INSTRUCTIONS (read them!)

- Do all work that you want graded in a bluebook. The graders will not look at the exam sheets.
- Write your name clearly on the front of all your bluebooks.
- Number your bluebooks (1-of-2, 2-of-2) if you use more than one so that the graders don’t miss some of your work.
- Total point values for each question and a breakdown of how much each part is worth are given with the question.
- This exam is closed book. You are not allowed a note sheet. Useful formulas are given below.
- If you have questions during the exam, please ask the proctors.
- Hand in your bluebook(s) and the exam sheet at the front of the room when you have finished.

USEFUL FORMULAS

- **Geometry/Trigonometry**
  - \( v^2 = v_x^2 + v_y^2 \)
  - \( v_x = v \cos \theta \)
  - \( v_y = v \sin \theta \)
  - \( \tan \theta = \frac{v_y}{v_x} \)

- **Constants**
  - \( e = 1.6 \times 10^{-19} \text{ C} \)
  - \( c = 3.0 \times 10^8 \text{ m/s} \)
  - \( 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \)
  - \( k_E = 1/4 \pi \epsilon_0 = 9.0 \times 10^9 \text{ N m}^2 / \text{ C}^2 \)
  - \( k_B = \mu_0/4 \pi = 1.0 \times 10^{-7} \text{ N s}^2 / \text{ C}^2 \)

- **Energy**
  - \( K = \frac{1}{2} m v^2 \)  \text{ Kinetic Energy} \)
  - \( \Delta K = -\Delta U \)  \text{ Energy Conservation} \)

- **Electricity**
  - \( \vec{F} = q \vec{E} \) \text{ Force-Field Eqn} \)
  - \( F = k_E \frac{q_1 q_2}{r^2} \) \text{ Coulomb’s Law} \)
  - \( E = k_e \frac{q}{r^2} \) \text{ Point Charge Field} \)
  - \( \Delta U = q V \) \text{ Potential E. from Voltage} \)
  - \( U(r) = k_E \frac{q_1 q_2}{r} \) \text{ P.E. from 2 charges} \)

- **Magnetism**
  - \( \vec{F} = q(\vec{v} \times \vec{B}) \) \text{ Magnetic Force} \)
  - \( p = m v = q B r \) \text{ Motion in a B-Field} \)

- **Waves and Light**
  - \( c = f \lambda \) \text{ speed-frequency-wavelength} \)
  - \( I = P_{\text{src}} / (4\pi r^2) \) \text{ intensity from pt. source} \)
  - \( p = E / c \) \text{ momentum of light}
PROBLEM 1: Multiple Choice [10 pts – 1 pt each]

i. If I bring a positively charged object near to (but not touching) a metal bar that is insulated from its surroundings. The side of the metal bar nearest the charged object will acquire a charge that is
   a. positive  
   b. negative  
   c. zero      
   d. can’t tell from the information given

Answer: b)
The positive charge will attract negative charge in the bar, causing the side of the bar nearest the charge to be positive and the other side to be negative.

ii. In the above problem, the total charge on the metal bar will be:
   a. positive  
   b. negative  
   c. zero      
   d. can’t tell from the information given

Answer: c)
No charge has actually been put on the bar - only a charge separation is induced.

iii. The change in kinetic energy of a charged particle traveling through a region of uniform electric field is
   a. positive  
   b. negative  
   c. zero      
   d. can’t tell from the information given

Answer: d)
The charged particle’s energy will change in the electric field, but we can’t say whether that change will be positive or negative until we know the direction of the field and the charge of the particle.

iv. The change in kinetic energy of a charged particle traveling through a region of uniform magnetic field is
   a. positive  
   b. negative  
   c. zero      
   d. can’t tell from the information given

Answer: c)
Uniform magnetic fields only change the direction of a charged particle's motion, not its speed.
v. Which of the following presented the largest obstacle to the development of the understanding of magnetism?
   a. the general weakness of magnetic forces
   b. the complicated form of the magnetic field
   c. the small impact of magnetism on everyday life
   d. the rareness of magnetic materials

Answer: a)
Small forces are much more difficult to study experimentally than large ones.

vi. Which of the following would not produce an electric field at a large distance from the object in question
   a. a single, stationary electron
   b. a single, moving electron
   c. an electric current flowing in a wire
   d. three charges of the same magnitude, two of which are positive and one negative, forming a tiny triangle

Answer: c)
Of the above choices only the current carrying wire has no net electric charge. It is therefore the only one that will not produce a large-scale electric field.

vii. Which of the following (if it existed) would not produce a magnetic field?
   a. an electric current
   b. a constant electric field
   c. a time-varying electric field
   d. a magnetic monopole

