Light and Telescopes

- Light
- Optics
- Telescope Design
Light as a wave

\[ h = h_o \sin \left(\frac{2\pi}{\lambda}(x - vt)\right) \]

1 oscillation occurs in \( \frac{\lambda}{v} \)

\[ \Rightarrow \text{Frequency of oscillation } \nu = \frac{v}{\lambda} \]

\[ \lambda \nu = v \]

\( \lambda = \text{wavelength} \)

\( h_o = \text{height of wave} \)

\( v = \text{velocity of wave} \)

For light, \( v = c \)

\[ c = 2.99792 \times 10^5 \text{ km/s} \]
Optics

- **Reflection:**

  \[ i = r \]

- **Refraction:**

  \[ n_1 \sin i = n_2 \sin r \]

  Snell’s law

  For air to glass,
  \[ n_1 = 1.0 \quad n_2 \sim 1.5 \]
  \[ \Rightarrow \sin i = 1.5 \sin r \]

  Note: \( n \) is really \( n(\lambda) \)
The Very Large Array
Kitt Peak National Observatory
Palomar Observatory
Radio Telescopes
Simple Telescope Designs

Reflecting telescope: mirrored surface reflects light to a common focus.

Refracting telescope: lenses are used to form image. (old style)
Refracting Telescopes
f-ratios and plate scale

\[ \text{f-ratio} = \frac{f}{D} \]

Plate scale: \[ s = 0.01745 f \]  
Units: cm/degree

For example: WIYN 0.9m at KPNO is at f/7.5

\[ \Rightarrow f/D = 7.5 \]
\[ \Rightarrow f = 7.5 \times 90 \text{ cm} = 675 \text{ cm} = 6.75 \text{ m} \]

Plate scale: \[ s = 0.01745 \times 675 = 11.78 \text{ cm/degree} \]

For 24 \( \mu \text{m} \) pixels: how many arcsec/pix?
\[ 1/s = 0.08489^\circ/\text{cm} \times 24 \times 10^{-4} \text{ cm/pix} \times 3600^\prime/\circ \]
\[ 1/s = 0.73^\prime/\text{pix} \]
Telescope Design Considerations

- Light gathering power
- Resolution
- “Speed” = f-ratio = f/D
- Magnifying power: F/f (focal length of objective/focal length of eye piece)
- Aperture blockage
Reflecting Telescope Designs

(a) Newtonian focus  (b) Prime focus  (c) Cassegrain focus  (d) Coudé focus
Comparison of Palomar and Gemini

Palomar 5 m

Gemini 8 m
Comparison of Palomar and Gemini

Palomar 5 m

Gemini 8 m
Aberations

- Chromatic aberation
- Spherical aberation
- Coma
- Astigmatism

\{ \text{Off axis performance} \}
Chromatic Aberation

(a) The problem

Focal point for blue light

Focal point for red light

(b) The solution

Focal point for both colors
Focus is a line.

The Arecibo radio telescope corrects for spherical aberration by using “line feeds.” [A recent upgrade replaced most of the line feeds with Gregorian optics.]
Parabolic Mirror

On-axis, light focuses at a point.
Off-axis, light is out of focus: coma
astigmatism
(both are second order effects)
Richey-Chrétian Design

- Correct off-axis performance by introducing slight curvature in primary mirror

Most common design for modern telescopes.
Schmidt Telescope

- Schmidt telescopes are good for large fields-of-view.
- The physical size of the correcting lens limits the applicability of this design for large telescopes.
Light Gathering Power

• Light gathering power is proportional to the surface area of the telescope: \( \text{LGP} \propto D^2 \)

• Compare WIYN 0.9m with WIYN 3.5m:
  - Ratio of LGP:
    \[
    \frac{(3.5\text{m})^2}{(0.9\text{m})^2} = \frac{12.25}{0.81} = 15.12
    \]
Resolving Power

• Resolving power comes from the diffraction limit of the telescope:
  – For a circular aperture: $\theta_m = \frac{1.22 \lambda}{D}$
  – For example: WIYN 3.5m
    • $\theta_m = 1.22 \left( \frac{5500 \, \text{Å}}{3.5 \text{m}} \right) = 1.9 \times 10^{-7} \, \text{rad} = 0.04''$
    • But, atmospheric turbulence limits this to 0.5 – 1.0” (or worse)
  • NOTE: For IR observations, adaptive optics can make active corrections for phase shifts and recover most of the resolution (new technique).
Resolving Power continued

