1. [20 pts] Recall that the Schechter function is:

$$\phi(L) dL = n^* (L/L^*)^\alpha \exp(-L/L^*) \ d(L/L^*)$$

where $n^*$ is the number density per Mpc$^3$ of $L^*$ galaxies. $L^*$ characterizes the break in the luminosity function, and $\alpha$ is the powerlaw slope for $L << L^*$. Further, recall that the absolute magnitude $M^*$ is related to the luminosity $L^*$ by

$$\log(L/L^*) = -0.4(M - M^*)$$

(a) Derive the expression for $\phi(M) dM$.

(b) For an apparent magnitude limited sample, the distance to which you can see a galaxy of luminosity $L$ is $d \propto L^{1/2}$, and the volume $V_s \propto d^3$. The luminosity distribution of such a sample is $n(L) dL = \phi(L) dL V_s(L)$. Find how $n(L)$ depends on $L/L^*$ and $\alpha$, and then evaluate the peak luminosity in units of $L^*$ for such a sample if $\alpha = -1.25$.

(c) The mass–to–light ratio of galaxies is approximately $12 (M_\odot/L_\odot)$. Using this and the values for the B band luminosity function $n^* = 0.012 h $ Mpc$^{-3}$, $L^* = 10^{10} h^{-2} L_\odot$, and $\alpha = -1.25$, find the mass density of local galaxies in grams per cm$^3$ for $h = 0.7$.

2. [10 pts] The S´ersic profile is often written as

$$I(R) = I(R_e) \exp(-b[(R/R_e)^{1/n} - 1])$$

where the constant $b$ is chosen so that the effective radius, $R_e$, includes half the light. For $n > 1$, $b \sim 1.999n - 0.327$.

(a) Show that the $R^{1/4}$ formula yields a total luminosity

$$L = \int_0^\infty 2\pi RI(R) dR = 8! \ \frac{e^{7.67}}{(7.67)^8} \pi R_e^2 I(R_e) \sim 7.22\pi R_e^2 I(R_e).$$

(b) Show that half of this light comes from within radius $R_e$.

3. [20 pts] I Zw 18 is a nearby, extremely metal–poor galaxy. It has an apparent magnitude (blue) of 16.61, a gas-phase metallicity of 1/50th of solar $[12 + \log(O/H) = 7.2]$, and an observed HI flux density of 2.97 Jy km/s.

(a) Assuming a distance of 10 Mpc, what is its absolute magnitude and total hydrogen mass? Recall that the total hydrogen mass can be estimated for an optically thin gas as $M_H = 2.356 \times 10^3 d_{Mpc}^3 \int S \ dv$.

(b) Using the results from the Wilson (1995) paper, calculate the expected J=1 $\rightarrow$ 0 CO line intensity for I Zw 18. [Note, eq 4 of the paper is for $\log(\alpha/\alpha_{Gal})$.]

(c) The newly re-furbished ARO 12m came back online in 2015. It has a beam size of 55$''$ for observations of the J=1 $\rightarrow$ 0 line at 115 GHz. This is sufficiently large that a single pointing will include all of the emission from a tiny galaxy such as I Zw 18. The observed HI line width of I Zw 18 is 50 km/s. If the ARO 12m has a gain of 34 Jy/K at 115 GHz, what is the expected peak antenna temperature for an observation of I Zw 18?

(d) With a one hour observation, it is possible to achieve rms noise of approximately 6 mK with the 115 GHz receiver and 5.2 km/s velocity resolution. Based on your calculation above, comment on the feasibility of detecting CO in I Zw 18 with the newly re-furbished KPNO 12m telescope.