Energy Production

- Possibilities:
  - Gravitational collapse
  - Thermal
  - Chemical
  - Nuclear

- Timescale for collapse (dynamical time or free-fall time)
  \[ t_{\text{ff}} \propto \frac{1}{\sqrt{G\rho}} \]

Why? Consider point particle at large distance:

Conservation of energy: \[ \Delta T = \Delta U \]

- pt. source

\[ \frac{1}{2} m \, v^2 = \frac{GMm}{r} - \frac{GMm}{r_0} \]

\[ \frac{1}{2} m \, (dr/dt)^2 = \frac{GMm}{r} - \frac{GMm}{r_0} \]

\[ t_{\text{ff}} = \int_{r_0}^{0} \left( \frac{dt}{dr} \right) dr = -\int_{r_0}^{0} \left[ \frac{GM}{r} - \frac{GM}{r_0} \right]^{-\frac{1}{2}} \, dr \]

\[ = \sqrt{\frac{3\pi}{32 \, G \, \rho}} \]

For sun, \( t_{\text{ff}} = \frac{1}{2} \) hour \( \Rightarrow \) sun will react quickly to perturbations
Timescales

- **Kelvin-Helmholtz timescale**: $t_{\text{K-H}} = \frac{GM^2}{RL}$
  - Lifetime of the star if energy is supplied by gravity
    - For sun: $t_{\text{K-H}} \sim 30 \times 10^6$ yrs

- **Thermodynamic timescale**: $t_{\text{th}} = \frac{N_kT}{L}$
  - Lifetime of a star that is just cooling
    - $t_{\text{th}} \sim t_{\text{K-H}}$

- **Chemical Burning**: $t_{\text{CB}} = \frac{\varepsilon M}{L}$
  - Where $\varepsilon =$ energy/mass of some chemical process
  - For TNT, $\varepsilon \sim 5000$ J/gm
  - For sun, $t_{\text{CB}} \sim 1000$ yrs

- **Nuclear Burning**: $t_{\text{NB}} = \frac{\eta M c^2}{L}$
  - If $\eta \sim 7 \times 10^{-4}$ for sun, $t_{\text{NB}} \sim 10$ billion years
  - This is the solution. Recall, however, that earlier models of the sun didn’t know about nuclear energy production. This was a major puzzle in the mid-1800’s.
Energy Generation

• Gravitational potential energy
  – Important for pre-main sequence stars
• Nuclear Fusion
  – Energy is released when 2 nuclei fuse
  – Relevant forces are strong and weak forces
  – $E = mc^2$
Nuclear Binding Energy

Binding energy = difference between the mass of the nucleus and the sum of the individual masses of the protons and neutrons:

\[ \text{Binding energy} = \Delta m \ c^2 \]

Protons: \( 2 \times 1.00728 \text{ amu} \)

Neutrons: \( 2 \times 1.00866 \text{ amu} \)

Mass of parts: 4.03188 amu

Mass of alpha: 4.00153 amu

Where 1 amu = \( 1.66054 \times 10^{-27} \text{ kg} = 931.494 \text{ MeV/c}^2 \)
Proton-Proton Chain

• PP I

\[ ^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + \text{e}^+ + \nu \quad (1.44 \text{ MeV}) \]
\[ ^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \quad (5.49 \text{ MeV}) \]
\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H} \quad (12.9 \text{ MeV}) \]

• PP II

\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \quad (1.59 \text{ MeV}) \]
\[ ^7\text{Be} + \text{e}^- \rightarrow ^7\text{Li} + \nu \quad (0.861 \text{ MeV}) \]
\[ ^7\text{Li} + ^1\text{H} \rightarrow 2 \ ^4\text{He} \quad (17.3 \text{ MeV}) \]

• PP III

\[ ^1\text{H} + ^7\text{Be} \rightarrow ^8\text{B} + \gamma \quad (0.14 \text{ MeV}) \]
\[ ^8\text{B} \rightarrow ^8\text{Be} + \text{e}^+ + \nu \]
\[ ^8\text{Be} \rightarrow 2 \ ^4\text{He} \quad (18.1 \text{ MeV}) \]
The proton-proton chain

a  Step 1

b  Step 2

c  Step 3
Solar Neutrino Problem

- Neutrino’s produced in sun should escape without interacting with solar material.

- **Solar neutrino problem** = not enough neutrinos have been detected from the sun
  - We only see $\sim \frac{1}{3}$ to $\frac{1}{4}$ of that predicted.

- **Solution?**
  - Early neutrino experiments were set up to detect only electron neutrinos. If neutrinos can change flavors, then the deficit can be explained.

  $\Rightarrow$ Solar astrophysics pushing particle physics, not the other way around.
CNO Cycle

PP Chains are dominant energy source in low mass stars, like our sun.

In higher mass stars, the CNO cycle is more efficient:

\[
\begin{align*}
^{12}\text{C} + ^1\text{H} & \rightarrow ^{13}\text{N} + \gamma \\
^{13}\text{N} & \rightarrow ^{13}\text{C} + \text{e}^+ + \nu \\
^{13}\text{C} + ^1\text{H} & \rightarrow ^{14}\text{N} + \gamma \\
^{14}\text{N} + ^1\text{H} & \rightarrow ^{15}\text{O} + \gamma \\
^{15}\text{O} & \rightarrow ^{15}\text{N} + \text{e}^+ + \nu \\
^{15}\text{N} + ^1\text{H} & \rightarrow ^{12}\text{C} + ^4\text{He}
\end{align*}
\]

Note: Need carbon as a catalyst for this process.
Triple Alpha Process

At even higher temperatures, $^4$He is converted to heavier elements:

$$^4\text{He} + ^4\text{He} \rightarrow ^8\text{B} + \gamma$$

$$^8\text{B} + ^4\text{He} \rightarrow ^{12}\text{C} + \gamma$$
That’s all well and good, but how likely are any of these processes to take place in our sun?