• For radio telescopes:
  – Consider radiation at 1420.4058 MHz (neutral H)
    • Green bank 100 m: $\theta = 1.22 \left( \frac{21\text{cm}}{100\text{m}} \right) = 8.7'$
    • Arecibo 305m: $\theta = 1.22 \left( \frac{21\text{cm}}{100\text{m}} \right) = 2.9'$
  – Clearly, need a huge radio telescope to have similar resolution as optical telescopes
    • Impractical to do this as a single dish
    • But, can connect several telescopes together as an interferometer
Interferometers

For constructive interference:
\[ L \sin \theta = m \lambda \]

\[ \sin \theta = m \lambda / L \quad \text{where } \sin \theta \text{ varies as Earth rotates} \]

Fringes are separated by:
\[ \sin \theta_f = \lambda / L \quad \Rightarrow \text{resolution} \]
Resolution of Radio Interferometers

• VLA: maximum separation is 36 km
  $\theta = 1.2''$ at 21 cm

• VLBA: Mauna Kea – St. Croix = 8611 km
  $\theta = 5.03$ mas

• Space VLBI – 21000 km
  $\theta = 2.06$ mas
Theoretical Resolution of Several Telescopes

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Diameter</th>
<th>Wavelength</th>
<th>Resolution?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keck</td>
<td>10m</td>
<td>5000 Å</td>
<td>$5 \times 10^{-8}$ rad = 0.01”</td>
</tr>
<tr>
<td>Keck</td>
<td>10m</td>
<td>1.2 μm</td>
<td>$1.2 \times 10^{-7}$ rad = 0.02”</td>
</tr>
<tr>
<td>HST</td>
<td>2.4m</td>
<td>5000 Å</td>
<td>$2.1 \times 10^{-7}$ rad = 0.04”</td>
</tr>
<tr>
<td>LBT</td>
<td>22.8m</td>
<td>5000 Å</td>
<td>$2.2 \times 10^{-8}$ rad = 4.5 mas</td>
</tr>
<tr>
<td>(2 × 8.4m. Separated by 14.417m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OWL</td>
<td>100m</td>
<td>5000 Å</td>
<td>$5 \times 10^{-9}$ rad = 1 mas</td>
</tr>
<tr>
<td>VLA C config</td>
<td>3.4 km</td>
<td>21 cm</td>
<td>$6.2 \times 10^{-5}$ rad = 12.7”</td>
</tr>
</tbody>
</table>
Why use telescopes at different wavelengths?

- Astronomical objects emit radiation at all wavelengths, but some are “more visible” at certain wavelengths than others.

- Why?
  - Blackbody radiation at different Ts
    - $3 \, \text{K} \rightarrow 3 \, \text{mm}$
    - $300 \, \text{K} \rightarrow 9.9 \, \mu\text{m}$
    - $5800 \, \text{K} \rightarrow 5000 \, \text{Å}$
    - $27000 \, \text{K} \rightarrow 1100 \, \text{Å}$

  - Optical depth
    - Dust obscures regions of star formation in the optical
      - Easier to observe SF regions in radio
    - Dust re-radiates in the IR
      - Easier to study dust features in the IR

- Resolution
  - Better resolution with radio telescopes (operated as interferometers).
Spectral Lines and Redshift

- **Common lines:**
  
  - 21 cm       HI spin flip       radio
  - 2.6 mm   CO J=1→ 0       mm
  - 5007 Å   [O III]      optical
  - 1.78 Å   Fe XXVI Ly α   x-ray

- **Redshift (doppler shift) will move some of these lines out of original wavelength region.**

  For low redshift:
  
  \[
  \lambda = \lambda_o (1 + v/c)
  \]

  \[
  \nu = \frac{c}{\lambda} = \frac{\nu_o}{(1 + v/c)}
  \]

  If \( v > 0 \), light is redshifted;

  for \( v < 0 \), light is blueshifted.

