frac{\partial}{\partial \ln \tilde{p}_{fh_k}} \left\{ \sum_g (\tilde{p}_{hh,g} - p_{hh,g}) q_{fh,g} \right\}$$

$$= - \sum_g \left[ p_{hh,g} q_{hh,g} \left( \frac{\partial \ln p_{hh,g}}{\partial \ln \tilde{p}_{fh_k}} + \frac{\partial \ln p_{hh,g}}{\partial \ln q_{hh,g}} \right) \right].$$

where the second line implicitly assumes that by the second welfare theorem, $\tilde{p}^*_{hh,h} = p_{hh,h}$. Fifth, the effect of taxes on the consumer prices can be simplified using Roy’s identity, $-\frac{\partial v_i}{\partial \tilde{p}_{i,g}} = -q_{ji,g}$, as follows

$$\frac{\partial V_h}{\partial \ln \tilde{p}_{fh_k}} = \frac{\partial V_h / \partial Y_h}{\partial \ln Y_h} = -\tilde{p}_{fh,k} q_{fh,k}^c.$$

Finally, the effect of tariffs on wages can be determined by applying the implicit function theorem to the balanced trade condition, $D_h (\tilde{p}, w) = \sum_g (p_{fh,g} q_{fh,g} - \tilde{p}_{fh,g} q_{fh,g})$. Doing
so, implies \( \frac{dw_h}{dp_{f,h,k}} = -\frac{\partial D_h(.)}{\partial p_{f,h,k}} / \frac{\partial D_h(.)}{\partial w_h} \). Hence, defining \( \tau \equiv \left( \frac{\partial V_h}{\partial w_h} / \frac{\partial V_h}{\partial Y_h} \right) / \frac{\partial D_h}{\partial w_h} \),

\[
\frac{\partial V_h / \partial w_h}{\partial V_h / \partial Y_h} \frac{dw_h}{d \ln \hat{p}_{f,h,k}} = -\tau \sum_g \left\{ p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{f,h,g}}{\partial \ln \hat{p}_{f,h,k}} + \right. \\
+ \left. p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{h,h,g}}{\partial \ln \hat{p}_{f,h,k}} - \frac{\partial \ln E_f}{\partial \ln \hat{p}_{f,h,k}} \right\}
\]

Combing the above expression as well as noting that \( \partial V_h / \partial Y_h > 0 \) and \( q_{j,k} = q^*_{j,k} + q^c_{j,k} \), the F.O.C. can be simplified expressed as follows:

\[
\sum_g \left[ \left( \frac{\hat{p}_{f,h,g}}{p_{f,h,g}} - (1 + \tau) \right) p_{f,h,g} q_{f,h,g} \frac{\partial \ln q_{f,h,g}}{\partial \ln \hat{p}_{f,h,k}} - (1 + \tau) \left( \frac{\partial \ln p_{f,h,g}}{\partial \ln q_{f,h,g}} \frac{\partial \ln q_{f,h,g}}{\partial \ln \hat{p}_{f,h,k}} \right) p_{f,h,g} q_{f,h,g} \right] \\
+ \sum_g \left[ \left( 1 - (1 + \tau) \frac{p_{f,h,g}}{\hat{p}_{f,h,g}} \right) \hat{p}_{f,h,g} q_{f,h,g} \eta_{f,h,g} \right] \frac{\partial \ln E_{f,g}}{\partial \ln \hat{p}_{f,h,k}} = 0
\]

Given that \( 1 + t_g \equiv \frac{\hat{p}_{f,h,g}}{p_{f,h,g}} \) and \( 1 + x_g \equiv \frac{p_{f,h,g}}{\hat{p}_{f,h,g}} \), we can further simplify the above expression if we divide it by \( 1 + \tau \) and \( \sum_g p_{f,i,g} q_{f,i,g} \).

\[
\sum_g \left[ \left( \frac{1 + t_g}{1 + \tau} - 1 \right) \frac{\partial \ln p_{f,h,g}}{\partial \ln q_{f,h,g}} \frac{\partial \ln q_{f,h,g}}{\partial \ln \hat{p}_{f,h,k}} + \Delta(x) \frac{\partial \ln E_{f,g}}{\partial \ln \hat{p}_{f,h,k}} \right] = 0
\]

where \( \Delta(x) \equiv \sum_g \left[ \left( 1 - \frac{1}{(1+\tau)(1+x_g)} \right) \lambda_{f,h,g} \eta_{f,h,g} \right] \). Since \( a \) Foreign labor is assigned as the numeraire, \( w_f = 1 \), \( b \) Foreign does not collect tax revenue, and \( c \) the share of intermediate inputs in Foreign’s production is invariant to \( \hat{p}_{f,h,k} \), then \( \partial \ln E_{f,g} / \partial \ln \hat{p}_{f,h,k} = 0 \). Finally, we can determine \( \partial \ln p_{f,h,k} / \partial \ln q_{f,h,k} \) by applying the Implicit Function theorem to \( p_{f,h,k} = \tau_{f,h,C_f,k}(...; \sum_{j,i} \tau_{f,h,k} q_{f,i}) \), which yields \( \partial \ln p_{f,h,k} / \partial \ln q_{f,h,k} = \gamma_{f,k} r_{f,h,k} / (1 - \gamma_{f,k} r_{f,h,k} e_{f,h,k}) \). Combining these results reduces the F.O.C. to

\[
\sum_g \left[ \left( \frac{1 + t_g}{1 + \tau} - 1 - \frac{\gamma_{f,k} r_{f,h,k}}{1 - \gamma_{f,k} r_{f,h,k} e_{f,h,k}} \right) \frac{\partial \ln q_{f,h,g} \hat{r}_{f,h,k}}{\partial \ln \hat{p}_{f,h,k}} \right] = 0.
\]

