MATERIAL COVERED

- **Topics:** Quantum Mechanics
- **Problem Sets:** 3, 4
- **Format:** 5 multi-part questions
  - Multiple Choice ~10%
  - Short Answer ~60%
  - Physical Reasoning ~30%

- Questions will be based on material covered in:
  - Lectures
  - Problems Sets
  - Material not touched upon in the lectures or the homework will not appear on the exam, however, you will be expected to apply concepts you have learned to new problems.

- **Content**
  - Exam questions will be very similar to those given in the problem sets.
  - Questions about historical context will deal mainly with the development of concepts, rather than the sequence of events. Emphasis will be placed on:
    - experiments and their interpretation
    - ideas that changed the way physicists thought
  - Physical reasoning questions will ask you to apply the concepts you’ve learned in class to understand phenomena qualitatively
    - for example, determining the charge of a particle from its trajectory through a region of magnetic field
    - there will be no calculations, other than simple arithmetic

- The exam will be closed book and you will not be allowed to use notes.
  - I will supply a formula sheet with the exam (see end of this guide)
  - You do not need a calculator
GENERAL ADVICE

● Studying for the Exam
  − Do **all** the homework problems and understand the solutions.
  − Review your lecture notes – see more information on the Lectures web page.
  − Use the texts if you have difficulty understanding some aspect of the lectures or problems.
    Remember, exam questions will be based on the lectures and problem sets, not on the text.

● Exam-Taking Strategies
  − Before beginning the exam read over all the problems.
  − Before doing a problem read it carefully so you don’t miss anything.
  − Start with the easiest problem.
  − If you get stuck – don’t waste time. Go on to another problem.
  − Write legibly. If the grader can’t read your solution he/she can’t grade it.
  − Try not to be too verbose in the Short Answer questions. A few sentences should be sufficient.
  − Describe your logic in the Physical Reasoning problems. **Never just state the answer**
  − Draw detailed pictures - this can indicate to the grader that you understand the concept of the problem even if you don't get the details right.

● Partial Credit
  − Partial credit will be given on the Short Answer and Reasoning questions, but not on the Multiple Choice section.
  − You are much more likely to get partial credit if your answer follows clear and logical steps.
    ▶ Do not scribble down ideas/equations all over the page at random and expect the grader to guess at your thought processes.
    ▶ Do draw diagrams whenever possible and label them clearly with relevant information such as forces, trajectories, etc.
KEY CONCEPTS

WHAT TO REMEMBER FROM MIDTERM 1

- Classical electromagnetism will not be a focus of midterm 2, however, you should remember a few things from the last midterm, particularly...
  - acceleration of a charged particle by an electric field
  - bending of a charged particle's trajectory in a magnetic field
  - any classical concept that was modified by quantum mechanics (you will need to contrast new quantum ideas with old classical ones)
    - e.g. light as a wave: Young's experiment

