Endogenous Bank Networks and Contagion

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Motivation

Bank interconnectedness affects financial stability: The 2007 - 08 Crisis:
Lehman Brothers $\Rightarrow$ AIG $\Rightarrow$ European Sovereign Debt Crisis

Common asset holdings create another systemic important network layer: the 2007 - 08 Crisis; the Panic of 1873

The security holdings of commercial banks in Mexico, since 2005. 
This Paper

How financial institutions form connections in the first place?
- Micro-foundation of the banking sector
- Systemic externality

Model bank network formation as a portfolio selection problem
- Two network layers: interbank debts & common asset holdings → Complexity of interconnectedness
- How network formation depend on fundamentals: bank capital, risk aversion, bank debt risk

How direct connections and common asset holding affect financial stability?
- Extend Eisenberg and Noe (2001) → two network layers
Main Results

The topologies of the two network layers interact with each other.

Both layers of interconnectedness contribute significantly to systemic fragility in terms of higher probability of simultaneous multiple bank defaults.

- Banks’ risk tolerances increase $\Rightarrow$ higher integration, diversification, and network density
- Investments toward the private sector do not monotonically depend on risk aversions
- Higher asset correlations $\Rightarrow$ systemic fragility
Related Literature

- **Network formation**

Model: Set-up

- One period: $t = 0 \rightarrow t = 1$
- A finite set $B = \{1, \ldots, N\}$ of banks
- Exogenous bank capital: $e = (e_1, \ldots, e_b, \ldots, e_N)^\top$, heterogeneous
- A finite set $T = \{1, \ldots, K\}$ of risky non-financial firm assets with given expected return $\mu_T$ and variance-covariance $V_T$
- A bank can issue debt that can be purchased by other banks
- Possible to default $\Rightarrow$ bank debts are risky
- Bank default: proportionality, limited liability
- Expected asset returns are strictly positive
Bank Activities

- Bank can sell its own bank debt (borrowing), purchase \( N - 1 \) bank debts (lending), and invest in \( K \) firm assets

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lending to Banks ( \sum_{i \neq b} s_{ib} )</td>
<td>Borrowing from Banks ( q_b )</td>
</tr>
<tr>
<td>Investment in Firms ( \sum_{i \in T} s_{ib} )</td>
<td>Capital ( e_b )</td>
</tr>
</tbody>
</table>

- Bank \( b \)'s portfolio weights: \( \forall b \in B \)

\[
  w_{ib} := \begin{cases} 
  \frac{s_{ib}}{e_b} & \text{for } i \neq b, \ i \in B \cup T \\
  -\frac{q_b}{e_b} & \text{for } i = b, \ i \in B \cup T 
  \end{cases}
\]

- Two network layers:

\[
  W = [w_1, \ldots, w_b, \ldots, w_N] = \begin{pmatrix} W_B \\ W_T \end{pmatrix}
\]
Banks’ Portfolio Selection Problem

Each bank $b$ maximizes its expected equity value according to mean-variance rule: $\forall b \in B$

$$\min_{w_b} w_b^\top Vw_b$$

s.t. $\mu(W)^\top w_b = \mu^p_b$ \quad ($\lambda_b$)

$$1^\top w_b = 1$$

$$w_{ib} \geq 0, i \neq b$$

$$w_{bb} \leq 0$$

- $\lambda_b$: risk tolerance
- Assume the returns of bank debts and firm assets are jointly distributed with mean $\mu(W) = \begin{pmatrix} \mu_B(W) \\ \mu_T \end{pmatrix}$, and exogenous, positive definite variance-covariance $V \Rightarrow$ externality
Efficient Portfolios: the Critical Line Method

Based on Markowitz (1956) and Sharpe (1970, 1995), bank’s problem is equivalent to: \( \forall b \in B \)

\[
\max_{w_b} \quad U_b(W) = \lambda_b \mu(W)^\top w_b - w_b^\top V w_b \\
\text{s.t.} \quad 1^\top w_b = 1 \\
\quad lb_b \leq w_b \leq ub_b
\]  

- Vary \( \lambda_b \) from 0 to infinity to trace out the efficient frontier for every bank
- \( ub_b \): capital requirement
- Bank debt returns adjust to reach equilibrium
Interaction between $w_{Bb}$ and $w_{Tb}$

**Theorem** (Interaction between $w_{Bb}$ and $w_{Tb}$)

Assume $\sigma_{ib} = 0$ for any $i \in T$ and $b \in B$. In banks’ optimal portfolio selection problem (2) without short-selling, the allocation of portfolio weights within the firm assets, $w_{Tb}$, is related to the allocation of portfolio weights within the interbank debts, $w_{Bb}$. Furthermore, when all banks have investment opportunities into an unique common mutual fund $t$, a risk-averse bank’s portfolio weight on bank $j$’s debt is positively related to $\frac{\mu_j}{\mu_t}$, for $j \in B$. 
Banking Sector Equilibrium