Answer: b)
Magnetic fields only arise from moving charges (electric currents), time-varying electric fluxes/fields or magnetic monopoles (if such things exist).

viii. Which of the following is the primary mechanism for producing power in a battery (such as the one made by Volta)?
    a. production of electric charge
    b. a transformation of chemical energy to electrical energy
    c. a transformation of mechanical energy to electrical energy
    d. a time-varying magnetic flux producing an electric field

Answer: b)
Batteries use chemical reactions to produce a separation of charge.
ix. If light wave A has a longer wavelength than light wave B, and both are traveling in vacuum, the frequency of wave A is:
   a. larger than that of wave B
   b. smaller than that of wave B
   c. the same as that of wave B
   d. cannot tell from the information given
Answer: b)
Wavelength and frequency are related to the (constant) speed of light by: \( c = \lambda f \).
Since \( c \) is the same for both waves, the frequency of A must be smaller than the frequency of B for the above equation to be satisfied by both of them.

x. Which of the following properties of light made it difficult for Newton to observe its wavelike properties?
   a. the nature of the fields making up light
   b. the very large speed of propagation of light
   c. the extremely short wavelength of visible light
   d. none – it should have been obvious to Newton that light is wavelike
Answer: c)
The fact that visible light has wavelengths in the range of 400 – 700 nm meant that Newton would have had to create very small structures in order to observe its interference effects. He did not do this. Young, on the other hand, did.
PROBLEM 2 [20 pts]

a. [6] In 1766, Benjamin Franklin observed that an electrically charged object placed inside a charged, spherical shell experienced no electric force. What were Priestley and Cavendish able to deduce from this observation and how were they able to make that deduction?

Answer: Priestley and Cavendish were able to use this observation to deduce independently that the electrostatic force must have the same mathematical form as Newton's gravitational force. The property that Franklin observed is common to all central forces (of which Newton's gravity is one) that have the form,

\[ F = k \frac{q_1 q_2}{r^2}. \]

Note: you do not have to write down the form of this force to get credit for the problem, but you do have to indicate that only certain types of force have the property observed by Franklin.

b. [6] Sketch the magnetic field lines outside of a coil of wire with an electrical current flowing through it (a solenoid).

Why was this observation important, when first made by Ampère in 1820?

Answer: The sketch is shown to the right. You do not need to show the direction of the current or the field to get full credit for your sketch. This observation was important because it showed clearly that magnetism was related to electricity – in particular to the movement of charge (current).

c. [8] In the diagram to the right, a charged particle moves as shown through a region of uniform magnetic field, pointing out of the page. If we know the magnitude (and direction) of the magnetic field and measure the radius of curvature of the particle’s trajectory, we can deduce two important properties of the particle. What are these two properties and how, conceptually, would we find them? You can either use formulas to describe this, or discuss the relationships between the observables and the important properties in words.

Answer: The two properties that we can determine are the following.

i. The sign of the particle’s charge (positive or negative). This can be found by observing the direction the particle bends while moving in the field using the formula: \( F = q (v \times B) \).

ii. The particle’s momentum can be found from its relationship to the radius of curvature of the particle’s trajectory: \( p = q B r \).
PROBLEM 3 [25 pts]

Our understanding of the nature of electric charge evolved over several centuries. In the questions below, you will discuss several of the high points in this evolution.

a. [5] In the 18th century, Franklin postulated that charge is conserved. What does this mean?

Answer: Charge conservation means that the total amount of charge (positive + negative) always remains the same. Another way of stating this is to say that net charge cannot be created or destroyed.

b. [10] Also in the 18th century, Du Fay showed that there are only two types of charge, which Franklin named “positive” and “negative”. However, it wasn’t until 1879 that Hall determined conclusively which type of charge actually moves when current flows through a wire. Describe, conceptually, Hall's experiment. How did it distinguish between the two hypotheses of: (i) negative moving charge and (ii) positive moving charge? Which of these hypotheses is correct?

Answer: A diagram of Hall’s experiment (showing what would happen if current consisted of the flow of positive or negative charge) is shown below. The main components of it that you need to mention are:

i. The slab of metal through which current flowed.
ii. The magnetic field acting perpendicularly to the current direction. This caused the trajectory of the charged particles to bend.
iii. The buildup of charge (charge division) on the sides of the slab due to the curved trajectories of the charge carriers.
iv. The voltmeter measuring the voltage between the sides of the slab.

The positive and negative charge hypotheses were distinguished by whether the voltmeter read positive or negative voltage.