FOUNDATIONS OF QUANTUM MECHANICS

- Three observations (and their interpretations) that led to Quantum Mechanics
  - 1) Blackbody Radiation
  - 2) The Photoelectric effect
  - 3) Spectral lines of atoms
  - For each of these you should understand
    - a) the observation
    - b) how classical physics failed to explain it
    - c) the basic idea behind the quantum interpretation
  - See also: Lectures web page: “Summary of above 3 Phenomena”
- **Blackbody Radiation**: Planck
  - Understand what a blackbody is: an object that absorbs all radiation incident upon it
  - Examples of blackbodies
    - a box with a small hole in it
    - the Sun
  - Why are they interesting
    - the spectrum of emitted radiation is independent of the nature of the blackbody
      - allowed physicists to use something they understood (electromagnetic oscillators) to build a blackbody which they could calculate
  - Observation
    - the power radiated by the blackbody at different frequencies is peaked at a frequency that depends on the temperature of the body and falls off rapidly above and below that frequency
    - understand what the emission spectrum means
      - power emitted per unit area at a given frequency
      - or, energy emitted per unit volume at a given frequency
    - be able to sketch the shape
  - Classical Prediction
    - the amount of power should continue to rise with frequency leading to the prediction that infinite power would be emitted by the blackbody
  - Quantum Prediction: Max Planck – 1900
    - found good agreement with the observations if he assumed that the oscillators making up the blackbody emitting radiation of frequency $f$ could only have energies of:
      - $E = nhf$ ($n = 1,2,3,...$) [$h = Planck’s$ constant $= 6.63 \times 10^{-34} \text{ J.s}$]
    - Differences from classical picture
      - 1) Energy of oscillator depends only on frequency
        - classically depends on amplitude
      - 2) each oscillator has a minimum energy, which is non-zero
        - classically, zero energy should be possible
      - 3) only specific allowed energies for an oscillator
        - classically, any amplitude is possible $\rightarrow$ any energy is possible
  - Physical Reasoning
    - no real calculations possible here
    - but understand how blackbody spectrum changes with temperature
**Photoelectric Effect**: Einstein

- Observation: first by Hertz in 1886, most accurately by Lenard in 1902
  - see Homework 3, problem 3 for picture
  - observed that UV light striking a metal plate liberated electrons from the plate
  - observations that didn’t agree with classical predictions – see table below

- Quantum prediction by Einstein
  - photons (particles of light) with energy $E = hf$ knock electrons out of metal
  - Maximum energy of electrons
    - $K_{\text{max}} = E - W$
    - $E = $ energy of photon
    - $W = $ work function of metal = the minimum energy it takes to pull an electron off the surface of the metal
    - note: $K_{\text{max}}$ is the maximum electron energy – most will have smaller energies than this

- Physical Reasoning
  - sketch graphs of current in the circuit vs. battery voltage for various UV light intensities (see Homework 3, problem 3) and $K_{\text{max}}$ vs photon frequency
  - determine $K_{\text{max}}$, cutoff frequency, work function, etc.

<table>
<thead>
<tr>
<th>Observation</th>
<th>Classical Interpretation</th>
<th>Quantum Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $K_{\text{max}}$ did not depend on Intensity of light</td>
<td>Electrons should absorb $E$ continuously from EM waves. More intense EM should transfer $E$ faster to metal → e’s should have more $K$</td>
<td>$E$ is transferred to e’s in one shot by a photon with $hf$. Current increases w/ intensity though</td>
</tr>
<tr>
<td>2) Time b/w incidence of light and ejection of P.E.’s is ~instantaneous</td>
<td>For weak light need to absorb enough $E$ to gain escape energy</td>
<td>Absorption happens as single photon strikes electron</td>
</tr>
<tr>
<td>3) If freq of light was below a certain cutoff – no P.E.’s were emitted</td>
<td>Electrons should be emitted at any freq as long as intensity is high enough</td>
<td>Nothing emitted for $hf &lt; W$</td>
</tr>
<tr>
<td>4) $K_{\text{max}}$ increased with increasing freq</td>
<td>No relationship between $K$ and $f$ should exist. $K$ determined only by intens.</td>
<td>$K_{\text{max}} = hf - W$</td>
</tr>
</tbody>
</table>
• **Spectral Lines of Atoms:** Bohr
  - Difference between emission spectrum from a gas and from a blackbody
    - blackbody = continuous spectrum over frequency range
    - gas = only emission at specific frequencies
  - Balmer Formula for Hydrogen emission lines - worked well – but no one knew why
    - \[ f = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]
    - \( R = 3.3 \times 10^{15} \text{ Hz} \)
  - Classical Prediction: Rutherford – 1911
    - made possible by discovery of nucleus by Rutherford, Geiger, Marsden
    - small, heavy nucleus with all the positive charge
    - electrons orbiting the nucleus like planets around the Sun
  - Problems
    - 1) circling electrons would lose energy by radiation → spiral into nucleus (atom unstable)
    - 2) could not explain why there should only be specific energy states in atom
  - Quantum: Bohr Model – 1913
    - Assumptions
      - 1) Electrons only orbit nucleus at fixed radii (stationary states)
      - fixed energy levels: \( E_n \)
      - 2) Transitions b/w stationary states involve the emission or absorption of a quantum of EM radiation (photon) with energy
        - \( E(\text{photon}) = hf = E_1 - E_2 \)
  - Correspondence Principle
    - in certain regions of parameter space, classical and quantum predictions should agree
    - see Homework 3, problem 8
  - Physical Reasoning
    - use formulas for energies and radii of stationary states to understand properties of Hydrogen
    - ionization of Hydrogen: see Homework 3, problem 7
  - Angular Momentum Quantization
    - just understand what this means and why quantized radii give quantized angular momenta
  - Problems with the Bohr model
    - 1) no explanation of why electrons should be in stationary states other than this explained spectral lines
    - 2) why don't the electrons radiate when they move in a circle
NON-RELATIVISTIC QUANTUM MECHANICS

