Asymmetric Pricing across Channels

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Abstract

Among all the topics about price dispersion, asymmetric price dispersion between online and off-line channels and asymmetric pricing across traditional retailers, e-tailers, and multichannel retailers have received the most extensive empirical examination in the literature. However, in many cases, these studies generate contradictory results. This paper jointly models traditional retailer, e-tailer, and multichannel retailer into a market with both online and off-line channels and intends to provide a theoretical framework that may reconcile the conflicting empirical findings. The asymmetric channel use by the sellers and the different customer compositions online and off-line result in asymmetric pricings across the three types of sellers. E-tailer prices the most aggressively in the market. To balance its two channels, multichannel retailer partially gives up the online customers that search prices to e-tailer. The competitive pricing by multichannel retailer forces traditional retailer to heavily depend on its loyal customers. In comparison, the online channel has relatively lower prices but not necessarily less price dispersion than the offline channel. A data set of some most popular consumer electronics products at the leading price comparison site, Shopping.com, shows evidence consistent with the predicted relationship between e-tailers and multichannel retailers.

Introduction

Traditionally, studies of price dispersion have focused on the distribution and variation of prices that exist across the same items displayed in different stores. Price dispersion is thought to result from
the usually high cost of price searching and comparison. With the development of the Internet and the increasing maturity of online shopping as a viable purchasing option, buyers are discovering that their price search costs are dramatically falling while ease of online shopping expands and access to products increases. Rather than expending time and resources in physically traveling to brick-and-mortar stores to compare prices, or spending hours on the phone or scouring newspapers and other advertisements, customers can now easily use online price comparison sites to retrieve and compare prices at the click of a button. As the Internet has greatly reduced the cost of price search and comparison, many believe that price dispersion online should be eliminated or, at least, be reduced significantly. This belief is further strengthened as more and more traditional sellers enter the online marketplace and become multichannel retailers. However, overwhelming empirical evidence from various retail industries such as books, travel, electronics, computer parts, CDs and DVDs, health products, consumer products, insurance, and mortgages suggests that price dispersion online is significant, persistent, and, surprisingly, is not necessarily less than that seen off-line. For example, while Morton, Zettelmeyer, and Silva-Risso (2001) found less price dispersion online, Erevelles, Rolland, and Srinivasan (2001) spotted greater price dispersion online than offline. Scholten and Smith (2002) noticed comparable online and off-line price dispersion. Moreover, although it has been well established that traditional retailers, e-tailers, and multichannel retailers have different price levels, empirical investigations again reached conflicting conclusions about which type of retailer charges the highest price for the same goods.

The contradictory evidence suggests that price dispersion is asymmetric across channels and theories based on markets with unit channel are inadequate to explain price competition in markets with multiple channels. This paper examines the impact of asymmetric channel use and different online and off-line customer compositions on pricing. By placing all three types of retailers within these two channels, this paper illustrates price dispersion in a complete system rather than in an isolated branch. This placement allows for interaction between channels and thus accurately captures the distinct pricing problems faced by different types of sellers.

The model in this paper can be envisioned as follows: a homogeneous product market stretches through both off-line and online channels. The online and off-line channels consist of asymmetric

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fractions of customers who enjoy low search costs and actively search for and compare prices on the products they wish to buy. Customers encountering high search costs exclusively patronize certain retailers. Within the marketplace defined in this paper, there are three types of sellers: traditional retailer, e-tailer, and multichannel retailer. Traditional retailer, whose store is brick-and-mortar establishment with physical presence, only deals with off-line customers. E-tailer receives only online customers through its Web site. Multichannel retailer is visited by customers both off-line at its store’s physical location and online at its Web site.

This paper shows that pricing is significantly different across different types of sellers and hinges on the fractions of searchers across different channels. Both the traditional retailer and the e-tailer mix prices but the traditional retailer charges higher prices than the e-tailer because relatively fewer off-line customers search prices. Forced to balance its two channels, the multichannel retailer strategically mixes price to compete with both traditional retailer and e-tailer simultaneously. By doing so, the multichannel retailer not only remains competitive in both channels but also generates more profit than if it behaved solely like a traditional retailer or an e-tailer. The channel with relatively more searchers is more competitive. It is also true, however, that although the two channels have asymmetric price dispersions, the channel with relatively more searchers does not necessarily have less price dispersion.

