Basic 3-d graph:

\[ P(x,y,z) \text{ or } P(r,\theta,\phi) \]

\[ z = r \cos \theta \]
\[ y = r \sin \theta \sin \phi \]
\[ x = r \sin \theta \cos \phi \]
Geophysics Application: Earth based coordinate systems

Longitude = $\phi$
Latitude = $90^\circ - \theta$
Height = $r - R_E$

Define zero points:
- Greenwich, England
- Equator
Astronomy Application: Celestial Sphere

Right Ascension (RA) = φ
Declination (δ) = 90° − θ

Define zero points:
Vernal Equinox (= Point of Aries)
Equator

Units for RA: 24 hr = 360°
1 hr = 15°
1 min = 15’
1 s = 15”

NOTE: 1 min = 15’ cos(δ)
View from Bloomington:

\[ \chi = \text{altitude of NCP} \]
\[ \text{Z. A.} = \text{zenith angle} \]

For star: zenith angle + altitude = 90° (right triangle)
For NCP: angle between NCP and C.E. = 90°
North Pole: Latitude = 90° N

What is value of $\chi$?
$\chi = 90^\circ$

What is value of $\delta$ at zenith?
$\delta$ at zenith = $90^\circ$

What decl. range is circumpolar?
$\delta$(NCP) - $\chi < \delta < \delta$(NCP): $0^\circ < \delta < 90^\circ$
What is value of $\chi$?
What is value of $\delta$ at zenith?
What decl. range is circumpolar?

Equator: Latitude = 0° N

$\chi = 0°$

$\delta$ at zenith = 0°

$\delta$(NCP) – $\chi < \delta < \delta$(NCP): 90° and -90°
Bloomington: Longitude = 86° 31.5’ W, Latitude = 39° 09.9’ N

What is value of $\chi$?  $\chi = 39° 09.9’$
What is value of $\delta$ at zenith?  $\delta$ at zenith = 39° 09.9’
What decl. range is circumpolar?  $\delta(NCP) - \chi < \delta < \delta(NCP): 50.835° < \delta < 90°$
<table>
<thead>
<tr>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Height</th>
<th>Alt. of NCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Institute d’ Astrophysique</td>
<td>2° 20.2’</td>
<td>48° 50.2’</td>
<td>67m</td>
<td>48° 50.2’</td>
</tr>
<tr>
<td>WIYN Observatory</td>
<td>-111° 35.9’</td>
<td>31° 57.6’</td>
<td>2084m</td>
<td>31° 57.6’</td>
</tr>
<tr>
<td>Keck Observatory</td>
<td>-155° 28.7’</td>
<td>19° 49.7’</td>
<td>4160m</td>
<td>19° 49.7’</td>
</tr>
<tr>
<td>Byurakan Ast. Obs.</td>
<td>44° 17.5’</td>
<td>40° 20.1’</td>
<td>1500m</td>
<td>40° 20.1’</td>
</tr>
<tr>
<td>CTIO</td>
<td>-70° 48.9’</td>
<td>-30° 09.9’</td>
<td>2215m</td>
<td>-30° 09.9’</td>
</tr>
</tbody>
</table>
Solar Motion:

The apparent motion of the sun defines the **ecliptic**. The sun does not have the same RA (or Dec) all year. In 1 year, it will go around the ecliptic once.

Maximum declination: 23.5°
Minimum declination: -23.5°
At the vernal and autumnal equinox, the sun is at $\text{declination} = 0^\circ$. Therefore, 12 hour day and 12 hour night.
Seasons

Earth is inclined with respect to the ecliptic (plane of orbit):

Seasons are caused by tilt of the earth, not by the eccentricity of orbit (e = 0.017)
video from NASA’s Meteosat that shows the Earth at the same time of day for a year.

http://www.youtube.com/watch?v=MWwmP4obdwA&feature=player_embedded
Heating:

Incident radiation spread over larger area in winter:

\[ \text{Area} = \frac{A}{\sin(\text{elevation})} \]

(However, large bodies of water (oceans) are good thermal reservoirs and respond slowly to solar heating.)
Seasons

• At the equinoxes, the sun transits at $90^\circ$ (zenith) at the equator and $0^\circ$ (horizon) at the poles.
• At the solstices, the sun is $\pm 23.5^\circ$, so it transits directly overhead for observers at which latitudes?