- **Matter Waves:** de Broglie
  - de Broglie Wavelength: \( \lambda = \frac{h}{p} \)
  - understand how de Broglie arrived at this relationship – Homework 4, problem 1
    - integer number of wavelengths must fit in electron’s circular orbit
  - Experimental Confirmation of de Broglie: Davisson and Germer (1927)
    - observed interference patterns when scattering low-energy electrons off of crystals (like two slit experiment with light)
    - don’t need to know the details of the experiment – just that it used interference to show the wavelike nature of electrons

- **Two-Slit experiment with electrons**
  - be able to sketch the pattern observed on the screen for many wavelike electrons (quantum) vs many particle-like electrons (classical)
  - measuring device at one of the slits → wavefunction collapse → observe particle-like pattern on the screen
    - an example of complementarity of wave-like and particle-like behavior

- **Mathematical representations of Quantum Mechanics**
  - 1) **Matrix Mechanics:** Heisenberg
    - some pairs of variables behave like matrices: \( px - xp = \frac{h}{2\pi} i \)
  - 2) **Wave Mechanics:** Schrödinger
    - quantum states obey a wave equation
    - solutions are the superposition (addition) of several different waves
  - 3) **Quantum Electrodynamics:** Dirac
    - explicitly included relativity
  - understand the main conceptual differences b/w these: Homework 4, problem 4

- **The Wavefunction:** Schrödinger
  - Understand probabilistic interpretation of Schrödinger’s wavefunction: \( \psi(x,t) \)
    - \( |\psi(x,t)|^2 = \) probability density at point \( x \)
  - Prob. of observing the particle (or state) within \( dx \) of point \( x \) is: \( P = |\psi(x)|^2 dx \)

- **Uncertainty Principle:** Heisenberg
  - Concept of complementary variables
  - two variables that cannot be simultaneously measured with infinite accuracy
    - position, momentum
    - time, energy
    - \( L_x, L_z \): components of angular momentum
    - wave, particle: wave-particle duality is the basis of complementarity
  - Mathematics of complementarity: Heisenberg’s Uncertainty Principle
    - \( \Delta x \cdot \Delta p \geq \frac{h}{2\pi} \) position-momentum
    - \( \Delta E \cdot \Delta t \geq \frac{h}{2\pi} \) energy-time
  - Be able to reproduce the qualitative argument of how the wave-like nature of particles (de Broglie and Schrödinger) → position-momentum complementarity
    - see Homework 4, problem 5
• **Copenhagen Interpretation of Quantum Mechanics**: Bohr
  - 1) Observation is probabilistic by nature
  - 2) Observation fundamentally changes a system
    ▶ collapses the wavefunction
  - Understand how this differs from the classical (objective) view
    ▶ 1) Nature is deterministic (not governed fundamentally by probability)
    ▶ 2) Observation makes no difference to the state of a system (objectivity)
  - Schrödinger’s Cat & EPR Paradox (Einstein-Podolsky-Rosen)
    ▶ not necessary to understand in detail
    ▶ just realize that these highlight the fact that quantum mechanics is either:
      ○ a) non-casual: information travels faster than c, or
      ○ b) non-objective: observation changes the system
  - Hidden Variable theories
    ▶ attempt to preserve objectivity and causality by saying that there is a deterministic theory (that we have not yet found) underlying quantum mechanics
  - Bell’s Inequality
    ▶ allows to distinguish experimentally between QM and Hidden Variables
    ▶ Uses different predictions of correlations between complementary variables in two-particle states that are described by a single wavefunction
      ○ see Homework 4, problem 8 and Lectures page