Another remarkable finding is that the multichannel retailer does not compete with the e-tailer to its full capacity. Instead, it partially gives up online price searchers to the e-tailer and prices its product less aggressively. By doing so, the multichannel retailer can generate more profit from the off-line channel. Anyway, the multichannel retailer’s pricing is still too aggressive for the traditional retailer to match. Therefore, the traditional retailer has to heavily depend on its loyal customers who regularly shop at its physical location, and thus promotes the least often and offers the smallest discount.

**Motivation**

Since the emergence of e-retailing, growing number of buyers have switched from shopping via physical store visits to online shopping via Web sites. According to eMarketer, in August 2006
approximately 100 million people (1/3 of the population) in the U.S. were "heavy" Internet users\(^2\). Further, based on a sample of 1,000 heavy users, 84\% had engaged in online shopping and had made purchases online. Among the heavy users studied, price comparison was the second most-used Internet technology, next only to instant messaging. However, while the Internet provides sellers with an alternative marketplace, it also significantly reduces the cost for customers to search prices, making the online channel very attractive to buyers but more challenging to sellers.

E-retailing also has greatly shaken the established confidence of traditional sellers and led to the emergence of multichannel retailers. Online retailing challenges the practices of all the traditional retailers who hold onto old business model. These business practices may have worked extremely well in the past but are not as effective in a modern business environment heavily influenced by the constant presence of Internet commerce and the vastness of the World Wide Web. Retailers who cling to outmoded practices run the risk of becoming obsolete and being edged out of their niche by Web-savvy businesses that take full advantage of the online environment. As a result, except in the initial years of e-retailing, dotcoms are not the only significant participants online; inspired by the success of e-tailers such as eBay.com, amazon.com and buy.com, many traditional retailers have extended their presence to the online channel and evolved into multichannel retailers. For example, the study performed in 2002 by National Association of German Retailers (HDE) found 29\% of the German retailers offered their products and services online as well as in their physical brick-and-mortar stores.

The distinct characteristics of online and offline channels and the interplay between them have inspired many questions. The most tested conjecture is that reduced search costs and heightened competition online may lower price levels and diminish price dispersion. However, empirical investigations have found varied results. For example, Brynjolfsson, and Smith (2000) examined prices of books and CDs in 1998 and 1999. After weighting price by market share, they found more price dispersion and higher price level offline. However, Clay, Krishnan, Wolff, and Fernandes (2002) observed higher price levels and more dispersion in the online book industry from 1999 to 2000. Ancarani and Shankar (2004) demonstrated that price dispersion is a delicate subject; the price dispersion in the book and CD market studied was sensitive to the measure used. Prices charged

\(^2\)A "heavy" internet user is defined by The New Digital Divide study at Universal McCann as "someone who has accessed the Web at least 11 times in the previous seven days".
by e-tailers varied across a wider range while those by traditional retailers demonstrated greater standard deviation.

As more and more traditional retailers develop into multichannel retailers, recent works focus on comparing prices across traditional retailers, multichannel retailers and e-tailers. Ancarani and Shankar (2004) showed, when shipping costs were included, multichannel retailers charged the highest prices, followed by e-tailers and then by traditional retailers. Pan, Ratchford, and Shankar (2002) suggested that multichannel retailers have higher average prices than e-tailers. Consistent with Pan, Ratchford, and Shankar (2002), Xing, Yang, and Tang (2006) noted higher prices among multichannel retailers than among e-tailers in the DVD industry and also pointed out that the difference diminishes over time, though very slowly.

To summarize, the empirical literature repeatedly documents conflicting evidence of price level and price dispersion among the three types of retailers. While some of the inconsistencies can be explained by the evolution of the e-market to maturity, most are left tangled and unresolved. The evidence suggests that this issue pertains to specific industries in specific time periods and responds to specific measures.