In order for 2 protons to fuse, they have to get within a nuclear radii, $R_{\text{nuc}}$, of each other. At this distance, they experience a large coulombic repulsion.
\( R_{\text{nuc}} \sim 1 \text{ fermi} \sim 10^{-13} \text{ cm} \)

\( E = \frac{e^2}{R_{\text{nuc}}} \sim 2 \times 10^{-6} \text{ ergs} \)

For \( T = 12 \times 10^6 \text{ K} \) (central temp for our sun)

\( E = kT \sim 2 \times 10^{-9} \text{ ergs} \)

Thus, the probability for finding nucleus with enough energy to get over barrier is:

\[ \sim e^{-1000} \sim 10^{-400} \]

There are only \( 2 \times 10^{30} \text{ kg} / 1.67 \times 10^{-27} \text{ kg} \sim 10^{57} \) nuclei in the whole star.

\( \rightarrow \) Unlikely that any one will get over barrier!!
But, the sun shines! So this must work somehow…

**Quantum tunneling:**

Probability that particle of energy $E$ will tunnel through classical barrier is:

$$P_{\text{tunnel}} = \exp\left(- \int_{R_{\text{nuc}}}^{R_E} 2 \left(2m \frac{[V(r) - E]}{\hbar}\right)^{\frac{1}{2}} \, dr\right)$$

Do some algebra:

$$P_{\text{tunnel}} = \exp\left[- \left(\pi \frac{e^2}{\hbar}\right) \left(2m/E\right)^{\frac{1}{2}} \right]$$

To get proper probability, multiply above by energy distribution for gas at temperature $T$ (Maxwell-Boltzmann).
$$\frac{dP}{dE}_{\text{M-B}} = \frac{2\pi}{(\pi kT)^{3/2}} \exp(-E/kT) \ E^{1/2}$$

$$\Rightarrow \frac{dP}{dE} = 2\pi E^{1/2} \exp\left(-\frac{E}{kT} + (\pi \frac{e^2}{\hbar})(2m/E)^{1/2}\right) \frac{(\pi kT)^{3/2}}{(\pi kT)^{3/2}}$$

The peak of this function is called the Gamow peak.

$$E_{\text{peak}} = \left(\pi \frac{e^2}{\hbar}\right) \sqrt{\frac{m}{2}} \ kT \sim 6 \text{ keV for } T \sim 12 \times 10^6 \text{ K}$$

Play some math tricks…

$$P = 4 \left(\frac{\pi}{3}\right)^{1/2} \left[\pi^2 \frac{e^4}{2} \frac{m}{\hbar}\right]^{1/3} \exp\left(-3 \left[\pi^2 \frac{m}{2} \frac{e^4}{2\hbar^2kT}\right]^{1/3}\right)$$

Using approximate numbers, $P \sim 2 \times 10^{-5} \Rightarrow \text{much better!}$
Can work out similar probabilities for CNO cycle.
Lifetime on Main Sequence

\[ t_\odot \sim 10 \text{ billion years} \]

\[ \frac{L_*}{L_\odot} = (\frac{M_*}{M_\odot})^4 \]
\[ \frac{t_*}{t_\odot} = (\frac{M_*}{M_\odot})/(\frac{L_*}{L_\odot}) \]
\[ \frac{t_*}{t_\odot} = (\frac{M_*}{M_\odot})^{-3} \]

→ Massive stars are shorter lived than low mass stars.

→ Can age-date a cluster by looking for Main Sequence turn off region.

After hydrogen burning comes He burning, etc. Everything up to iron is exothermic. Try to burn iron, and it is endothermic.
Abundance relative to silicon = 10^6

Mass numbers:
- H, BB, He-H burning, He burning, C burning, Si burning, nuclear statistical equilibrium, neutron capture.

Elements:
- Li, Be, B, C, O, Ne, Si, S, Ca, Fe, Ni, Ge, Sr, Xe, Ba, Pt, Pb.
Iron-56 is the most abundant and most stable isotope. It does not have Z or N equal to a magic number!

Note the oscillations of abundance depending upon whether Z and N are odd or even.

Abundances peak for Z or N equal to a magic number.

Doubly Magic
Z = 82
N = 126
Pb

10^6
10^4
10^2
10^0

Relative Abundance

56
Fe

50
100
150
200

Mass Number A
A diagram showing a Hertzsprung-Russell (HR) diagram with different stars of various masses (1 M⊙, 2 M⊙, 3 M⊙, 5 M⊙, 9 M⊙) plotted against luminosity and temperature. The diagram includes a line indicating the zero-age main sequence and another line marking the termination of core hydrogen fusion.
Color ratio \( (b_v/b_B) \)

Apparent brightness

Apparent visual magnitude

Surface temperature (K)

Horizontal branch

Turnoff point

Main sequence

Red giants