<table>
<thead>
<tr>
<th>Season</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>$23.5^\circ$ = Tropic of Cancer</td>
</tr>
<tr>
<td>Winter</td>
<td>$-23.5^\circ$ = Tropic of Capricorn</td>
</tr>
</tbody>
</table>

• Where is the sun circumpolar?

<table>
<thead>
<tr>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ - 23.5^\circ = 66.5^\circ$ = Artic Circle</td>
</tr>
<tr>
<td>$-90^\circ + 23.5^\circ = -66.5^\circ$ = Antarctic Circle</td>
</tr>
</tbody>
</table>

• At the poles, 6 month “days” begin/end at the equinox.
Back to Celestial Motion:

Objects “move” from East to West.

Where in the US should you go to see sunrise over the ocean? Where in the US should you go to see sunset over the ocean?
Celestial Motion:

**Transit** = object crosses the meridian

**Hour Angle** = time difference before/since transit

**Hour Angle** = local sidereal time – right ascension
What is the time?

Local solar time = hour angle of the sun + 12 hours

Universal time (UT) = Local solar time in Greenwich, England

Time Zones

Eastern Standard time
= UT – 5 hours

Eastern Daylight time
= UT – 4 hours

Central Standard time
= UT – 6 hours
What is sidereal time?

Local sidereal time (LST) = hour angle of the vernal equinox

Because the earth is in orbit around the sun, it takes a little longer to get back to “same point.” Thus, the solar day is slightly longer than the sidereal day.

1 year = 365.25 days (solar)
   = 366.25 days (sidereal)

Therefore,
1 sidereal day = 365.25/366.25 solar day

1 sidereal day = 0.99727 solar days
1 sidereal day = 23h 56m 4.1s (solar)
How to calculate where an object will be in the sky at any given time?
Transformation equations:

\[
\begin{align*}
\cos a \sin A &= \cos \delta \sin h \\
\cos a \cos A &= -\sin \delta \cos \phi + \cos \delta \cos h \sin \phi \\
\sin a &= \sin \delta \sin \phi + \cos \delta \cos h \cos \phi \\
\cos \delta \cos h &= \sin a \cos \phi + \cos a \cos A \sin \phi \\
\sin \delta &= \sin a \sin \phi - \cos a \cos A \cos \phi
\end{align*}
\]

\[h = \text{LST} - \alpha = \text{Hour Angle}\]

A = azimuth, toward west from south

a = altitude

\(\phi = \text{observers latitude}\)

\(\alpha = \text{Right Ascension}\)

\(\delta = \text{Declination}\)
Rise and Set are special: \( a = 0 \)

Therefore, \( \cos a = 1 \) and \( \sin a = 0 \)

Above simplify to:
\[
\begin{align*}
\sin A &= \cos \delta \sin h \\
\cos A &= -\sin \delta \cos \phi + \cos \delta \cos h \sin \phi \\
0 &= \sin \delta \sin \phi + \cos \delta \cos h \cos \phi \\
\cos \delta \cos h &= \cos A \sin \phi \\
\sin \delta &= -\cos A \cos \phi
\end{align*}
\]

Use eq. 3:
So, \( \cos h = -\tan \delta \tan \phi \)

Or, \( h = \pm \cos^{-1}(-\tan \delta \tan \phi) \)
Specific application: calculate rise and set times

(1) Determine LST at midnight
(2) Determine transit time (EDT)
(3) Determine the hour angle of rise and set
(4) Compute the rise and set time (EDT)