The remainder of the paper proceeds as follows: relevant literature is reviewed first, then a Triopoly model mapping all the three types of retailers into online and off-line channels is established to characterize the pricing by each retailer and the price dispersion in each channel. Finally, the paper empirically tests the theoretical prediction that multichannel retailers set their prices higher and have wider price ranges than pure online e-tailers.

Literature Review

In sharp contrast to the rich studies of price dispersion in homogeneous product markets with one channel, research fitting asymmetric customer search into homogeneous product market with multiple channels is quite rare. Instead, identical products from sellers using different channels are usually modeled as differentiated products. For example, Pan, Shankar, and Ratchford (2002) modeled the competition between an e-tailer and a multichannel on a same Hotelling (1929) main

[^3]: Earlier works usually ignored multichannel retailers and divided prices into online and offline. For example, although data in Brynjolfsson and Smith (2000) included both multichannel retailers and e-tailers, the difference between the two was not investigated.

[^4]: For a thorough list of empirical works of online price dispersion, refer to Pan, Ratchford and Shankar (2004).
street" and found that the multichannel, which offered better service quality, had higher price in equilibrium. In their model, customers lived at different locations along the street and incurred transaction costs that were positively related to the distances from their homes to the stores. Given the prices by the e-tailer and the multichannel retailer, every customer picked the store that maximized his utility. Given this strategy by customers, the two stores set prices to maximize their own profits. Druehl and Porteus (2006)’s Duopoly game analyzed competition between an e-tailer and a traditional retailer. Viswanathan (2005) extended Salop (1979)’s circular city model and arranged an e-tailer, a traditional retailer, and a multichannel retailer around two unit circles representing online and off-line channels. Note that in these models, after taking transaction cost or quality into consideration, customers generate different utilities from identical products with the same price.

Unlike all the works mentioned above, this paper models customer search in a homogeneous product market with two channels, and thus is closely related to literature on search and price dispersion such as Varian (1980), Stahl (1989), and Narasimhan (1988). In this literature, customers are divided into two types: loyal and switcher, or uninformed and informed. Price dispersion is rationalized as a strategy for firms to balance the profits from the two types. Varian (1980) differentiated customers with different search costs and demonstrated that price dispersion arose when sellers were trying to attract those informed customers. Stahl (1989) showed how smooth migration from "marginal cost pricing" to "monopoly pricing" might arise when search cost or/and the fraction of consumers searching costlessly changed. Narasimhan (1988)’s Duopoly model studied customer preference and switch cost.

This paper is different from extant literature on customer search because it simultaneously models asymmetric competition in two channels. Asymmetric channel use has never been considered in search models. Instead, sellers are simplified to be identical and competing in a single channel. Such simplification limits our understanding when we approach multichannel pricing. By modeling all three types of retailers that asymmetrically use the two channels, this paper gives a complete portrait integrating competition as it exists among retailers online and off-line.

This paper also differs from extant literature on product differentiation in that it captures and

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5 For a thorough review of theoretical and empirical works of price dispersion, refer to Baye, Morgan and Scholten (2006)
accounts for the existence of the customers who search prices. These customers are those who, for example, visit price comparison sites when buying books online or those who check newspapers for automobile prices by different local dealers. The product differentiation literature assumes that all customers have heterogeneous preferences or transaction costs, and thus do not treat homogeneous products offered by different sellers as perfect substitutes. However, the presence of flourishing price comparison services such as Shopping.com, PriceGrabber.com, and Nextag.com suggests such shoppers do exist. Baye, Gatti, Kattuman, and Morgan (2006) estimated that 13% of Kelkoo.com customers buy only on price. Moreover, the pure strategy equilibriums predicted by the literature on product differentiation are not consistent with the temporal online price dispersion as observed in Baye, Morgan, and Scholten (2004a).

Lastly, except for Viswanathan (2005), the extant studies model competition between only two types of retailers, which may potentially bias the conclusion. This paper addresses this potential weakness by assigning all three types of retailers onto the online and off-line channels.