**Definition (Equilibrium Bank Network)**

Given the expected returns on firm assets $\mu_T$, the variance-covariance matrix $V$ of $N$ bank debt returns and $K$ firm asset returns, bank capitals $\{e_b\}_{\forall b \in B}$, and banks’ risk tolerances $\{\lambda_b\}_{\forall b \in B}$, the network of inter-bank debts and banks’ investments in firms are determined by the portfolio matrix $W^*$ and the expected returns of bank debts $\mu_B(W^*)$, where $W^*$ and $\mu_B(W^*)$ solve the mean-variance optimization problem (2) for all banks with associated risk tolerances. Meanwhile, portfolio weights on interbank debts $W^*_B$ satisfy the market clearing conditions for all bank debts:

$$\sum_{j \in B} w_{bj} e_j = 0, \quad \forall b \in B. \quad (3)$$
Example: Multiple Equilibria

2 banks and 1 firm; unbinding leverage ratio; $e = [100, 100]^\top$; $\mu_t = 0.13$; $V$ given; $\lambda = [10, 1]^\top$; adjust the expected bank debt returns in $[0.00001, 0.2]$

- Continuum of equilibria: $\mu_1 = \mu_2 \in [11.75\%, 12.99\%]$
- Another equilibrium at $\mu_1 = 10.93\%$, $\mu_2 = 10.67\%$; bank 1 borrows 57.01% of $e_1$ from bank 2
Identification of Equilibrium Network

Adopt one factor model to adjust $\mu_B$

- Assume returns of bank debts are correlated with a common factor $f$ with given $\mu_f$ and $\sigma_f^2$: $r_b = \alpha_b + \beta_b r_f + \epsilon_b$, $b \in B$
- First and second moments of bank debts are determined by the factor:

  \[ \mu_b = \alpha_b + \beta_b \mu_f, \quad b \in B \]  

  \[ \sigma_b^2 = \beta_b^2 \sigma_f^2 + \sigma_{\epsilon_b}^2, \quad b \in B \]  

  \[ \sigma_{ij} = \beta_i \beta_j \sigma_f^2, \quad i \neq j, \quad i, j \in B \]  

- Equilibrium may not exist, but given it exists, will be unique
- In computational examples, pick combinations of parameter values such that equilibrium exists
Comparative Statics: Incomplete Network

- $e = [10, 10, 10, 10]^\top$; unbinding upper bound for leverage
- Factor model: $\mu_1 = 0.0011 + 0.6\mu_f$, $\mu_2 = 0.0011 + 0.6\mu_f$,
  $\mu_3 = 0.0013 + 0.601\mu_f$, $\mu_4 = 0.0012 + 0.601\mu_f$;
  $\mu_f \in [0.01\%, 50\%]$; $\sigma_f = 18\%$; $\sigma_\epsilon^2 = [0.001, 0.005, 0.0051, 0.008]^\top$
- Firm asset: $\mu_T = 2.5\%$; $\sigma_T = 7\%$
- Risk tolerances: $\lambda_1 \in [1, 4.5]$; $\lambda_2 \in [1.5, 5]$; $\lambda_3 \in [2.5, 500.5]$; $\lambda_4 \in [2.11, 45]$
Comparative Statics: Incomplete Network (continue)
Comparative Statics: Star Network

- \( e = [50, 10, 10, 10]^\top \); unbinding upper bound for leverage
- Factor model: \( \mu_1 = 0.005 + 1.7 \mu_f, \mu_2 = 0.001 + 0.711 \mu_f, \)
  \( \mu_3 = 0.001 + 0.7 \mu_f, \mu_4 = 0.001 + 0.7 \mu_f; \)
  \( \mu_f \in [0.01\%, 50\%]; \sigma_f = 18\%; \sigma_e^2 = [0.03, 0.06, 0.055, 0.05]^\top \)
- Firm asset: \( \mu_T = 2.5\%; \sigma_T = 7\% \)
- Risk tolerances: \( \lambda_1 \in [1, 14] \); increase risk tolerances of banks 2, 3, and 4 from 0.01 to 0.4

![Diagram showing the network transition with nodes 1, 2, 3, 4.]
Comparative Statics: Star Network (continue)
Comparative Statics: Robustness Check

Graphs showing comparative statics of endogenous network formation.
Obligation and Realized Payment