Hall’s measurements indicated that it is negative charges that are moving when current flows.
c. [5] J. J. Thomson is given credit for discovering the electron in his study of cathode rays. What property of the electron did Thomson measure and what about the measurement allowed him to claim discovery of a new particle? 

Answer: Thomson measured the charge to mass ratio \( \frac{q}{m} \) of these particles. The fact that it was some 2000 times larger than \( \frac{q}{m} \) for the lightest particle known at the time (ionized Hydrogen) was strong evidence that Thomson was seeing something new.

d. [5] Millikan measured the charge of the electron in his famous “oil drop” experiment. What did he observe about the charge on the drops that allowed him to deduce the electron charge? What did this measurement say about the variation of charge from electron to electron?

Answer: He observed that the drops always had a charge that was an integer multiple of \(-e\) \((-1.6 \times 10^{-19} \text{ C})\). This led him to conclude that a drop with charge \(-2e\) had two extra electrons on it, a drop with charge \(-3e\) had three extra electrons on it, etc. So the electron charge was simply \(-e\). Because the drops always had charge of \(-ne\), Millikan was able to say that every electron has a charge of exactly \(-e\).
PROBLEM 4 [20 pts]

a. [5] The phenomenon of interference can be used to distinguish particles from waves. What happens when two waves interfere destructively? Describe one way that two, initially coherent, waves can be made to interfere destructively at some point in space.

Answer: When two waves interfere destructively the result is that they cancel each other out and nothing is observed. There are several ways that two initially coherent waves can be made to interfere with each other.

i. They can be made to travel through different distances between their source and the point of observation. Although you don’t have to mention this in your answer, destructive interference will happen if their path length difference is an odd multiple of half a wavelength.

ii. They can be made to travel through materials with different indices of refraction.

iii. One of them can be reflected off of a substance (like a mirror) that introduces a phase shift.

b. [9] Young’s experiment provided the first strong evidence that light is a wave. What, physically, did Young observe in his experiment? What would this observable have looked like if light were (i) a particle, or (ii) a wave?

Answer: Young observed intensity patterns of light on a screen in his experiment. If light were a particle, Young would have observed the following.

On the other hand, wave-like light would produce an interference pattern at the screen as shown below.

The important distinction for this problem is that particles produce two bright bands, while waves produce multiple bright bands.
c. Maxwell was able to describe Young's light wave as made up of electric and magnetic fields. Make a sketch of an electro-magnetic wave, showing clearly how the $E$ and $B$ fields vary, the spatial relationship between their directions, and how the directions of $E$ and $B$ compare to the direction of propagation of the wave.

Answer: The sketch is shown below. The important points that you need to include in your sketch to receive full credit are the following:

i. $E$ and $B$ must vary in phase with each other

ii. $E$ and $B$ must be perpendicular to each other

iii. The direction of propagation must be shown and it must be perpendicular to $E$ and $B$. 

![EM Wave Diagram]
PROBLEM 5 [25 pts]

a. [8] List all the ways that electric and magnetic fields can be produced.
Answer:

Electric Fields are produced by:
  i. Electric charge
  ii. Time varying magnetic fields (actually time-varying magnetic flux, but you
      don't have to say that explicitly to get full credit)

Magnetic Fields are produced by:
  i. Moving electric charge (electric current)
  ii. Time varying electric fields (actually time-varying electric flux, but you
      don't have to say that explicitly to get full credit)

b. [8] Describe briefly, in words, how an electric charge that is oscillating up and down
   in space produces electromagnetic radiation.
Answer:
  i. Since the electric charge is moving up and down, the electric field
     associated with that charge at some point in space will also change.
  ii. The changing electric field will induce a magnetic field
  iii. Since we now have a magnetic field where none previously existed, we
       have a changing magnetic field. This will induce an electric field.
  iv. This new electric field will, in turn, induce a magnetic field.
  v. etc., etc., etc.
  vi. Since information takes time to travel through space, these changing
      fields will propagate outward as a wave.

Answer: Maxwell's theory predicts that electro-magnetic radiation comes in a nearly infinite range of wavelengths/frequencies. It was the existence of radiation outside the wavelength range of visible light that was shown experimentally by Hertz.

A simplified diagram of his apparatus is shown below. In your answer, you should include

i. The transmitter (spark-gap connected to a voltage source). This produced sparks that created EM radiation

ii. The receiver (spark-gap not connected to a voltage source). Sparks were produced in this spark gap when the EM radiation from the transmitter passed by it.

iii. The wavelength (and other properties) of the radiation causing the spark at the receiver was determined using interference-type experiments (not shown below).