This paper should not be considered as challenging the validity of prior works. To the contrary, product differentiation models successfully predict that certain customers have strong preferences for certain type of retailers. Such customers are modeled as loyal customers in the literature on search.

Model

Suppose there are two channels in a homogeneous product market: a traditional off-line channel and an online channel. As in Varian (1980), there are two types of customers found within each channel: loyals and switchers. Loyals are those customers who encounter high search costs in making price comparisons and thus do not search prices but just buy from certain sellers up to their reservation prices. On the other hand, switchers search prices costlessly and then buy at the lowest price found. For simplicity, let loyals be evenly divided among sellers in each channel: $l_f$ for each seller in the off-line channel, and $l_n$ for each in the online channel. Suppose there are $s_f$ switchers off-line and $s_n$ switchers online. Both loyals and switchers have unit demand and

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$^6$In this paper, shoppers are assumed trapped in one channel and cannot compare price across channels. If cross-searching is allowed, it can be shown that those shoppers who have access to both channels will use the online channel since the lowest price found online is never higher than that offline. Therefore, these customers will behave exactly the same as switchers in the online channel.
common reservation price $V$ for the product. Since online search cost is greatly reduced, assume there are relatively no fewer *switchers* in the online channel, i.e., $\frac{s_m}{s_n} \geq \frac{s_f}{s_l}$.

Since the model intends to capture pricing across different types of sellers, only one representative retailer of each type is modeled. Seller $t$ is a traditional retailer off-line, seller $e$ is an e-tailer only operating online, and seller $m$ is a multichannel retailer selling both online and off-line. All three sellers are risk neutral and maximize their individual expected profits given perfect information about customer composition across channels and reservation price. However, sellers are not able to distinguish customers with different search costs and thus cannot engage in price discrimination. Furthermore, seller $m$ is committed to charge same prices along the two channels. Let all sellers have the same constant marginal cost, which is assumed to be zero without loss of generality, and assume there is no other transaction cost.

This Triopoly game is the most abridged version of competition across traditional retailers, e-tailers, and multichannel retailers. However, it adequately captures the nature of competition in a typical market with two channels. Differentiating customers into *loyal* segment and *switcher* segment is standard in price dispersion literature. What makes this paper distinct is that it juxtaposes asymmetric competitions along the two channels and links them through the pricing by the multichannel retailer.

In the following section, competition with symmetric fractions of *switchers* in the two channels is studied first, followed by the analysis of competition with asymmetric fractions of *switchers*. And then, predictions from the asymmetric model are tested using a data set collected from a leading price comparison site in the United States.

**Results**

Let $(\underline{p}_i, \overline{p}_i)$ be the lower and upper bound of the price interval over which seller $i$ mixes prices. When $\underline{p}_i = \overline{p}_i$, it means seller $i$ plays pure strategy. Let $F_i(p)$ be the cumulative distribution function of seller $i$’s pricing, $\Pi_i(p)$ the expected profit when it charges price $p$, and $\pi_i$ the expected profit in the resulted equilibrium.

**Proposition 1** *There exists no pure strategy Nash equilibrium.*
Proof. First, note that given the competitor’s price, each seller’s profit is not continuous. For example, if switchers are evenly divided when the price tie happens and seller $m$ does not charge over reservation price, seller $t$’s profit conditional on seller $m$’s price is

$$\Pi_t(p_t|p_m) = \begin{cases} p_t(s_f + l_n) & \text{if } p_t < p_m \leq V \\ p_t(\frac{s_f}{2} + l_f) & \text{if } p_t = p_m \leq V \\ p_t l_f & \text{if } p_m < p_t \leq V \\ 0 & \text{if } p_m \leq V < p_t \end{cases}$$

The non-existence of pure strategy Nash equilibrium in the setting of loyal versus switcher stems from the discontinuity of the profit function above. The proof is standard in the literature of price dispersion and thus is skipped. ■

Pure strategies cannot be in equilibrium because sellers lose money when their prices can be successfully predicted by their rivals. If a seller repeats the same price, its rival can anticipate this price and will undercut the price by one cent to get all switchers plus the rival’s loyals. This seller can then further undercut the rival’s price and win back all the switchers. Such undercutting happens in turn until it is not profitable to compete for switchers. Then one seller will price high to monopolize its loyals, but when this happens, its rival will again undercut the monopoly price by one cent, thus beginning a new round of undercutting that persists until it is no longer profitable for either competitor.