\textsuperscript{20}Since Foreign does not collect tax revenue, gross expenditure in Foreign, \( E_f \), also equals gross output revenue.
Step 2. Deriving the F.O.C. for Export Taxes. The F.O.C. with respect to sector $k$’s export tax can be stated as

$$\frac{dW_h (\hat{p}; w)}{d \ln \hat{p}_{hf,k}} = \frac{\partial V_h}{\partial Y_h} \left\{ \frac{\partial \Pi_h}{\partial \ln \hat{p}_{hf,k}} + \frac{\partial R^X_h}{\partial \ln \hat{p}_{hf,k}} + \frac{\partial R^D_h}{\partial \ln \hat{p}_{hf,k}} \right\} + \frac{\partial V_h / \partial \ln \hat{p}_{hf,k}}{\partial Y_h / \partial Y_h} d \ln \hat{p}_{hf,k} \left\{ \frac{\partial \Pi_h}{\partial \ln \hat{p}_{hf,k}} + \frac{\partial V_h / \partial \ln \hat{p}_{hf,k}}{\partial Y_h / \partial Y_h} d \ln \hat{p}_{hf,k} \right\} = 0,$$

In the above expression, $\frac{\partial V_h}{\partial \ln \hat{p}_{hf,k}} = 0$. The remaining non-zero elements can be expressed as follows. First, The effect of export taxes on producer surplus, $\partial \Pi_h / \partial 1 + x_k$, can be expressed as

$$\frac{\partial \Pi_h (\hat{p}; w)}{\partial \ln \hat{p}_{hf,k}} = \sum_g \sum_{i=h,f} \left( \frac{\partial \Pi_{hi,g}}{\partial p_{hi,g}} \frac{\partial p_{hi,g}}{\partial \ln \hat{p}_{hf,k}} \right) + \frac{\partial \Pi_h}{\partial \ln \hat{p}_{hf,k}} = \sum_g \sum_{i=h,f} \left( p_{hi,g} q_{hi,g} \frac{\partial p_{hi,g}}{\partial \ln \hat{q}_{hf,g}} \frac{\partial \ln q_{hf,g}}{\partial \ln \hat{p}_{hf,k}} \right),$$

where the last line follows from Hotelling’s lemma that $\partial \Pi_{hi,g} / \partial p_{hi,g} = q_{hi,k}$ and $\partial \Pi_h / \partial p_{hf,g} = 0$. Second, the effect of export taxes on import tax revenues, $R_h^M \equiv \sum_g (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g}$, can be expressed as

$$\frac{\partial R^M_h (\hat{p}; w)}{\partial \ln \hat{p}_{hf,k}} = \frac{\partial}{\partial \ln \hat{p}_{hf,k}} \left\{ \sum_g (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g} \right\} = - \sum_g \left[ (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g} \frac{\partial \ln E_h}{\partial \ln \hat{p}_{hf,k}} + p_{hf,g} q_{hf,g} \frac{\partial \ln p_{hf,g}}{\partial \ln \hat{p}_{hf,k}} \right].$$

Third, the effect of export taxes on export tax revenues, $R_h^X \equiv \sum_g (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g}$, can be expressed as

$$\frac{\partial R^X_h (\hat{p}; w)}{\partial \ln \hat{p}_{hf,k}} = \frac{\partial}{\partial \ln \hat{p}_{hf,k}} \left\{ \sum_g (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g} \right\} = \sum_g \left[ (\hat{p}_{hf,g} - p_{hf,g}) q_{hf,g} \frac{\partial \ln q_{hf,g}}{\partial \ln \hat{p}_{hf,k}} - p_{hf,g} q_{hf,g} \frac{\partial \ln p_{hf,g}}{\partial \ln \hat{q}_{hf,g}} \frac{\partial \ln q_{hf,g}}{\partial \ln \hat{p}_{hf,k}} \right].$$
Fourth, the effect of import taxes on domestic tax revenues, $R_h^X \equiv \sum_g (\bar{p}_{hh,g} - p_{hh,g})q_{hh,g}$, can be expressed as

$$\frac{\partial R^D (\bar{p}; w)}{\partial \ln \bar{p}_{hf,k}} = \frac{\partial}{\partial \ln \bar{p}_{hf,k}} \left\{ \sum_g (\bar{p}_{hh,g} - p_{hh,g})q_{fh,g} \right\}$$

$$= - \sum_g \left[ \bar{p}_{hh,g} q_{hh,g} \frac{\partial \ln p_{hh,g}}{\partial \ln q_{hf,g}} \frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{hf,k}} \right]$$

where the second line implicitly assumes that by the second welfare theorem, $\bar{p}_{hh,k} = p_{hh,k}$. Finally, the effect of export taxes on wages can be determined by applying the implicit function theorem to the balanced trade condition, $D_h = \sum_g p_{fh,g} q_{fh,g} - \bar{p}_{hf,g} q_{hf,g}$. Doing so, implies $\frac{dw_h}{d\bar{p}_{hf,k}} = -\frac{\partial D_h}{\partial \bar{p}_{hf,k}} / \frac{\partial D_h}{\partial w_h}$. Hence, adopting our earlier definition, $\tau \equiv \left( \frac{\partial V_h}{\partial w_h} \right) / \frac{\partial V_h}{\partial Y_h}$.