- \( I = (I_1, \ldots, I_N)^\top \): the vector of total dollar amounts invested in firm assets by each bank
- \( L = (L_1, \ldots, L_N)^\top \): the vector of total nominal contractual obligation of each bank to repay all its creditors
- Interbank debt network: an \( N \times N \) matrix \( M \) computed from \( W_B \), with element \( m_{ij} \)
- Firm asset holdings: an \( N \times K \) matrix \( A \) computed from \( W_T \), with element \( a_{ij} \)
- \( p = (p_1, \ldots, p_N)^\top \): banks’ realized payment towards its debt obligations to all other banks
Bank Payment: Assumptions

Extended from Eisenberg and Noe (2001):

- **Proportionality**: if bank $b$ defaults, all its creditors are paid in proportion to the size of their nominal loans to bank $b$, $m_{bj}$

- **Limited Liability**: $\forall b \in B,$
  \[
  p_b \leq \sum_k a_{bk} r_k I_b + \sum_j m_{jb} p_j + e_b
  \]

- **Absolute Priority**: $\forall b \in B,$
  \[
  p_b = \begin{cases} 
  L_b & \text{bank } b \text{ does not default}, \\
  \sum_k a_{bk} r_k I_b + \sum_j m_{jb} p_j + e_b & \text{bank } b \text{ defaults}
  \end{cases}
  \]
Payment Vector and Clearing Equilibrium

\[ pb = \min \{ \sum_k a_{bk} r_{T_k} I_b + \sum_j m_{jb} p_j + e_b, L_b \}, \quad \forall b \in B \]

Clearing equilibrium: \( p^*(r_T, I, L, M, A) = \Phi(\text{Ar}_T \circ I + M^\top p^* + e, L) := \min \{ \text{Ar}_T \circ I + M^\top p^* + e, L \} \)

Extend theorems in Eisenberg and Noe (2001), a clearing equilibrium payment vector exists. Furthermore, it is unique for all \( A, I, M, \) and \( L \) provided that the asset rates of return are strictly positive.

Fictitious default algorithm from Eisenberg and Noe (2001) to compute \( p^* \)
Example: Circle Network vs. More-Connected Network

Assume all banks completely diversify in investments in “local” firms. Same portfolio weights on firm assets; same returns, and total obligations of each bank in both networks.
Perfectly Diversified Firm Investments: Uncorrelated vs. Correlated Assets

- 5 firm assets outside banking sector; 100,000 simulations
- Uncorrelated firm asset returns: Draw $r_{Ti} \sim N(1.1, 0.3)$ independently
- Correlated firm asset returns: multivariate $N(1.1, 0.3)$ with correlation between any two asset returns equals 0.8

Table: Probability of Default: Uncorrelated vs. Correlated Firm Assets

<table>
<thead>
<tr>
<th></th>
<th>Default Bank</th>
<th>Uncorrelated</th>
<th>Corr = 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle Network</td>
<td>Bank 2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0.002%</td>
<td>0.78%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>25.43%</td>
<td>35.55%</td>
</tr>
<tr>
<td>More Connected Network</td>
<td>Bank 2</td>
<td>2.13%</td>
<td>15.23%</td>
</tr>
<tr>
<td></td>
<td>Bank 3</td>
<td>1.28%</td>
<td>10.63%</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0.005%</td>
<td>0.78%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>25.47%</td>
<td>35.55%</td>
</tr>
</tbody>
</table>
Widening of Financial Crisis: Uncorrelated vs. Correlated Assets

More connectedness and higher correlation among assets cause the banking system becoming less stable.

Table: Contagion Effect: Uncorrelated vs. Correlated Firm Assets

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Default Event</th>
<th>Uncorrelated</th>
<th>Corr = 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle Network</td>
<td>Bank 4,5</td>
<td>0.001%</td>
<td>0.782%</td>
</tr>
<tr>
<td></td>
<td>Default</td>
<td></td>
<td></td>
</tr>
<tr>
<td>More Connected Network</td>
<td>Bank 2,3,5</td>
<td>0.018%</td>
<td>0.879%</td>
</tr>
<tr>
<td></td>
<td>Default</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bank 2,3,4,5</td>
<td>0</td>
<td>0.045%</td>
</tr>
<tr>
<td></td>
<td>Default</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Deepening of Financial Crisis: Uncorrelated Assets
Deepening of Financial Crisis: Correlated Assets (Circle Network)
Deepening of Financial Crisis: Correlated Assets (More-Connected Network)

Empirical Distribution of Loss/Obligation Given Default: Bank 2 (More-Connected Network)

Empirical Distribution of Loss/Obligation Given Default: Bank 3 (More-Connected Network)

Empirical Distribution of Loss/Obligation Given Default: Bank 4 (More-Connected Network)

Empirical Distribution of Loss/Obligation Given Default: Bank 5 (More-Connected Network)
Common Asset Holding: Low-Risk Firm Assets

- Common asset holdings: 6 firm assets; each bank invest 70% in the 6th firm and 30% in its local firm.
- Correlated firm asset returns: multivariate $N(1.1, 0.1)$ with correlation between asset returns equals 0.8.

