An experimental test of the Newtonian inverse square law below 50 µm is in progress at Indiana University. The experiment uses 1 kHz planar oscillators as test masses with a metal membrane stretched between them to suppress backgrounds, a technique showing potential for probing exceptionally small distances and operation at the limit of instrumental thermal noise. Previous data from this experiment, which set new constraints on short-range phenomena motivated by string models, are being re-analyzed for possible signals of Lorentz violation in the Standard Model Extension.

1. Introduction

Laboratory-scale tests of the Newtonian inverse square law have received a revival of attention in the last decade. This is especially true of experiments in the regime below a few hundred microns, where gravity is poorly measured. Predictions of new effects in this range have arisen in several contexts, including attempts to describe gravity and the other fundamental interactions in the same theoretical framework.

Deviations from the inverse square law are commonly parameterized by a Yukawa interaction, the potential for which is given by:

\[ V(r) = \frac{Gm_1m_2}{r} \alpha e^{-r/\lambda}. \]  

(1)

Here, \( G \) is the Newtonian Gravitational Constant, \( m_1 \) and \( m_2 \) are test masses, \( r \) is their separation, \( \alpha \) is the strength of a new interaction relative to gravity and \( \lambda \) is the range. Fig. 1, adapted from Refs. 1 and 2, shows the limits in the range between 1 µm and 1 cm. The published limits on new forces above 10 µm are defined by classical gravity measurements (curves...
labeled “Irvine” and “Washington”), including the E"ot-Wash torsion pendulum experiment. Below a few microns the best limits are derived primarily from Casimir force measurements. The limits in the intermediate range are the result of two high-frequency (kilohertz-range) experiments (“Stanford”, “Colorado”). The latter experiment, published by one of the authors in 2003, has been re-located to Indiana University and is described below. The limits allow for new forces in nature several million times stronger than gravity at ranges resolvable by the unaided eye.

Figure 1. Parameter space for Yukawa-type forces in which the strength relative to gravity ($\alpha$) is plotted versus the range ($\lambda$). The experimentally excluded region is above and to the right of the bold, solid curves. Fine and dashed lines are theoretical predictions.

In addition to this very general motivation for new experiments, a number of specific theoretical predictions of new physics has arisen in the short-range regime, represented by the fine and dashed lines in Fig. 1. Most notable is the line indicating Yukawa corrections to the inverse square law which arise from compact extra dimensions. Corrections are also predicted from massive scalars in string theories, such as moduli and dilatons. The other predictions shown are motivated by the cosmological constant problem and the strong CP problem of QCD.
2. The Indiana Short–Range Experiment

For an experiment to be sensitive to gravity or possible new forces in nature at short range, the test masses must in general be scaled to that range, so as not to be overwhelmed by Newtonian forces at larger scales. The highly sensitive, linear response of thin fibers under tension has made the torsion pendulum the instrument of choice for laboratory gravity measurements. However, it tends to be limited by backgrounds including vibrations, tilts and mechanical relaxation—which would be expected to be exacerbated at small test mass separations—and it is to the credit of the Eötvös group for having made that technology successful below 100 µm. The Indiana experiment takes a different approach, showing potential for greater sensitivity at shorter ranges.

Fig. 2, adapted from Ref. 7, shows the central components of the experiment. The detector is driven by the source mass on resonance near 1 kHz, placing a heavy burden on vibration isolation. The 1 kHz operation is chosen since at this frequency it is possible to construct a simple, passive vibration isolation system. The entire apparatus is enclosed in a vacuum chamber and operated at 10^{-7} torr to further reduce the acoustic coupling. Detector oscillations are read out with a capacitive transducer and lock-in amplifier. This design has proven effective for suppressing all background forces to the extent that the only effect observed is thermal noise due to dissipation in the detector mass. At the time of publication, this experiment set the strongest limits on new forces of nature between 10 and 100 µm, and improved on previous searches by up to a factor of 1000.

The 100 µm test mass separation in the 2003 experiment was determined largely by the shield thickness (60 µm) and not the amplitude of the test mass oscillations; test mass separations as small as ~30 µm could be achieved with a thin enough shield. Since the relocation of the experiment to Indiana, shield prototypes 10 µm thick have been made using stretched metal membranes, reducing the test mass gap to below 50 µm.

First data with the membrane shield in 2007 revealed a large signal that was traced to electronic pick-up. This signal has been suppressed below the detector thermal noise and more sensitive force data are now being obtained with the smaller test mass gap. Assuming the backgrounds can be controlled, the projected limits attainable with the current experiment are about two orders of magnitude more sensitive than the best limits in Fig. 1 at 10 µm. A cryogenic version of the experiment could attain sensitivity to gravitational strength forces at 20 µm.
2.1. Search for Lorentz Violation in the 2002 Data Set

Recently, a new analysis was begun to look for evidence of the violation of Lorentz symmetry, based on the data set used to extract the Yukawa limits published in 2003. Violation of Lorentz symmetry would manifest itself in the Indiana short-range experiment as a sidereal modulation of the force signal.

Fig. 3 is a first look at the 2002 data set as a function of time. While it may hardly be suggestive of any periodicity in the data set, a sidereal fit could yield useful limits on Lorentz violating effects.

