INSTRUCTIONS (read them!)

- Do all work that you want graded in a bluebook. The graders will not look at the exam sheets.
- Write your name clearly on the front of all your bluebooks.
- Number your bluebooks (1-of-2, 2-of-2) if you use more than one so that the graders don’t miss some of your work.
- Total point values for each question and a breakdown of how much each part is worth are given with the question.
- This exam is closed book. You are not allowed a note sheet. Useful formulas are given below.
- If you have questions during the exam, please ask the proctors.
- Hand in your bluebook(s) and the exam sheet at the front of the room when you have finished.

USEFUL FORMULAS

Geometry/Trigonometry

- \( v^2 = v_x^2 + v_y^2 \)
- \( v_x = v \cos \theta \)
- \( v_y = v \sin \theta \)
- \( \tan \theta = \frac{v_y}{v_x} \)

Planetary Motion and Gravity

- \( T^2 = ka^3 \)
- \( F = GMm/r^2 \)

Constants

- \( g = 9.8 \text{ m/s}^2 \) (near earth)
- \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \)

Motion Equations

- \( \langle a \rangle = \Delta v / \Delta t \)
- \( \langle v \rangle = \Delta x / \Delta t \)
- \( \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \) (const \( a \))

Energy

- \( W = F \cdot d \cos \theta \) (work)
- \( P = W / t \) (power)
- \( K = \frac{1}{2} mv^2 \) (kinetic energy)
- \( W_{\text{tot}} = \Delta K \) (Work-K.E. theorem)
- \( E_{\text{mech}} = K + U \) (mechanical Energy)
- \( \Delta K = -\Delta U \) (mech E conservation)
- \( U(r) = -GMm/r \) (grav potential E)
- \( \Delta U = mgh \) (near earth)
PROBLEM 1: Multiple Choice [10 pts – 1 pt each]

i. Prior to Brahe’s observations, whose theory agreed best with observation?
   a. Ptolemy’s
   b. Copernicus’
   c. neither agreed very well with observations
   d. both agreed reasonably well with observations
   Answer: d)
   Before Brahe, positions of celestial objects were known to an accuracy of ~12’.
   Both Ptolemy’s and Copernicus' predictions turned out to be different from the
   true position of the planets and stars by up to ~8’, well within the observational
   limits of the time.

ii. In which direction does the acceleration of the earth point as it moves around the sun?
    a. in the same direction as the earth’s motion
    b. opposite to the earth’s motion
    c. toward the sun
    d. away from the sun
    Answer: c)
    This follows from the direction of the force of gravity in Newton's Law.

iii. The speed of a planet, in an elliptical orbit, at perihelion (closest to the Sun) is:
     a. less than the speed at aphelion
     b. greater than the speed at aphelion
     c. equal to the speed at aphelion
     Answer: b)
     This follows from Kepler’s 2nd Law.

iv. As you move closer to a planet, the gravitational potential energy of the you-planet
    system:
    a. decreases
    b. increases
    c. remains the same
    d. can’t tell with the information given
    Answer: a)
    Since $U = -G\frac{m}{r}$, if $r$ decreases $U$ decreases (becomes more negative) as well.
v. A measurement of a certain quantity yields 3.1 with an uncertainty of 0.1. Which of the following are possible true values of the quantity?
   a. 3.1
   b. 3.05
   c. 2.9
   d. all of the above
   e. none of the above

Answer d)
Because of the statistical nature of uncertainty, any of the above numbers are possible true values of the quantity in question. Some (like 2.9) are very improbable though.

vi. Which is the most probable true value of the quantity in the question above?
   a. 3.1
   b. 3.05
   c. 2.9
   d. all of the above
   e. none of the above

Answer: a)
For correctly made measurements, the measured value is the best estimate of the true value of the quantity in question.

vii. Who does more work
   a. Sisyphus, rolling a 200 kg stone up a 100 m hill.
   b. Atlas, supporting the world (mass 5.98\times 10^{24} \text{ kg}) on his shoulders
   c. they both do the same amount of work
   d. impossible to tell without more information

Answer: a)
Atlas does no work, since he is not moving while he holds up the Earth.

viii. Which of the following is a conservative force
   a. kinetic friction
   b. gravity
   c. the force I exert picking up a book
   d. none of these forces is conservative

Answer: b)
Of the above, only the work done by gravity is independent of the path through which the object moves.
ix. According to the caloric theory of heat
   a. temperature is the result of random motion of molecules in an object
   b. heat and work are both forms of energy
   c. different objects all have the same capacity to store heat
   d. heat can neither be created nor destroyed
   Answer: d)
In caloric theory, heat was a mysterious fluid of which there was a fixed quantity in the universe. It had no particular relation to work or motion, but it could be stored in objects. To agree with measurements of specific heat, however, caloric theory required that different objects have the capacity to store different amounts of heat.