Instead, sellers can avoid their prices being efficiently predicted by rivals if they mix prices. By mixing prices, every seller has positive probability to win business from switchers. When using mixed strategy pricing, the seller’s task is to balance its profit from loyals and switchers. Note that sellers have to compete for switchers. Cutting price has two effects; it will decrease the profit from loyals but increase the chance to win purchases from the switchers. Since the loss from loyals can be recovered by the expected profit from switchers, sellers may price below the reservation price and have the same amount of expected profit. However, no seller is willing to price overly low if the expected profit from switchers will not offset the profit loss from loyals.

**Proposition 2** For each seller $i$, there is a threshold price $d_i$ under which seller $i$ has no incentive to charge. And the threshold price is the highest for seller $t$, and the lowest for seller $e$, i.e., $d_t \geq d_m \geq d_e$. 

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Proof. At any price \( p \), seller \( t \) can acquire at most \( l_f + s_f \) customers and thus the profit from them is at most \( p(l_f + s_f) \). Because seller \( t \) can secure a profit of \( l_f V \) if it charges the reservation price, it is not willing to charge prices less than \( \frac{l_f V}{l_f + s_f} \). In other words, the threshold price for seller \( t \),

\[
d_t = \frac{l_f V}{l_f + s_f}.
\]

Similarly, \( d_e = \frac{l_n V}{l_n + s_n} \), and \( d_m = \frac{(l_f + l_n) V}{l_f + s_f + l_n + s_n} \). Then, it follows \( d_t \geq d_m \geq d_e \) because \( \frac{s_n}{l_n} \geq \frac{s_f}{l_f} \). Note that at prices below \( d_i \), seller \( i \) cannot generate enough profit from switchers to compensate for the decreased profit from loyals even if it wins switchers with probability one. ■

Equilibrium with Symmetric Fractions of Switchers

Assume the two channels have the same proportions of switchers, i.e., \( \frac{s_n}{l_n} = \frac{s_f}{l_f} \). It follows that \( d_t = d_e = d_m \equiv d \). In this case, all three sellers have incentive to compete over \([d, V]\). Because the competition is modeled in a symmetric environment where seller \( t \) and \( e \) have the same proportion of switchers, it is reasonable to assume seller \( t \) and \( e \) employ symmetric pricing strategies. Therefore, it can be shown, among other asymmetric Nash equilibriums, there exists a unique symmetric Nash equilibrium.

Proposition 3 There exists a unique symmetric mixed strategy Nash equilibrium where each seller has the same pricing strategy

\[
F_t(p) = F_e(p) = F_m(p) = \begin{cases} 
0 & \forall p \in [0, d) \\
1 - \frac{l_f}{s_f} \left( \frac{V}{p} - 1 \right) & \forall p \in [d, V] \\
1 & \forall p \in (V, \infty] 
\end{cases}
\]

Proof. First, consider the competition in the off-line channel. Since every seller’s expected profit in mixed strategy equilibrium is constant, for seller \( t \), \( \Pi_t(p) = \Pi_t(V) \), which implies \( p l_f + ps_f (1 - F_m(p)) = l_f V \). Then it follows \( F_m(p) = 1 - \frac{l_f}{s_f} \left( \frac{V}{p} - 1 \right) \). Similarly, for seller \( m \)’s expected profit to be constant, \( p (l_f + l_n) + ps_f (1 - F_t(p)) + ps_n (1 - F_e(p)) = (l_f + l_n) V \). Under the assumption that pricing strategies between seller \( t \) and \( e \) are symmetric, \( F_t(p) = F_e(p) = 1 - \frac{l_f}{s_f} \left( \frac{V}{p} - 1 \right) \) because \( \frac{s_n}{l_n} = \frac{s_f}{l_f} \). For the above cumulative distribution functions to be valid pricing strategies in equilibrium, it is sufficient to demonstrate that, given the strategies by other sellers, the expected profit of each