  **Relativistic motion:**

  \[
  \lambda = \lambda_o \left[\frac{(1 + v/c)}{(1 - v/c)}\right]^{1/2}
  \]

  \[
  \nu = \nu_o \left[\frac{(1 - v/c)}{(1 + v/c)}\right]^{1/2}
  \]
Atomic Structure

• Nucleus consists of protons and neutrons, bound by the **strong** force.

• A neutral atom has equal numbers of protons and electrons
  
  \[ Z = \text{number of protons} = \text{atomic number} \]
  
  – An isotope has same atomic number (Z) but differing numbers of neutrons (N)

  \[ ^{Z+N}X = AX \] where \( A = Z+N = \text{atomic mass} \)

  e.g., \(^1\text{H} \) Hydrogen
  
  \(^2\text{H} \) Deuterium
  
  \(^3\text{H} \) Tritium
Bohr Model of the Atom

- Electrons do not move randomly around the nucleus; their orbits are quantized.

$\rightarrow$ quantum mechanics.

Permitted orbits are those with $L = \hbar/2\pi \times n = \hbar n$

where $L =$ angular momentum; $n =$ integral number
Bohr Model continued

• \( mv^2/r = (1/4\pi \varepsilon_o) q^' q/r^2 \)
  
  [Centripetal force = columbic attraction]

• Additional constraint that: \( L = \hbar n = mv r \)

• So, \( r = (1/4\pi \varepsilon_o) q^' q/mv^2 = n\hbar/mv \)

• \( r = n^2 (\varepsilon_o/\pi) (h^2/m_e q^' q) \rightarrow \text{quantized radii} \)
Energy

- \( E = K.E. + P.E. \)
  - \( P.E. = \int \frac{1}{4\pi\varepsilon_o} (q' q \: dr/r^2) \)
  - K.E. = \( \frac{1}{2} m \: v^2 \)

\[ \Rightarrow E = \frac{mv^2}{2} - \left( \frac{1}{4\pi\varepsilon_o} \right) \left( q \: q' /r \right) \]

\[ \Rightarrow E(n) = -\frac{1}{8} \: m_e (q \: q' )^2 /\varepsilon_o^2 \: n^2 \: h^2 \]

Negative, so electrons are bound

For Hydrogen, \( E(n) = - \frac{1}{8} \left( m_e e^4 /\varepsilon_o^2 \: h^2 \right) \left( 1/n^2 \right) = -R'/n^2 \)

where \( R' = 2.18 \times 10^{-18} \) J
Energy Differences

• Since $E = \hbar \nu$, can express energy level differences:

$$\Delta E = E(n_a) + \hbar \nu = E(n_b)$$

$$\hbar \nu = E(n_a) - E(n_b) = -R' \left( \frac{1}{n_a^2} - \frac{1}{n_b^2} \right)$$

Since $\frac{1}{\lambda} = \nu/c \rightarrow$

$$\frac{1}{\lambda_{ab}} = -\left[ \frac{R'}{hc} \right] \left( \frac{1}{n_a^2} - \frac{1}{n_b^2} \right)$$

$$= R \left( \frac{1}{n_b^2} - \frac{1}{n_a^2} \right)$$

where $R = \text{Rydberg's constant}$

$$R = 1.0968 \times 10^7 \text{ m}^{-1}$$
Energy Level Diagram

Lyman series: $n_b = 1$
Balmer series: $n_b = 2$
Paschen series: $n_b = 3$
Brackett series: $n_b = 4$
Pfund series: $n_b = 5$
“Hydrogen-like” atoms

- Singly ionized Helium (He$^+$)
- $E_n = -4 \frac{R}{n^2}$

More generally:

$$\frac{1}{\lambda_{ab}} = R Z^2 \left( \frac{1}{n_b^2} - \frac{1}{n_a^2} \right)$$
Molecules

• Electronic transitions can also occur here:
  – \( \text{H}_2 \rightarrow \text{H}_2^+ + e^- \) (ionization of molecular Hydrogen)

• Vibrational transitions (IR)
  – “bands” – many transitions at similar frequencies

• Rotational transitions (Radio)
  – Require a dipole

\[ I = \text{moment of inertia} \]
\[ E = I\omega^2/2 = \mu r^2 \omega^2 /2 \]

where \( \mu \) is reduced mass:
\[ \mu = mM/(m + M) \]

\[ I\omega = \hbar J \]

quantization

\[ E = \hbar^2 J(J+1)/(2\mu r^2) \]