x. I attempt to sell a perpetual motion machine to you, which, according to my claim, extracts energy from the Earth’s gravitational field, while going through a cycle. My extremely complicated machine ends up in exactly the same configuration after a cycle as before. Why won’t this machine work?
   a. It would have to violate momentum conservation.
   b. It would have to violate energy conservation.
   c. It would have to create heat.
   d. It could work. You should invest.
   Answer: b)
The only way that the gravitational potential energy of the machine-earth system can change is if the machine changes position. Claiming that the machine’s gravitational potential has increased when it starts and ends at the same position is therefore a violation of the concept of energy conservation.
PROBLEM 2 [25 pts]
The Wishy-Washy Washtub Co. of Walla Walla, Washington has two types of employees: workers and supervisors. Workers pick up washtub pieces from a bin, inspect them and put them back in the bin. Supervisors push papers around desks all day.

a. [5] Why is it difficult to predict the effort that the workers and supervisors expend in their tasks using only the forces that they apply?
Answer: It is impossible to know beforehand exactly what force a person will apply before he or she actually applies it.
More information (not required to get credit for this part): forces like gravity and friction are predictable — since they don't depend on whim. So they are much easier to deal with. The Work-Kinetic Energy theorem allows us to relate these easily-calculable effects to the difficult concept of effort for the workers and supervisors.

b. [5] The path through which a worker moves a washtub piece as he picks it up, inspects it and replaces it in the bin is shown above as diagram a. If the piece begins and ends at rest and at the same vertical position, how much work does the worker do on it? Justify your answer.
Answer: The work done by the worker is 0.
To justify this you need to use the Work-K.E. theorem.
\[ \Delta K = 0 = W_{\text{tot}} = W_{\text{grav}} + W_{\text{worker}} \]
Since gravity is a conservative force, and since the washtub piece begins and ends at the same vertical position, we know that \( W_{\text{grav}} = 0 \). This means we must have: \( W_{\text{worker}} = 0 \). Note: because the worker exerts a non-conservative force, you cannot simply argue that the work he/she does is zero because the piece starts and ends at the same position.
c. [10] A supervisor pushes a stack of paper in a horizontal, circular path on a desk as shown in diagram b. above. Draw a free-body diagram of all the forces (including their directions) acting on the paper at some point on its path. You may neglect air resistance. Which of the forces acting on the stack of paper do work? For those force doing non-zero work, indicate whether that work is positive or negative.

Answer: See diagram below.

- \( W_{\text{normal}} \) 0 normal force perpendicular to direction of motion
- \( W_{\text{supervisor}} \) >0 supervisor's force acts in direction of motion
- \( W_{\text{gravity}} \) 0 gravity perpendicular to direction of motion
- \( W_{\text{friction}} \) <0 friction opposite to direction of motion

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d. [5] The concepts of Work and Mechanical Energy allow us to predict the speeds and positions of objects. What is one important variable related to motion that Work/Mechanical Energy cannot be used to find?

Answer: Three possibilities are given below
   i. time
   ii. acceleration
   iii. direction of motion

Although you don’t have to include this to get credit, you would have to solve Newton’s equations of motion to get the first two of the above. The third (direction) is often obvious from the problem, but again could be found mathematically using Newton.
PROBLEM 3 [20 pts]

a. [5] Why was it necessary for proponents of a geocentric (earth-centered) universe to introduce the concept of epicycles?

Answer: The thing that couldn’t easily be explained in simple geocentric theories, where everything moved in circles around the earth, was the motion of the planets. As seen from Earth, the planets occasionally seem to reverse the direction of their motion (called retrograde motion). Note: you don’t need to mention retrograde motion to get full credit for the problem – simply say that the issue was the motion of the planets.

b. [5] What was the importance of Brahe’s measurements of the positions of stars in the heliocentric vs. geocentric universe debate? What did they allow astronomers to realize?

Answer: The main importance of Brahe’s observations was that they were significantly more precise than any made before him. In fact, they were precise enough to allow astronomers to distinguish between the predictions of heliocentric (Copernican) and geocentric (epicycles) theories of the universe. It turned out that neither Copernicus nor epicycles agreed with Brahe’s measurements, but you don’t have to mention that to get credit.

c. [5] In order to achieve agreement with Brahe’s measurements, Kepler had to make a revolutionary new hypothesis about the motion of objects in the heavens that forms the basis of his laws of planetary motion. What was that hypothesis?

Answer: Kepler made the radical assumption that planets move in ellipses around the Sun, rather than in circles as Copernicus (and everyone else) had previously believed. Kepler's other laws are also important, but it is the shift to elliptical orbits which was the truly new concept.

d. [5] You discussed one of Kepler’s three laws in part c) of this problem. Describe one of his other two laws in this part. You can use formulas if you want, but be sure to discuss what the variables in them mean.

Answer: Either describing Kepler’s 2nd or his 3rd law is enough to get credit for this part of the problem

- **Kepler’s 2nd:** This law states that planets sweep out equal areas in equal times during their orbits. It also implies that a planet’s speed varies along its orbit, being fastest at perihelion (closest approach to the sun) and slowest at aphelion (furthest from the sun).
- **Kepler’s 3rd:** This law gives the relationship between the size of a planet’s orbit and its period:
  \[ T^2 = K a^3 \]
  where \( T \) is the period of the orbit, \( a \) is the length of the semi-major axis of the ellipse describing the orbit, and \( K \) is a constant set by the body the planet is orbiting.