\footnote{If the assumption of symmetric pricing of seller \( t \) and \( e \) is removed, there also exist infinite asymmetric equilibriums where pricings by seller \( t \) and \( e \) have asymmetric distributions. However, seller \( m \) always has the above pricing strategy.}
seller is constant over \([d, V]\) and is reduced when it prices out of this interval.

\[
\forall p \in [0, d), \Pi_t(p) = p(l_f + s_f) < d(l_f + s_f) = l_f V. \quad \forall p \in [d, V], \Pi_t(p) = pl_f + ps_f \left( \frac{d}{s_f} \left( \frac{V}{p} - 1 \right) \right) = l_f V, \forall p \in (V, \infty], \Pi_t(p) = 0 \text{ because no customer buys above the reservation price. Therefore, seller t's expected profit is constant over interval } [d, V], \text{ and is reduced when it prices out of this interval.}
\]

Similarly, it can be shown that the expected profits of both seller \(e\) and seller \(m\) are constant over interval \([d, V]\), and are reduced when the sellers price out of the intervals. For this reason, the mixed strategies above are valid.

Therefore, when there is a symmetric fraction of switchers online and off-line, the resulting price dispersions in the two channels are also symmetric. In the equilibrium, the traditional retailer, e-tailer, and multichannel retailer follow a same pricing strategy and do not distinguish themselves from one another. Because customer compositions in both channels are the same, a price cut has the same effect online and off-line on the multichannel’s expected profit. Therefore, the multichannel retailer does not coordinate its pricing strategy to differentiate between the online and off-line channels.

**Equilibrium with Asymmetric Fractions of Switchers**

Since the Internet has greatly reduced customers’ search cost, it is reasonable to assume relatively more switchers exist online than off-line, i.e., \( \frac{s_n}{l_n} > \frac{s_f}{l_f} \), then it follows \( d_t > d_m > d_e \). This illustrates the fact that pricing strategy hinges on the relative size of switchers. Profit from switchers is more attractive to sellers in the online channel because of the relatively larger number of switchers. Therefore, the e-tailer is willing to cut price quite deeply to attract purchases by the switchers. On the other hand, the traditional retailer is the most conservative in competing for switchers because it does not encounter the same relative number of switchers as the traditional retailer and multichannel retailer. When the multichannel retailer takes account of its customers in both channels, the proportion of switchers in its customer base is higher than that for the traditional retailer but is lower than that for the e-tailer. Therefore, the multichannel retailer is less aggressive than the e-tailer but more aggressive than the traditional retailer.

**Proposition 4**  There exists no common interval over which seller \(t\) and seller \(e\) mix price.
Proof. It is standard that the price intervals over which sellers mix prices are convex and under the reservation price. Suppose there exists an interval $[a, b]$ such that $[a, b] \subseteq \left( \bar{p}_t, \bar{p}_t \right) \cap \left( \bar{p}_e, \bar{p}_e \right)$. It must be true that $[a, b] \subseteq \left( \underline{p}_m, \bar{p}_a \right)$. Otherwise, seller $t$ and $e$ encounter no competition in their individual channels over interval $(a, b)$. Therefore, pricing at $b$ brings them more profit than pricing at $a$. That is, $\Pi_t(a) < \Pi_t(b)$ and $\Pi_e(a) < \Pi_e(b)$. However, these imply $a$ and $b$ cannot be jointly in the supports over which $t$ or $e$ mix prices, because the two prices generate different expected profits.