$$\frac{\partial V_h}{\partial \bar{p}_{hf,k}} \frac{dw_h}{d \ln \bar{p}_{hf,k}} = -\tau \left\{ \sum_g \left[ p_{fh,g} q_{fh,g} \left( \frac{\partial \ln p_{fh,g}}{\partial \ln \bar{p}_{hf,k}} + \eta_{fh,g} \frac{\partial \ln E_h}{\partial \ln \bar{p}_{hf,k}} \right) \right] - \bar{p}_{hf,k} q_{hf,k} - \sum_g \bar{p}_{hf,g} q_{hf,g} \frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{hf,k}} \right\}$$

Combing the above expression as well as noting that $\partial V_h / \partial Y_h > 0$ and $q_{ji,k} = q_{ji,k}^T + q_{ji,k}^C$, the F.O.C. can be simplified expressed as follows:

$$(1 + \tau) \sum_g \left( p_{fh,g} q_{fh,g} \tilde{e}_{fh,k} \right) + \sum_g \left( \left[ (1 + \tau) - \frac{\bar{p}_{hf,g}}{\bar{p}_{hf,k}} \right] \bar{p}_{hf,g} q_{hf,g} \frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{hf,k}} \right)$$

$$+ \left( \sum_g \left[ \frac{\bar{p}_{hf,g}}{p_{fh,g}} - (1 + \bar{\tau}) \right] p_{fh,g} q_{fh,g} \eta_{fh,g} \right) \frac{\partial \ln E_h}{\partial \ln \bar{p}_{hf,k}}$$

where recall that $\frac{\partial \ln q_{hf,g}}{\partial \ln \bar{p}_{hf,k}} = e_{hf,k} + \eta_{hf,g} \frac{\partial \ln E_f}{\partial \ln \bar{p}_{hf,k}}$. Given that $1 + t_g \equiv \frac{\bar{p}_{fh,g}}{p_{fh,g}}$ and $1 + x_g \equiv \frac{\bar{p}_{fh,g}}{p_{fh,g}}$, we can further simplify the above expression if we divide it by $1 + \tau$ and $E_f$ (noting that $p_{fh,k} q_{fh,k} = \hat{r}_{fh,k} E_f$ and $\bar{p}_{hf,g} q_{hf,g} = \hat{\lambda}_{hf,g} E_f$):
\[ \lambda_{hf,k} + \sum_g \left( 1 - \frac{1}{(1 + x_g)(1 + \tau)} \right) \lambda_{hf,g} \varepsilon_{hf,g}^{h,f,k} \]

\[- \sum_g \left( \hat{r}_{fh,g} \alpha_{fh,g}^{h,f,k} \right) \frac{Y_f}{E_f} + \Delta_h(t) \frac{\partial \ln E_h}{\partial \ln p_{hf,k}} + \Delta_f(x) \frac{\partial \ln E_f}{\partial \ln p_{hf,k}} = 0, \]

where \[ \Delta_h(t) \equiv \sum_g \left( \frac{1 + t_g}{1 + \tau} - 1 \right) \hat{r}_{fh,g} \varepsilon_{hf,g} \] is a uniform term. As noted before, given our choice of numeraire, \[ \partial \ln E_f / \partial \ln p_{hf,k} = 0. \] Given the Lerner symmetry and the multiplicity of the optimal trade tax, there always exists a solution to the above problem where \[ \Delta_h(t) = 0. \] Henceforth, we restrict our attention to solving for this particular solution. Once we do that, the remaining solutions can be identified with a basic multiplicative transformation of the import and export tax vectors. yields the following optimality condition:

\[ \hat{\lambda}_{hf,k} + \sum_g \left( 1 - \frac{1}{(1 + x_g)(1 + \tau)} \right) \hat{\lambda}_{hf,g} \hat{\varepsilon}_{hf,g}^{h,f,k} - \sum_g \left( \hat{r}_{fh,g} \hat{\alpha}_{fh,g}^{h,f,k} \right) = 0. \]  

(11)

**Step 3: Simultaneously Solving the System of F.O.C.** As a final step, we simultaneously solve the system of F.O.C.s for all tax instruments:

\[ \sum_g \left( \frac{1 + t_g}{1 + \tau} - 1 \right) \varepsilon_{fh,g} \frac{\partial \ln q_{fh,g}}{\partial \ln p_{hf,k}} r_{fh,g}, \quad \forall k \in K \]

\[ \sum_g \left( 1 - \frac{1}{(1 + x_g)(1 + \tau)} \right) \alpha_{hf,g}^{h,f,k} = 1 - \sum_g \frac{\hat{r}_{fh,g} \alpha_{fh,g}^{h,f,k}}{\hat{\lambda}_{hf,k}}, \quad \forall k \in K \]