Note: you will not get full credit for this part if you mention Kepler’s 1st law (planetary orbits are ellipses) here.
PROBLEM 4 [25 pts]

Newton’s Law of Gravity describes how the force of gravity between two objects depends on their masses and the distance between them and has the following form:

\[ F = G \frac{Mm}{r^2} \]

a. [8] What was one aspect of this law that was strongly supported by experiment or observation? What was that experimental or observational evidence?
Answer: Mentioning any one of the following is sufficient to get credit for this part of the problem.

1. The \( 1/r^2 \) part: this was required by Kepler's Laws and therefore supported by Brahe's observations on which those Laws were based.
2. The direction of the force (attractive and pointing on a line between the two objects): this was also require be Kepler's Laws.
3. The dependence on the small mass, \( m \): this agreed with Galileo's observation that all objects fall with the same acceleration, in a medium devoid of resistance, regardless of their mass.
4. The dependence on the large mass, \( M \), is required by Newton's 3rd Law.

b. [8] Cavendish was the first person to measure accurately the value of the constant, \( G \), in Newton's Law of Gravity. Why was it so hard to measure \( G \), and what device did Cavendish use to allow him to do it?
Answer: Because gravity is such a weak force (the value of \( G \) is very small), the force between two masses that could be set up in a laboratory is extremely small and thus difficult to measure. Cavendish dealt with this problem by using a Torsion Pendulum, which is a device capable of measuring very small forces. If you don't mention the term torsion pendulum explicitly, then you need to describe briefly what it does (or draw a sketch of it). The important points here are that the torsion pendulum is a rod with weights on its ends suspended by a thin wire from its center. Force is derived from a measurement of the twist of the pendulum when another weight is brought near it.

c. [9] Astronauts in orbit around the Earth appear to be weightless yet the force of gravity acting on them is only slightly less than that on the surface of the Earth. How is this possible? In your answer be sure to mention the characteristic of orbital motion that leads to weightlessness, but you don't need to derive how this works.
Answer: The astronauts definitely feel a force due to its gravitational attraction to the Earth. Thus, they are constantly accelerating toward the center of the Earth. (They don't hit the Earth because they also have a transverse (horizontal) component to their orbital motion.) The fact that the only force acting on them is the force of gravity means that they are in free fall, like an apple dropped from a tree, a cannonball flying through the air, or a person in a freely falling elevator. We showed in class that the weight of a freely falling object, as measured by a scale, is zero.
PROBLEM 5 [20 pts]

a. [4] How was the concept of Latent Heat (the amount of heat emitted or absorbed when a substance changes phase, e.g. ice to water, or water to steam) explained in the caloric theory of heat?

Answer: Calorists claimed that a substance required different amounts of caloric fluid to exist in its different phases. For example, liquid water at 0° C needed more caloric fluid than an equal mass of ice at 0° C. In order for ice to change into water, the ice had to absorb a certain amount of caloric fluid from some outside source. Similarly, water had to give off some of its caloric fluid to become ice.

b. [4] What did Rumford find while boring out cannons that disagreed with this view?

Answer: Rumford found that the amount of heat released from the cannon while it was being bored was much more than would have been required to melt the cannon. From a calorist point of view, the caloric fluid released when the cannon was bored had to come from the cannon itself. However, if this amount of caloric fluid was present, initially, in the cannon, then it could not have existed in its solid state.
c. [8] One of Joule’s experiment on heat, that we covered in class, consisted of an insulated container full of fluid that was agitated by a paddle-wheel. In this experiment, Joule compared two quantities that he measured (or, more precisely, that he derived from his measurements). What were these quantities and what did he find about them?

Answer: You do not have to include it in your answer to get credit, but I show a diagram of Joule’s experiment below for the sake of clarity in the following:

The two quantities were:
1. The mechanical work done on the system, measured using the falling weight.
2. The heat produced in the liquid, calculated from the measured temperature rise in the liquid and its known specific heat.

He found two main things about these quantities:
1. Heat was being produced in the liquid, not flowing from outside the system.
2. The amount of heat produced was related to the amount of work done and that relationship was the same, regardless of the type of liquid.

Note: you do not have to discuss how these quantities were measured/calculated to get full credit for this part.
d. [4] List one way in which the results of this experiment disagreed with the predictions of the caloric theory of heat.

Answer: Any of the following is sufficient for this part.

1. The caloric theory claimed that heat (caloric fluid) could not be produced or destroyed, it could only move from one place to another. In Joule’s experiment mechanical work was being transformed into heat.

2. In the caloric theory, there was no relationship between heat and work since they were two different things. Joule’s experiment showed a clear, universal relationship.

3. The caloric theory postulated that heat was a fluid; it had nothing to do with molecular motion. Joule’s experiment indicated that the temperature increase (caused by the added heat) in the fluid was the result of its increased random motion when agitated by the paddle wheel.