Therefore, we have $[a, b] \subseteq \left( \bar{p}_t, \bar{p}_t \right) \cap \left( \bar{p}_e, \bar{p}_e \right) \cap \left( \underline{p}_m, \bar{p}_a \right)$. Then the competition off-line determines $F_m(p) = 1 - \left( \frac{\pi_t}{s_f} - \frac{t_f}{s_f} \right)$ while that online implies $F_m(p) = 1 - \left( \frac{\pi_e}{s_n} - \frac{t_n}{s_n} \right)$. However, equation $\frac{\pi_t}{s_f} - \frac{t_f}{s_f} = \frac{\pi_e}{s_n} - \frac{t_n}{s_n}$ does not allow for any interval solution unless $\frac{\pi_t}{s_f} = \frac{\pi_e}{s_n}$. Recall that $\frac{\pi_t}{s_f} = \frac{\pi_e}{s_n}$ implies symmetric fractions of switchers online and off-line. Therefore, there is no interval solutions that may simultaneously satisfy both $F_m(p) = 1 - \left( \frac{\pi_t}{s_f} - \frac{t_f}{s_f} \right)$ and $F_m(p) = 1 - \left( \frac{\pi_e}{s_n} - \frac{t_n}{s_n} \right)$ when fractions of switchers are asymmetric.

**Proposition 5** There exists a unique mixed strategy Nash equilibrium where:

1). Distributions of pricing are

\[
F_t(p) = \begin{cases} 
0 & \forall p \in [0, p^*) \\
1 - \left\{ \frac{(l_f + s_f + l_n + s_n)(l_f + s_f + l_n)l_fV}{s_f[(l_f + s_f)^2 + l_f(l_n + s_n)]} \right\} p^{-1} - \frac{l_f + l_n}{s_f} & \forall p \in [p^*, V) \\
1 & \forall p \in [V, \infty) 
\end{cases}
\]

\[
F_e(p) = \begin{cases} 
0 & \forall p \in [0, p) \\
1 - \left\{ \frac{(l_f + s_f + l_n + s_n)(l_f + s_f + l_n)l_fV}{s_n[(l_f + s_f)^2 + l_f(l_n + s_n)]} \right\} p^{-1} - \frac{l_f + l_n}{s_n} & \forall p \in [p, p^*) \\
1 & \forall p \in (p^*, \infty) 
\end{cases}
\]

\[
F_m(p) = \begin{cases} 
0 & \forall p \in [0, p] \\
1 - \left\{ \frac{(l_n + s_n)(l_f + s_f + l_n)l_fV}{s_n[(l_f + s_f)^2 + l_f(l_n + s_n)]} \right\} p^{-1} - \frac{l_n}{s_n} & \forall p \in [p, p^*) \\
1 - \frac{l_f}{s_f} \left( \frac{V}{p} - 1 \right) & \forall p \in [p^*, V) \\
1 & \forall p \in [V, \infty) 
\end{cases}
\]

Where $p = \frac{(l_f + s_f + l_n)l_fV}{(l_f + s_f)^2 + l_f(l_n + s_n)}$ and $p^* = \frac{(l_f + s_f + l_n + s_n)l_fV}{(l_f + s_f)^2 + l_f(l_n + s_n)}$. And

2). In the equilibrium,
\[
\begin{align*}
\pi_t & = \ell_f V \\
\pi_e & = \frac{(l_n + s_n)(l_f + s_f + l_n)l_f V}{(l_f + s_f)^2 + l_f (l_n + s_n)} \\
\pi_m & = \frac{(l_f + s_f + l_n + s_n)(l_f + s_f + l_n)l_f V}{(l_f + s_f)^2 + l_f (l_n + s_n)}
\end{align*}
\]

**Proof.** See Appendix. ■

Proposition 5 suggests that the three types of sellers have totally different pricing strategies as illustrated in Figure 1. Note that \(F_t\) first-order stochastically dominates \(F_m\), and then \(F_e\). Therefore, the e-tailer is the most aggressive seller in the market. It always offers prices lower than the traditional retailer and charges low prices with higher probability than the multichannel retailer. The intuition is that winning *switchers* is more attractive to the e-tailer than to any other seller because it encounters the highest fraction of *switchers*.