First, note that by the Lerner symmetry, the value of \( \tau \) is redundant. In other words, replacing \( 1 + \tau \) with any \( 1 + \tilde{\tau} \in \mathbb{R}_+ \) identifies an optimal tax schedule. To derive a simplified expression for optimal export taxes, define \( \Omega_{hf,k} \equiv 1 - \sum_g \left( \frac{r_{fh,g} \alpha_{fh,g}^{h,f,k}}{\hat{r}_{fh,g} \alpha_{fh,g}^{h,f,k}} \right) \) and suppose the optimal export tax has the following formulation:

\[ 1 + x_k^* = \frac{\varepsilon_{hf,k}}{\varepsilon_{hf,k} + \xi_{hf,k} + \Omega_{hf,k}}. \]

Plugging the above expression back into the F.O.C. implies that that

\[ \sum_g \left( \xi_{hf,g} + \Omega_{hf,g} \right) \frac{\hat{\lambda}_{hf,g} \varepsilon_{hf,g}^{h,f,k}}{\hat{\lambda}_{hf,k} \varepsilon_{hf,k}^{h,f,k}} = \Omega_{hf,k}. \]

This equation can be written in matrix notation as \( \Xi (\xi + \Omega) = \Omega \), where \( \xi \equiv [\xi_{hf,k}]_k \) and \( \Omega \equiv [\Omega_{hf,k}]_k \) are \( K \times 1 \) vectors and
Ξ ≡ [ξ_{hf,k}]_{k,g} and I_K are K × K matrixes. Inverting the above system implies that

\[[ξ_{hf,k}]_k = (Ξ^{-1} - I_K) Ω. \quad (12)\]

So, altogether the following tax schedule corresponds to an optimal policy for an arbitrary choice of $1 + \bar{t} \in \mathbb{R}_+$:

\[
1 + t_k^* = \frac{1 + \frac{1}{α_{hf,k}} (1 + \bar{t})}{1 + α_{hf,k} + ζ_{hf,k} - \sum_g \frac{f_{hg}}{α_{hf,k}} ζ_{hf,k}} (1 + \bar{t})^{-1},
\]

where ζ_{hf,k} is given by Equation 12.

### C Proof of Theorem 2

The F.O.C. with respect to sector k’s tariff can be expressed as

\[
\frac{dW_h(\bar{p}; w)}{d \ln \bar{p}_{fh,k}} = \frac{∂V_h(.)}{∂Y_h} \left[ \frac{∂Y_h}{∂ \ln \bar{p}_{fh,k}} + \frac{∂Y_h}{∂ w_h} \frac{d w_h}{d \ln \bar{p}_{fh,k}} \right] = 0.
\]

It should be noted upfront that the difference between the present setup and the pure Ricardian case, is that the (conditional) tariff pass-through $σ_{fh,g}^{\bar{t}h} = \frac{∂ \ln \bar{p}_{fh,g}}{∂ \ln (1 + t_k)}$ can be non-zero (even if $g \neq k$) due to (i) the upward sloping supply curve in industry g, plus (ii) the cross-substitutability between industry k and industry g goods. Plugging $Y_h = w_h L_h + \mathcal{R}_h^M$, into the F.O.C. yields the following:

\[
\frac{dW_h(\bar{p}; w)}{d \ln \bar{p}_{fh,k}} = \frac{∂V_h(.)}{∂Y_h} \left\{ \frac{∂\mathcal{R}_h^M}{∂ \ln \bar{p}_{fh,k}} + \frac{∂V_h}{∂ \bar{p}_{fh,k}} \frac{∂ \bar{p}_{fh,k}}{∂Y_h} + \frac{∂V_h}{∂ w_h} \frac{d w_h}{d \ln \bar{p}_{fh,k}} \right\} = 0,
\]

By Roy’s identity

\[
\frac{∂V_h}{∂ \ln \bar{p}_{fh,k}} = -\bar{p}_{fh,k} q_{fh,k}^C - \sum_g \bar{p}_{hh,g} q_{hh,g}^C \frac{∂ \ln \bar{p}_{hh,g}}{∂ \ln \bar{p}_{fh,k}}
= -\bar{p}_{fh,k} q_{fh,k}^C - \sum_g \bar{p}_{hh,g} q_{hh,g}^C \bar{r}_{fh,k}
\]

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where \( \hat{\alpha}_{i,k}^{f,h} \equiv \partial \ln \hat{p}_{ij,g} / \partial \ln \hat{p}_{fj,h,k}^{T} \) denotes the general equilibrium effect of a change in \( \hat{p}_{fj,h,k} \) on the price industry \( g \)’s output in country \( i \). The \( K \times K \) matrix for \( \hat{\alpha}_{i,k}^{f,h} \) can be calculated by applying the Implicit Function Theorem to \( \hat{p}_{fj,k} = (1 + t_{f,j,k})C_{f,k}(w_{f}, \hat{p}_{ff}^{T}, \hat{p}_{hf}^{T}) \), which implies that

\[
\left[ \hat{\alpha}_{f,g}^{f,k} \right]_{k,g} = \left( I_{K} - \left[ \alpha_{f,g}^{h,k} \right]_{k,g} \right) \left[ \alpha_{f,g}^{f,k} \right]_{k,g} \cdot
\]