**Table:** Probability of Default: Diversified Investments vs. Common Asset Holding (Low-Risk Firm Assets)

<table>
<thead>
<tr>
<th></th>
<th>Default Bank</th>
<th>Diversified Investments</th>
<th>Common Asset Holding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Circle Network</strong></td>
<td>Bank 2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0</td>
<td>0.006%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>20.14%</td>
<td>20.14%</td>
</tr>
<tr>
<td><strong>More Connected Network</strong></td>
<td>Bank 2</td>
<td>0.46%</td>
<td>2.05%</td>
</tr>
<tr>
<td></td>
<td>Bank 3</td>
<td>0.23%</td>
<td>0.25%</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0</td>
<td>0.006%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>20.14%</td>
<td>20.14%</td>
</tr>
</tbody>
</table>
Widening of Financial Crisis: Low-Risk Firm Assets

Common asset holdings significantly contribute to financial fragility

Table: Contagion Effect: Diversified Investments vs. Common Asset Holding (Low-Risk Firm Assets)

<table>
<thead>
<tr>
<th></th>
<th>Diversified Investment</th>
<th>Common Asset Holding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Circle Network</strong></td>
<td>Bank 4,5 Default</td>
<td>0</td>
</tr>
<tr>
<td><strong>More Connected Network</strong></td>
<td>Bank 2,3,5 Default</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 2,3,4,5 Default</td>
<td>0</td>
</tr>
</tbody>
</table>
Common Asset Holding: High-Risk Firm Assets

Increase the standard deviation of firm asset returns to 0.4; others remain the same as the low risk firm asset case

Table: Probability of Default: Diversified Investments vs. Common Asset Holding (High-Risk Firm Assets)

<table>
<thead>
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<th>Default Bank</th>
<th>Diversified Investment</th>
<th>Common Asset Holding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle Network</td>
<td>Bank 2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0.84%</td>
<td>0.88%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>32.84%</td>
<td>32.84%</td>
</tr>
<tr>
<td>More Connected</td>
<td>Bank 2</td>
<td>8.96%</td>
<td>13.89%</td>
</tr>
<tr>
<td>Network</td>
<td>Bank 3</td>
<td>5.54%</td>
<td>5.58%</td>
</tr>
<tr>
<td></td>
<td>Bank 4</td>
<td>0.84%</td>
<td>0.88%</td>
</tr>
<tr>
<td></td>
<td>Bank 5</td>
<td>32.84%</td>
<td>32.84%</td>
</tr>
</tbody>
</table>
Widening of Financial Crisis: High-Risk Firm Assets

More connectedness and common asset holdings both contribute to financial fragility.

Table: Contagion Effect: Diversified Investments vs. Common Asset Holding (High-Risk Firm Assets)

<table>
<thead>
<tr>
<th></th>
<th>Diversified Investment</th>
<th>Common Asset Holding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle Network</td>
<td>Bank 4,5 Default</td>
<td>0.084%</td>
</tr>
<tr>
<td>More Connected Network</td>
<td>Bank 2,3,5 Default</td>
<td>0.305%</td>
</tr>
<tr>
<td></td>
<td>Bank 2,3,4,5 Default</td>
<td>0.005%</td>
</tr>
</tbody>
</table>
Prospective Policy Implications: A Simple Example

![Graph showing the relationship between Bank 1 Risk Tolerance and Equilibrium Expected Asset Returns](image)

- **Prospective Policy Implications and Future Research**

- **Endogenous Bank Networks and Contagion**

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Future Works To Do

- Empirical analysis of multi-layered bank networks (work in progress)
  - Bank networks during the National Banking Era
  - Data: bank-level data from *Reports of the Several Banks and Savings Institutions of Pennsylvania, 1876 - 1894*
  - Network visualizations and centrality measures
  - Test the main results from the theoretical model

- Financial contagion model with multi-layer bank network: interbank debts, Repo, bonds, securities, etc.

- Regulatory policy and central bank bail-out

- Asset prices and fire-sales: a dynamic model
Wrap Up

- Network structures of interbank debts and common asset holdings interact with each other
- Banks tend to take higher leverage and form more links as they become more risk tolerant, and yet, investments in firms do not monotonically related to risk aversion
- When network is incomplete, the banking sector becomes less stable when integration and network density increase
- Higher correlations among firm asset returns not only increase default probabilities of individual banks, but also increase probability of contagion in terms of widening and deepening of financial crisis
- Common asset holding increases the default probability of individual banks by a relatively small amount. However, it increases probability of systemic failure significantly