• **Quantum State of Electron**
  - Any electron can be completely described by four quantum variables
  - In an atom these variables have only certain allowed values

<table>
<thead>
<tr>
<th>Quant No.</th>
<th>Corresponds to</th>
<th>Equation</th>
<th>Possible Values</th>
<th>No. of States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Energy (orbital)</td>
<td>$E_n = -\frac{13.6}{n^2} \text{ eV}$</td>
<td>1,2,3,...</td>
<td>infinite</td>
</tr>
<tr>
<td>$l$</td>
<td>Angular Momentum</td>
<td>$L^2 = \hbar^2 l(l+1)$</td>
<td>0,1,...,$n$-1</td>
<td>$n$</td>
</tr>
<tr>
<td>$m$</td>
<td>Ang. Mom. Comp</td>
<td>$L_z = m \hbar$</td>
<td>-1,...,0,...1</td>
<td>2$l$+1</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Spin Component</td>
<td>$S_z = m_s \hbar$</td>
<td>-$\frac{1}{2}$ , $\frac{1}{2}$</td>
<td>2</td>
</tr>
</tbody>
</table>

  - Pauli Exclusion principle
    ▶ no more than one electron can be in any specific quantum state at a time
    ▶ use this to count the number of electrons in an orbital: see Homework 4, prob 7
    ▶ yields all observed chemical properties of elements
  - Discovery of Spin
    ▶ proposed to be the 4\textsuperscript{th} quantum variable describing the electron
    ▶ behaves like angular momentum – but intrinsic to particle
    ▶ two components allowed: $+\frac{1}{2}$ , $-\frac{1}{2}$ (in units of $\hbar/2\pi$)

• **Stern-Gerlach Experiment**: experimental confirmation of spin
  - understand basics of experiment: see Homework 4, problem 6 and Lectures page
  - Classical prediction
    ▶ thin beam should be spread into a broad band at the screen because spin/angular momentum can have any orientation with respect to the B-field
  - Quantum prediction
    ▶ beam should be split into several distinct bands at screen because only certain orientations of spin/angular momentum are allowed
    ▶ would expect an odd number of bands if the silver atoms have non-zero angular momentum (which they don’t)
    ▶ observed two bands because of 2 spin orientations of outermost electron in silver atom
  - demonstrated that spin was a real property of the electron and that spin (and also, by extension, angular momentum) are quantized
QUANTUM ELECTRODYNAMICS (part 1)

- Development of QED: Dirac
  - Problem it was designed to address: relativistic (Lorentz) invariance
    - other versions of quantum mechanics did not obey Lorentz transformations (didn’t work properly at high speeds)
  - Method
    - describe problem with a Hamiltonian that explicitly included the quantum nature of the electromagnetic field and its interaction with matter
      - $H(\text{total}) = H(\text{matter}) + H(\text{field}) + H(\text{interaction})$
      - no need to understand the details of the Hamiltonian – just its 3 components
      - previous theories had glossed over quantum nature of field
  - Early Successes
    - a) successfully included Lorentz invariance (special relativity)
    - b) made all the same (correct) predictions that other versions of quantum mechanics had
      - atomic spectra, etc.
    - c) spin of electron was required mathematically for QED to work
      - put in \textit{ad hoc} in other versions of quantum mechanics
  - New Prediction: negative energy electrons (interpreted as anti-matter)
    - theory vindicated with the discovery of the positron
  - Problems with QED: infinities
    - gave predictions of infinite: energy, mass, interactions, charge for electrons
    - understand qualitatively how these arise – see Homework 4, problem 9