Likewise, \( \hat{\alpha}_{f,g}^{f,k} \) can be calculated by applying the Implicit Function Theorem to \( \hat{p}_{hf,k} = \tau_{hi,k}C_{hi,k}(w_{f}, \hat{p}_{fh}^{T}, \hat{p}_{hh}^{T}) \), which yields the following:

\[
\left[ \hat{\alpha}_{h,g}^{f,k} \right]_{k,g} = \left( I_{K} - \left[ \alpha_{h,g}^{h,k} \right]_{k,g} \right) \left[ \alpha_{h,g}^{f,k} \right]_{k,g} \cdot
\]

With the above definition in mind, the effect of import taxes on import tax revenues, \( R_{h}^{M} \equiv \sum_{g}(\hat{p}_{fh,g} - p_{fh,g})q_{fh,g} \), can be expressed as

\[
\frac{\partial R_{h}^{M}(\hat{p},w)}{\partial \ln \hat{p}_{fh,k}} = \frac{\partial}{\partial \ln \hat{p}_{fh,k}} \left\{ \sum_{g}(\hat{p}_{fh,g} - p_{fh,g})q_{fh,g} \right\} \\
= \hat{p}_{fh,q}q_{fh,k} + \sum_{g} \left( (\hat{p}_{fh,g} - p_{fh,g})q_{fh,g} \left( \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} + \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{hh,g}} \hat{\alpha}_{h,g}^{f,k} \right) - p_{fh,g}q_{fh,g} \hat{\alpha}_{f,g}^{f,k} \right),
\]

where as before (with a slight abuse of notation) \( \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} = \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} + \eta_{fh,g} \frac{\partial \ln E_{h,g}}{\partial \ln \hat{p}_{fh,k}} \cdot \) The income driven term, however, can be dropped as it has a second-order effect, since \( \frac{\partial \ln E_{h,g}}{\partial \ln \hat{p}_{fh,k}} \propto (R_{h}^{M}/Y_{h})_{i,h,k} \), where \( \delta_{h} \approx \lambda_{i,h,k} \approx 0 \).

\[
\frac{\partial V_{h}}{\partial w_{h}} = \frac{d w_{h}}{d \ln \hat{p}_{fh,k}} = -\tau \sum_{g} \left\{ p_{fh,g}q_{fh,g} \left( \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{fh,k}} + \frac{\partial \ln q_{fh,g}}{\partial \ln \hat{p}_{hh,g}} \hat{\alpha}_{h,g}^{f,k} \right) + p_{fh,g}q_{fh,g} \hat{\alpha}_{f,g}^{f,k} - \hat{p}_{hf,g}q_{hf,g} \frac{\partial \ln q_{hf,g}}{\partial \ln \hat{p}_{hf,g}} \hat{\alpha}_{h,g}^{f,k} \right\}
\]

Finally, we have to plug Equations back into the F.O.C.. To simplify some terms we can use the zero profits condition, which entails that

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21To keep things simple, we are exercising a slight abuse of notation here. A more elaborate choice of notation would be \( \hat{\alpha}_{i,k}^{f,h} = \partial \ln \hat{p}_{ij,g} / \partial \ln \hat{p}_{fj,h,k}^{T} \), which clarifies that \( \hat{\alpha}_{i,k}^{f,h} \neq \hat{\alpha}_{i,k}^{f,k} \).
\[
\frac{\partial \Pi_h(p; \omega)}{\partial \ln \hat{p}_{fh,k}} = \sum_s \sum_{i=h, f} \left( \frac{\partial \Pi_h}{\partial \ln p_{hi,g}} \frac{\partial \ln p_{hi,g}}{\partial \ln \hat{p}_{fh,k}} \right) + \frac{\partial \Pi_h}{\partial \ln \hat{p}_{hh,k}} + \sum_s \left( \frac{\partial \Pi_h}{\partial \ln \hat{p}_{hh,k}} \frac{\partial \ln \hat{p}^T_{hh,k}}{\partial \ln \hat{p}_{fh,k}} \right)
\]

\[
= \sum_s \sum_{i=h, f} \left( \hat{p}_{hi,g} q_{hi,g} \tilde{\alpha}_{h,g}^{f,k} \right) - \hat{p}_{fh,k} q_{fh,k} - \sum_s \left( \sum_{i=h, f} \hat{p}_{hh,g} q_{hh,g} \tilde{\alpha}_{h,g}^{f,k} \right) = 0,
\]

Combining the above expressions, the F.O.C. can be stated as

\[
\sum_s \left( \left[ \frac{1 + t_g}{1 + \tau} - 1 \right] \left( \epsilon_{fh,g} + \epsilon_{fh,g} \tilde{\alpha}_{h,g}^{f,k} \right) - \hat{\alpha}_{h,g}^{f,k} \right) \lambda_{fh,g} = 0.
\]

where note that \( \hat{p}_{fh,g} / p_{fh,g} = 1 + t_k \). To further simplify the above expression, we can divide by \( 1 + \tau \) and gross expenditure in Foreign (which equal gross production revenue since Foreign collects no tax revenue, i.e., \( \sum_k E_{f,k} = \sum_k Y_{f,k} \)), which yields

\[
\sum_s \left( \left[ \frac{1 + t_g}{1 + \tau} - 1 \right] \left( \epsilon_{fh,g} + \epsilon_{fh,g} \tilde{\alpha}_{h,g}^{f,k} \right) - \hat{\alpha}_{h,g}^{f,k} \right) \lambda_{fh,g} = 0.
\]

