1. [15 pts] For this problem, consider an idealized spiral galaxy which only has a disk component.
   (a) Using simple dynamical arguments (e.g., Newton’s Laws), derive the scaling relationship
   between the mass, $M$, of the spiral galaxy and the maximum rotation velocity, $V_{\text{max}}$, and size,
   $h_R$, for a bulge-less disk galaxy.
   (b) Recall that $L = 2\pi I(0)h_R^2$ for an exponential disk galaxy. If the $M/L$ ratio and central surface
   brightness are constant for all galaxies, find the scaling relationship for luminosity and maximum
   rotation velocity.
   (c) Discuss the implications if low surface brightness galaxies follow the same scaling relations
   as high surface brightness galaxies. In particular, derive the relation between the mass-to-light
   ratio and the central surface brightness if all disk galaxies follow the same scaling relation, e.g.,
   the Tully-Fisher relation.

2. [20 pts] (a) Assuming virial equilibrium, show that if all ellipticals had the same mass–to–light
   ratio and surface brightness $I(R_e)$, they would follow the Faber-Jackson relation.
   (b) One equation for the fundamental plane is
   
   $$\log R_e = 0.36(<I>_e/\mu_B) + 1.4\log \sigma_0$$

   where $R_e$ is in kpc, $<I>_e$ is surface brightness in $\mu_B$ magnitudes per square arcsec, and $\sigma_0$ is
   in km/s. Find an edge-on view of the fundamental plane where $\log \sigma_0$ is plotted horizontally.
   What linear combination of $\log R_e$ and $<I>_e$ should be plotted vertically?
   (c) Show that the above is equivalent to
   
   $$R_e \propto \sigma^{1.4} I_e^{-0.9}$$

   where $I_e$ is expressed in intensity units (e.g., $L_\odot$ pc$^{-2}$).
   (d) Assuming virial equilibrium and the above scaling relation, find the dependence of the mass-
   to-light ratio on $\sigma$ and $R_e$.

3. [15 pts] Write a short abstract of your selected class project. You should write the abstract
   in a similar style as a AAS meeting abstract (brief and to the point). Just a reminder, at this
   stage in the semester you should have a well defined approach to your topic and have preliminary
   results from your analysis.