- Discovery of the Positron
  - Two experimental ingredients to this discovery
    - 1) Cosmic Rays
      - high energy particles produced in space that strike the earth’s atmosphere
      - these interact with atoms in the atmosphere causing “showers” of particles
    - 2) Cloud Chamber
      - understand basic principle of operation
      - water vapor condenses along the track of a particle because ionization caused by particle provides condensation centers
  - Anderson’s experimental setup to track cosmic rays
    - 1) Cloud Chamber – photos taken at random times → cosmic ray tracks
    - 2) Magnetic Field
      - momentum of track from size of its curvature: $p = qB\ell$
      - sign of track from direction of curvature: $F = q\mathbf{v} \times \mathbf{B}$
    - 3) Iron Plate → tell direction track is moving
      - track loses energy going through iron
    - 4) Type of particle causing track determined by
      - thickness of track: electrons = thin, protons=thick
      - amount of energy loss in iron plate
    - see Homework 4, problem 10
### USEFUL FORMULAS

#### Geometry/Trigonometry
- \( v^2 = v_x^2 + v_y^2 \)
- \( v_x = v \cos \theta \)
- \( v_y = v \sin \theta \)
- \( \tan \theta = \frac{v_y}{v_x} \)

#### Constants
- \( e = 1.6 \times 10^{-19} \) C, fundamental charge
- \( c = 3.0 \times 10^8 \) m/s, speed of light
- \( 1 \text{ eV} = 1.6 \times 10^{-19} \) J
- \( k_E = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \) N m^2/C^2, electric const
- \( k_B = \frac{\mu_0}{4\pi} = 1.0 \times 10^{-7} \) N m/A, magnetic const
- \( h = 6.63 \times 10^{-34} \) J s, Planck’s const
- \( a_o = 5.3 \times 10^{-11} \) m, Bohr radius

#### Energy & Momentum
- \( p = m v \), Momentum
- \( K = \frac{1}{2} m v^2 \), Kinetic Energy
- \( \Delta K = -\Delta U \), Energy Conservation
- \( E^2 = (pc)^2 + (mc^2)^2 \), Relativistic Energy

#### Quantum
- \( E = hf \), Photon Energy
- \( K_{\max} = E_i - W \), Photo-Electron Energy
- \( E_n = -13.6 \text{ eV} \), Hydrogen Energy
- \( r_n = n^2 a_o \), Hydrogen radii
- \( \lambda = h/p \), de Broglie wavelength
- \( \Delta x \Delta p \geq \frac{h}{2\pi} \), Heisenberg Uncertainty

#### Electricity
- \( \vec{F} = q \vec{E} \), Force-Field Eqn
- \( F = k_e \frac{q_1 q_2}{r^2} \), Coulomb’s Law
- \( E = k_e \frac{q}{r^2} \), Point Charge Field
- \( \Delta U = qV \), Potential E. from Voltage
- \( U(r) = k_e \frac{q_1 q_2}{r} \), P.E. from 2 charges

#### Magnetism
- \( \vec{F} = q(\vec{v} \times \vec{B}) \), Magnetic Force
- \( p = m v = q B r \), Motion in a B-Field

#### Waves and Light
- \( c = f \lambda \), speed-frequency-wavelength
- \( I = P_{src} / (4\pi r^2) \), intensity from point source
- \( p = E / c \), momentum of light