The next step is to characterize \( \tau \equiv \frac{-\partial (W_f / \partial w_f)}{\partial \sum_g (p_{fh,g} q_{fh,g} - p_{fh,g} \eta_{fh,g}) / \partial w_f} \), where the partial derivative denotes the derivative with respect to \( w_f \) holding \( \hat{p}_{fh,k} \) fixed.

\[
\tau = -\sum_s p_{fh,k} q_{fh,g} \frac{\partial \ln p_{fh,g}}{\partial \ln w_f} - \sum_s p_{fh,k} q_{fh,g} \left( \frac{\partial \ln q_{fh,g}}{\partial \ln p_{fh,k}} \frac{\partial \ln p_{fh,k}}{\partial \ln w_f} + \frac{\partial \ln p_{fh,g}}{\partial \ln E_{f,k}} \frac{\partial \ln E_{f,k}}{\partial \ln w_f} \right)
\]

\[
= -\sum_s p_{fh,k} q_{fh,g} \frac{\partial \ln p_{fh,g}}{\partial \ln w_f} - \sum_s p_{fh,g} q_{fh,g} \left( \epsilon_{fh,g} + \eta_{fh,g} \right) \delta_{f,g}
\]

\[
= \sum_s p_{fh,g} q_{fh,g} \delta_{f,k} + \sum_s p_{fh,g} q_{fh,g} \epsilon_{fh,k} \delta_{f,k} = -\frac{1}{1 + \sum_s \omega_{fh,g} \epsilon_{fh,g}} = -\frac{1}{1 + \varepsilon_{fh}},
\]

where the second line follows from the well-known result in consumer theory that \( \sum_i (\epsilon_{h_i,f,g}^{f_i,g}) + \eta_{h,f} = 0 \); \( \varepsilon_{fh} \) denotes the elasticity of foreign demand for labor as defined under Definition D5; and \( \omega_{fh,g} \equiv \delta_{f,k} \lambda_{fh,k} / \sum_k \delta_{f,k} \hat{p}_{fh,k} \) is the weight assigned to industry.
when calculating the elasticity of labor demand. \( \delta_{f,g} \), meanwhile, denotes the total contribution of Foreign labor to industry-level output. As discussed in Section 4, the vector \( \delta_f \equiv [\delta_{i,k}]_k \) can be calculated as

\[
\delta_f = (I_K - \alpha_{ff})^{-1} \bar{\alpha}_f,
\]

where recall that \( \bar{\alpha}_f \equiv [\bar{\alpha}_{f,g}, \bar{\alpha}_{g,f}]_k \) is a \( K \times 1 \) vector and \( \alpha_{ff} \equiv [\alpha_{f,f}]_g \) is a \( K \times K \) input-output matrix. Also, note that if Home is sufficiently small compared to Foreign (which is often the case since Foreign is an aggregate of the rest of the world), then \( \delta_{f,k} \approx 1 \) in which case \( \omega_{hf,g} \approx \lambda_{hf,g}/\lambda_{hf} \). Finally, plugging the all the above expressions back into the Equation 13, and inverting the system specified by this equation implies that

\[
1 + t^*_k = \frac{\tilde{\epsilon}_{hf}}{1 + \tilde{\epsilon}_{hf}} \left(1 + \frac{\tau_k}{\tilde{\epsilon}_{fh,k}(1 + \tilde{\epsilon}_{hf})}\right),
\]

where \( \tau \equiv [\tau_k] \) is given by

\[
\tau = \left[ 1_{k=g} + \frac{\epsilon_{ff,g} \bar{\alpha}_{f,k}}{\epsilon_{fh,k} \bar{\alpha}_{f,k}} \right]^{-1} \left[ \sum_{g \in K} \left( \bar{\alpha}_{f,k} - [1 + \tilde{\epsilon}_{hf,g} \lambda_{hf,g}] \bar{\alpha}_{f,k} \lambda_{hf,g} \right) \right].
\]

D The Multiple Country Case

Suppose there are arbitrarily many countries. Then, the effect of country \( i \)'s import tax on own welfare can be expressed as

\[
\frac{dW_i(.)}{d \ln (1 + t_{ji,k})} = \frac{\partial W_i(.)}{\partial \ln (1 + t_{ji,k})} + \sum_{j \neq i} \frac{\partial W_i(.)}{\partial \ln w_j} \frac{d \ln w_j}{d \ln (1 + t_{ji,k})}.
\]

It is straightforward to show that \( \frac{\partial W_i(.)}{\partial \ln w_j} \frac{d \ln w_j}{d \ln (1 + t_{ji,k})} \propto \lambda_{ji,r_{ji,k}} \lambda_{ji,k} \), and that based on actual trade data, \( \lambda_{ji,r_{ji,k}}/\lambda_{ii,r_{ii,k}} \approx 0 \) for \( j \neq i \). So, treating labor in country \( j \) as the numeraire, changes in welfare can be approximated to a first-order as

\[
\frac{dW_i(.)}{d \ln (1 + t_{ji,k})} \approx \frac{\partial W_i(.)}{\partial \ln (1 + t_{ji,k})} + \frac{\partial W_i(.)}{\partial \ln w_i} \frac{d \ln w_i}{d \ln (1 + t_{ji,k})}.
\]