Derivation of a fitting function, based on the most general Lorentz violating force compatible with the Standard Model and the known behavior of gravity, has been made possible by the recent extension of the Standard Model Extension (SME) into the gravitational sector.\textsuperscript{14} The fitting function has the form

\[ F = C_1 + C_2 \sin(\omega_\oplus T_\oplus) + C_3 \cos(\omega_\oplus T_\oplus) + C_4 \sin(2\omega_\oplus T_\oplus) + C_5 \cos(2\omega_\oplus T_\oplus), \]  

(2)

where the \( C_i \) are linear combinations of the coefficients \( s^{JK} \) controlling Lorentz violation in the SME gravity sector, \( \omega_\oplus \) is the sidereal angular
frequency of the earth, and \( T_{\oplus} \) is the time as measured in the sun–centered celestial equatorial frame.\(^{15}\)

The SME expression for the gravitational potential between two test masses \( m_1 \) and \( m_2 \) is given by:

\[
V = -\frac{G m_1 m_2}{|\vec{x}_1 - \vec{x}_2|} \left( 1 + \frac{1}{2} \hat{x}^i \hat{x}^k \bar{s}^{ijk} \right),
\]

(3)

where \( \vec{x}_1 - \vec{x}_2 \) is the vector separating \( m_1 \) and \( m_2 \), \( \hat{x}^i \) is the projection of the unit vector along \( \vec{x}_1 - \vec{x}_2 \) in the \( i \)th direction, and \( \bar{s}^{ijk} \) is a set of 9 dimensionless coefficients of Lorentz violation in the standard laboratory frame.\(^{16}\) As pointed out in Ref. 16, Lorentz violation leads to a misalignment of the force associated with Eq. 3 relative to the vector \( \vec{x}_1 - \vec{x}_2 \), however, the inverse–square behavior is preserved. Therefore it is expected that the sensitivity of the Indiana experiment to the \( \bar{s}^{ijk} \) will be poor (at least relative to the anticipated sensitivity of dedicated torsion pendulum experiments, which the authors of Ref. 16 estimate at \( 10^{-4} \) to \( 10^{-15} \)). However, it is possible that the Indiana experiment may be able to set fairly long–standing, if weak, limits on unique combinations of the \( \bar{s}^{ijk} \) due to the specific geometry.
The force corresponding to Eq. 3 has been integrated over the experimental test mass geometry using a Monte Carlo technique. The next step, in progress as of this writing, is to transform the results into the sun-centered celestial equatorial frame to calculate expressions for the $C_i$ in Eq. 2 in terms of the $s^{JK}$. In the meantime, the approximate sensitivity of the experiment to the $\hat{s}^{jk}$ can be estimated from the lab frame calculation.

The result of the calculation can be expressed as the sum of terms each consisting of a numerical term in Newtons times an individual coefficient of Lorentz violation:

$$F_{LV} = (-8.7 \times 10^{-16}N)s^{11} + (-8.7 \times 10^{-16}N)s^{22} + (9.4 \times 10^{-16}N)s^{33} + (-4.2 \times 10^{-16}N)s^{23}.$$  \hspace{1cm} (4)

The value of each numerical term is, unsurprisingly, approximately equal to the Fourier amplitude of the (unmodified) Newtonian force on the detector mass at the resonant frequency. The numerical terms proportional to $s^{12}$ and $s^{13}$ are about 1-2 orders of magnitude smaller than the terms in Eq. 4, and appear to be very sensitive to various run control parameters in the integration. The Monte Carlo is being further optimized to make these results more robust.

Sensitivity to the $s^{jk}$ can be estimated by comparison of the Lorentz violating signal to the limiting thermal noise. The thermal noise force is found from the mechanical Nyquist theorem to be:

$$F_T = \sqrt{\frac{4kT}{\tau} \left( \frac{m\omega}{Q} \right)},$$  \hspace{1cm} (5)

where $m$ is detector mass, $\omega$ is the resonant frequency, $Q$ is the detector quality factor, $T$ is the temperature and $\tau$ is the measurement integration time. For typical $Q$ values ($2.5 \times 10^4$), a temperature of 300 K, and an integration time of 720 s (equal to the averaging time of a single point in Fig. 3), this force is about $1 \times 10^{-14}$ N.

Keeping only the diagonal terms in Eq. 4, setting the ratio $SNR = |F_{LV}/F_T|$ to unity results in the surfaces shown in Fig. 4. The basic structure of the surfaces in Fig. 4 is a consequence of the signs of the terms in Eq. 4, and indicative in part of the combination of the $s^{jk}$ to which the Indiana short–range experiment will be sensitive. The $s^{11}$ and $s^{22}$ terms are of the same sign, and opposite in sign to the $s^{33}$ term. In the case when the signs of the $s^{11}$ and $s^{22}$ coefficients themselves are opposite, all of the signal is in the $s^{33}$ term, the limits on which might be constrained to order unity or better.
Figure 4. Relationship between diagonal elements of $\tilde{s}^{ij}$ matrix in the case where $|F_{LV}/F_T| = 1$. Allowed and excluded values of the $\tilde{s}^{jj}$ are shown assuming no evidence of sidereal variation in the force signal.

References