Bloomington: Longitude = 86° 31.5’ W, Latitude = 39° 09.9’ N

Longitude in hours = -86.525°/15°/hr = -5.768333 hr

EDT = UT – 4 hours

→ Midnight in Bloomington is 4h UT.
LST at midnight?

\[ \text{LST} = \text{GMST} + \text{longitude} \]

(1) **Look up GMST** at 0h UT (tabulated in the Astronomical Almanac)
   For night of Aug 27 → 28: GMST is 22:26:27.74 at 0h UT

(2) **Calculate GMST** at midnight in Bloomington:
   Eastern Daylight Time (EDT) is 4 hours West
   \[ \text{GMST} = \text{GMST}(0h) + 4 \text{ hours (solar)} / 0.99727 \text{ (solar/sidereal)} \]
   \[ \text{GMST} = 22.44103889 + 4.010949893 = 26.45198878 \text{ (-24)} \]

(3) **Calculate LST at midnight**:
   \[ \text{LST} = \text{GMST} + \text{longitude} \]
   \[ \text{LST} = 26.45198878 - 5.768333 \]
   \[ = 20.68365878 \]
   \[ = 20:41:01.2 \]
Transit time? Vega: RA = 18 37 21.7 Dec = 38 47 45

(1) Calculate hour angle at midnight:
   = (time difference before/since transit in sidereal hours)

   \[ HA = LST - RA \]
   = 20:41:01.2 – 18:37:21.7
   = 20.68365878 – 18.622694444
   = 2.06096434 \rightarrow Vega transits before midnight

(2) Convert time difference to solar hours:
   \[ \Delta T = 2.06096434 \text{ (sidereal)} \times 0.99727 \text{ (solar/sidereal)} \]
   = 2.05533791 \text{ (solar)} before midnight

(3) Calculate time in EDT:
   \[ \text{Transit} = 24 - \Delta T \]
   what would be the equation if transit after midnight?
Rise/Set Hour Angle?  
Vega: RA = 18 37 21.7  Dec = 38 47 45

Calculate hour angle for altitude = 0:

\[ h = \pm \cos^{-1}(- \tan \delta \tan \phi) \]

\[ h = \pm \cos^{-1}(- \tan (39.165) \tan (38.79583333)) \]
\[ = \pm \cos^{-1}(-0.654828271) \]
\[ = \pm 130.9066299 \text{o} / 15 \text{o/hr} \]
\[ = \pm 8.72710866 \]
\[ = \pm 8 43 37 \]
Rise/Set Times? Vega: RA = 18 37 21.7 Dec = 38 47 45

Rise/Set = Transit + h : But h must be in solar hours

Rise: 21:56:41 EDT – h
= 21.94466209 – 8.72710866 sidereal × 0.99727 (solar/sidereal)
= 21.94466209 – 8.70328365
= 13.24137844
= 13:14:29 EDT

Set: 21:56:41 EDT + h
= 21.94466209 + 8.72710866 sidereal × 0.99727 (solar/sidereal)
= 21.94466209 + 8.70328365
= 30.64794574 – 24
= 6.647945743
= 06:38:53 EDT
## Vocabulary Review

<table>
<thead>
<tr>
<th>HA</th>
<th>RA</th>
<th>transit</th>
</tr>
</thead>
<tbody>
<tr>
<td>LST</td>
<td>Dec</td>
<td>ecliptic</td>
</tr>
<tr>
<td>EDT</td>
<td>altitude</td>
<td>equator</td>
</tr>
<tr>
<td>EST</td>
<td>zenith</td>
<td>vernal equinox</td>
</tr>
<tr>
<td>GMST</td>
<td>horizon</td>
<td>first point of aries</td>
</tr>
<tr>
<td>UT</td>
<td>meridian</td>
<td></td>
</tr>
</tbody>
</table>