The same applies to export taxes. We can, thus, cast the Country \( i \)'s optimal policy problem as one that maximizes \( W_i(\tilde{p},w_i) \) by choosing \( \{\tilde{p}_{ij,k}\} \) and \( \{\tilde{p}_{ji,k}\} \) subject to feasibility constraints. Following the same steps taken in Appendix B, the F.O.C. corresponding to
price \( \hat{p}_{j,i,k} \) (i.e., import tax, \( t_{j,i,k} \)) can be expressed as:

\[
\sum_g \sum_j \left[ \frac{1 + t_{j,i,g}}{1 + \tau_i} - 1 - \gamma_{j,i,g} \right] \frac{\partial \ln q_{j,i,g}^*}{\partial \ln \hat{p}_{j,i,k}} p_{j,i,g} q_{j,i,g} = 0.
\]

\( \gamma_{j,i,k} \) can be derived by applying the Implicit Function Theorem to \( p_{j,i,k} = \tau_{j,i,k} C_i (..., \sum_g \sum_d \tau_{j,i,g} q_{j,i,g} ) \), which implies the following:

\[
\gamma_{j,i,k} = \frac{\gamma_{j,k}^* r_{j,i,k}}{1 - \sum_{i \neq j} \gamma_{j,k}^* r_{j,i,k} \varepsilon_{j,i,k}}.
\]

Likewise, the F.O.C. corresponding price \( \hat{p}_{ij,k} \) (i.e., export tax, \( x_{ij,k} \)) can be expressed as:

\[
\sum_g \sum_j \left[ \frac{1}{(1 + x_{ij,g})(1 + \tau_i)} - 1 \right] \left[ \hat{\lambda}_{ij,g} \varepsilon_{ij,g} \right] = 1 - \sum_g \sum_j \frac{\Lambda_{ij,g} \hat{\lambda}_{ij,k}^{i,j_k}}{\hat{\lambda}_{ij,k}} \forall k \in K
\]

\[
\sum_g \left( \frac{1}{(1 + x_{ij,g})(1 + \tau_i)} - 1 \right) \hat{\lambda}_{ij,g} \varepsilon_{ij,g} = 1 - \sum_g \sum_j \frac{\Lambda_{ij,g} \hat{\lambda}_{ij,k}^{i,j_k}}{\hat{\lambda}_{ij,k}} \forall k \in K
\]

where \( \Lambda_{j,i,g} \equiv p_{j,i,g} q_{j,i,g} / \sum_{j \neq i} \sum_k p_{j,k} q_{j,k} \); \( \hat{\lambda}_{ij,g} \equiv \hat{p}_{ij,g} q_{ij,g} / \sum_{j \neq i} \sum_k p_{j,k} q_{i,j,k} \); and \( \hat{\lambda}_{j,i,g}^{i,j_k} = \alpha_{j}^i a_{j}^j \) if \( j \neq j \), with \( \hat{\lambda}_{j,i,g}^{i,j_k} = \alpha_{j}^i \). Combining the above expressions implies

\[
1 + t_{j,i,k}^* = \left( 1 + \frac{\gamma_{j,k}^* r_{j,i,k}}{1 - \sum_{i \neq j} \gamma_{j,k}^* r_{j,i,k} \varepsilon_{j,i,k}} \right) (1 + \bar{r})
\]

\[
1 + x_{ij,k}^* = \frac{\varepsilon_{ij,k}}{1 + \varepsilon_{ij,k} + \varepsilon_{ij,k} - \sum_g \sum_j \frac{\Lambda_{ij,g} \hat{\lambda}_{ij,k}^{i,j_k}}{\hat{\lambda}_{ij,k}}} \hat{\lambda}_{ij,k} (1 + \bar{r})^{-1}.
\]

**Uniformity of Import Taxes.** Here, we establish the uniformity of import taxes (when \( \gamma_{j,k} = 0 \)) without invoking any first-order approximation. Analogous to the two-country model, welfare in country \( i \) can be expressed as \( W_i = \partial V_i (Y_i, \hat{p}_i) \), where \( Y_i = w_i L_i + \sum_k (t_{j,i,k} p_{j,i,k} q_{j,i,k} + x_{ij,k} p_{ij,k} q_{ij,k}) \). Correspondingly, \( W_i \) is uniquely determined by the vector of import and export taxes, \( t_i = \{ t_{j,i,k} \} \) and \( x_i = \{ x_{ij,k} \} \), plus the vector of country-level wages, \( w = \{ w_i \} \):

\[
W_i (t_i, x_i; w) = V_i (Y_i (t_i, x_i; w), \hat{p}_i (t_i, x_i; w))
\]
Defining $D_i(t_i, x_i; w) = w_i L_i - \sum_k \sum_\ell (p_i, k q_{i, \ell, k})$, the equilibrium vector of aggregate wages, $w$, solves the following system of equations:

$$
\begin{align*}
D_1 (t_1, x_1; w) &= 0 \\
& \vdots \\
D_N (t_N, x_N; w) &= 0
\end{align*}
$$

(14)