For CO, \( J = 1 \rightarrow 0 \) and \( J=2 \rightarrow 1 \) are most commonly observed.
(115.2715 GHz) and (230.5424 GHz)
Absorption and Emission

\[ n = 2 \]

\[ n = 3 \]

a Absorption  

b Emission
Excitation

- **Excitation**: atom not in the ground state
- **De-excitation**: drop back towards the ground state
- **2 ways to be excited:**
  - **Radiative** excitation: photon absorbed by atom
    - Photon energy must equal energy difference between 2 levels
    - Absorption lines
  - **Collisional** excitation: transfer of kinetic energy via collision
    - $E = \frac{m(v_i^2 - v_f^2)}{2}$
    - If $E =$ energy of electronic transition, atom can be excited to higher state.
    - Return to ground state via emission of photons

\[ \lambda_0 = \text{Energy difference} \]
Excitation

• 2 ways to be de-excited:
  – **Radiative** de-excitation: spontaneous emission of a photon
    • Most radiative processes are fast: $t \sim 10^{-8}$ s
    • Some transitions are not favored and thus take place more slowly
      – Quadrupolar transitions result in “forbidden” lines
      – Collisional excitation more rapid in most terrestrial situations, so only see these lines in diffuse, astrophysical situations.
      – Examples: [OIII] 4959, 5007, 4363
        [NII] 6548, 6584
        [SII] 6717, 6731
  – **Collisional** de-excitation: inverse of collisional excitation

$\lambda_0 = \text{Energy difference}$
Ionization

• Ionization: \( X + \text{energy} \rightarrow X^+ + e^- \)

• Nomenclature:
  - Neutral atom: \( H \) or \( H \ I \)
  - \( He \) or \( He \ I \)
  - Singly ionized: \( H^+ \) or \( H \ II \)
  - \( He^+ \) or \( He \ II \)
  - Doubly ionized: \( O^{++} \) or \( O \ III \)

• Energy required to ionize atom depends on ionization state, the electron to be liberated, and excitation levels.

• For Hydrogen: electron from ground state can be removed if we supply 13.6 eV (ionization potential).
  - \( \text{I.P.}(n) = E(\infty) - E(n) = 13.6/n^2 \text{ eV} \) for Hydrogen
Recombination

• Reverse of ionization:
  \[ X^+ + e^- \rightarrow X + h\nu \]

• Results in emission lines
Line Intensities

• For an equilibrium state, the excitation ratio is:

\[
\frac{N_B}{N_A} = \frac{g_B}{g_A} \exp\left(\frac{(E_A-E_B)}{kT}\right) \quad \text{Boltzman eq.}
\]

\(N\) = number density
\(g\) = multiplicity of level
\(E\) = energy
\(T\) = temperature

• For ionization: \(X \rightarrow X^+ + e^-\):

\[
\frac{N_+}{N_0} = \left(\frac{A}{N_e}\right) (kT)^{3/2} \exp\left(-\frac{\chi_o}{kT}\right) \quad \text{Saha eq.}
\]

\(N\) = number density
\(A\) = atomic constants and probabilities
\(T\) = temperature
\(\chi_o\) = ionization potential
Kirchhoff’s Rules

- Hot blackbody
- Cloud of cooler gas
- Prism
- Absorption line spectrum
- Continuous spectrum
- Emission line spectrum
Kirchoff’s Rules

(1) A hot opaque solid, liquid, or highly compressed gas emits a continuous spectrum.

(2) A hot, transparent gas produces a spectrum of emission lines. The number and position of these lines depends on which elements are present in the gas.

(3) If light within a continuous spectrum passes through a transparent gas at a lower temperature, the cooler gas causes the appearance of absorption lines. Their position in the spectrum, their strength, and their number depend on the elements in the cooler gas.
Spectrograph Design

- Recall Snell’s law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

- Prism separates light into its component parts
Diffraction Gratings

- Constructive interference at \( \sin \theta = n \frac{\lambda}{d} \)

\( \lambda = \) wavelength  
\( n = \) order  
\( d = \) groove spacing
Building a Spectrograph