Keeping the above observation in mind, we can write the F.O.C. with respect to $t_{ji,k}$ as

$$
\frac{d W_i (t_i, x_i; w)}{d (1 + t_{ji,k})} = \frac{\partial V_i (Y_i, \tilde{p}_i)}{\partial Y_i} \left[ \frac{\partial Y_i}{\partial (1 + t_{ji,k})} + \frac{\partial Y_i}{\partial w_i} \frac{d w_i}{d (1 + t_{ji,k})} \right] \\
&+ \sum_{g \in K} \sum_{k \in C} \left( \frac{\partial V_i (Y_i, \tilde{p}_i)}{\partial \tilde{p}_{i,g}} \left[ \frac{\partial \tilde{p}_{i,g}}{\partial (1 + t_{ji,k})} + \frac{\partial \tilde{p}_{i,g}}{\partial w_{\ell}} \frac{d w_{\ell}}{d (1 + t_{ji,k})} \right] \right)
$$

Invoking Roy’s identity, $\frac{\partial V_i (Y_i, \tilde{p}_i)}{\partial \tilde{p}_{i,k}} / \partial Y_i$ yields the following optimality condition

$$
\frac{\partial Y_i}{\partial \ln (1 + t_{ji,k})} = \tilde{p}_{ji,k} q_{ji,k} + \sum_{\ell \in C, g \in K} \sum_{i \in G} \left( t_{\ell, i, k} p_{\ell, i, k} q_{\ell, i, k} \left( \varepsilon_{\ell, i, g} + \eta_{\ell, i, g} \frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})} \right) \right)
$$

Plugging the above equation back into the F.O.C. and defining $\Delta_{\ell, i, g}^{ji,k} \equiv \varepsilon_{\ell, i, g} + \eta_{\ell, i, g} \frac{\partial \ln Y_i}{\partial \ln (1 + t_{ji,k})}$, will yield the following optimality condition

$$
\frac{d W_i (t_i, x_i; w)}{d \ln (1 + t_{ji,k})} = \frac{\partial V_i (Y_i, \tilde{p}_i)}{\partial Y_i} \left[ \tilde{p}_{ji,k} q_{ji,k} - \tilde{p}_{ji,k} q_{ji,k} + \sum_{\ell \in C, g \in K} \sum_{i \in G} \left( t_{\ell, i, k} p_{\ell, i, k} q_{\ell, i, k} \Delta_{\ell, i, g}^{ji,k} \right) \right] - \sum_{\ell \in C} \left( v_{i, \ell} \frac{d \ln w_{\ell}}{d \ln (1 + t_{ji,k})} \right)
$$

where $v_{i, \ell} \equiv \partial W_i / \partial \ln w_{\ell}$. Applying the implicit function theorem to the System of Equations (14), we can solve for $d \ln w / d \ln 1 + t_i$ as follows:

$$
\frac{d \ln w_1}{d \ln (1 + t_{1,i,k})} \ldots \frac{d \ln w_1}{d \ln (1 + t_{N,i,k})} \\
\vdots \ldots \vdots \\
\frac{d \ln w_N}{d \ln (1 + t_{1,i,k})} \ldots \frac{d \ln w_N}{d \ln (1 + t_{N,i,k})}
$$

$$
= \left( \frac{\partial \ln D_1}{\partial \ln w} \right)^{-1} \left[ \frac{\partial \ln D_1}{\partial \ln (1 + t_{1,i,k})} \ldots \frac{\partial \ln D_1}{\partial \ln (1 + t_{N,i,k})} \\
\vdots \ldots \vdots \\
\frac{\partial \ln D_N}{\partial \ln (1 + t_{1,i,k})} \ldots \frac{\partial \ln D_N}{\partial \ln (1 + t_{N,i,k})} \right],
$$

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Letting $\tau_{\ell i}$ denotes element $\ell i$ of matrix $(\partial \ln D/\partial \ln w)^{-1}$, the above system implies that for every $\ell \in C$

$$\frac{d \ln w_\ell}{d \ln (1 + t_{ji,k})} = \sum_{\ell \in C} \left( \tau_{\ell n} \sum_{g \in K} \left( p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k} \right) \right).$$

Plugging the above expression back into the F.O.C. implies the following

$$\sum_{n \in C} \sum_{g \in K} \left( t_{ni,g} p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k} \right) - \sum_{\ell \in C} \left( \tau_{\ell n} \sum_{n \in C} \left( p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k} \right) \right) = 0.$$

The above expression can in turn be rearranged as

$$\sum_{n \in C} \sum_{g \in K} \left[ \left( t_{ni,g} - \tau_{ni} \right) p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k} \right] = \sum_{n \in C} \sum_{g \in K} \left[ \left( t_{ni,g} - \tau_{ni} \right) p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k} \right] = 0,$$

where $\tau_{ni} \equiv \sum_{\ell \in C} \tau_{\ell n} v_i \ell$, or, equivalently

$$T_i \Delta_i = 0,$$

where $T_i = [t_{ni,g} - \tau_{ni}]_{ng \in C \times K}$ and $\Delta_i = [p_{n_i g} q_{n_i g} \Delta_{n_i g}^{ji,k}]$ are respectively $1 \times N \cdot K$ and $N \cdot K \times N \cdot K$ matrices. If $\det \Delta_i \neq 0$, then $T_i = 0$ is the unique solution to the above system, which implies that the optimal import tax is uniform across products originating from the same exporting country:

$$t_{ji,k}^* = \tau_{ji}, \quad \forall j, k.$$
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European Union