![Graph showing F_t first-order stochastically dominates F_m, and then F_e. In addition, F_t has a mass point at \(\bar{p}\).](image)

When competing with the e-tailer and the traditional retailer at the same time, the multichannel must coordinate its two channels that have different levels of competition. Recall that the lower bound of the multichannel’s pricing is higher than \(d_m\) and the e-tailer has a greater chance of winning the *switchers*. This suggests that the multichannel retailer does not compete with the e-
tailer to its full capacity. Instead, it prices less aggressively and partially gives up online switchers to the e-tailer. This is because when the multichannel sets its prices, there is a chance for it to win the switchers in the off-line channel as well. Overly aggressive pricing, although increasing its chance to win the switchers online, hurts its profit from the switcher off-line. Therefore, partially giving up the switchers online earns the multichannel more profit than if it competed like either a retailer or an e-tailer. In turn, the e-tailer also benefits from the reduced competition from the multichannel retailer, although it may still make less than the traditional retailer.

The traditional retailer is the least competitive seller in the market. While the other two types of sellers do not monopolize their individual loyals, the traditional retailer charges the reservation price with positive probability and offers the smallest discount among the three types of sellers. This is because given the smallest fraction of switchers and the relatively more aggressive pricing by the multichannel retailer, the traditional retailer has to heavily exploit its loyals to offset its weakness in generating profit from the switchers.

Because online and off-line channels have different combinations of types of sellers, they also display asymmetric price levels and asymmetric price dispersion. Since prices by the e-tailer are lower than those by the traditional retailer and the multichannel is a "common factor" in both channels, the expected price online is lower than that off-line. However, the price dispersion of the e-tailer is not necessarily less than that of the traditional retailer, making the comparison of price dispersion between online and off-line ambiguous. However, we can find in Figure 1 that the multichannel retailer mixes prices over an interval wider than both the e-tailer and the traditional retailer.

Data

The data studied in this paper were collected from Shopping.com, a leading price comparison site in the United States. It was ranked by Hitwise as having the biggest market share (18.38%) in U.S. comparison shopping in the week ending November 19th, 2005. According to Nielsen/NetRatings, Shopping.com was the second among the top 5 companies ranked by sponsored link impressions in

\[ \text{Data} \]

\[ \text{The data studied in this paper were collected from Shopping.com, a leading price comparison site in the United States. It was ranked by Hitwise as having the biggest market share (18.38%) in U.S. comparison shopping in the week ending November 19th, 2005. According to Nielsen/NetRatings, Shopping.com was the second among the top 5 companies ranked by sponsored link impressions in} \]

\[ \text{Note that the multichannel's expected profit in the equilibrium is more than } (l_f + l_n) V. \]

\[ \text{Note that } p > d_n. \]

\[ \text{It is reasonable to assume the offline channel has a larger total number of customers and more loyals, i.e., } l_f + s_f > l_n + s_n \text{ and } l_f > l_n. \text{ Then it can be easily shown that } \pi_c < \pi_t. \]
the week ending August 21, 2005, surpassed only by auction giant eBay. On December 13th, 2006, according to Alexa.com, it ranked 354 among all web sites and generated 2,900 reaches per million users.

The consumer electronics market is considered one of the most developed and mature online retail markets. All of Dealerscope’s Top 50 Consumer Electronics Retailers have online presence. A joint study by Yahoo! and the Consumer Electronics Association in June 2006 found 77% of consumer electronics spending was influenced by the Internet. They also identified 47% of purchasers as searchers who were actively involved in price comparison and in seeking the lowest prices for their intended purchases.\footnote{However, the separate percentages of online and offline searchers were not reported.}

Price information on 39 products in computer section and electronics section was collected twenty times from October 21st, 2006 to January 7th, 2007. From October 21st, 2006 to December 17th, 2006, the prices were collected twice a week, and from December 17th, 2006 to January 7th, 2007, the prices were collected once a week.

The 39 products are from 6 categories: 7 flash memories, 7 printers, 4 monitors, 7 digital camcorders, 9 digital cameras, and 5 media players. The detailed product information is tabulated in Table 1 in the Appendix. The sampling plan used to determine these products was as follows: in each category, several brands were randomly chosen from the top 10 most popular brands. Then, for each brand chosen, at most three products were randomly selected from the top 5 most popular products. In addition to randomness, such a sampling also has other advantages. First, sampling is not biased by retailers because it exclusively depended on product and brand. Second, prices collected are for popular products, which usually require more deliberation by retailers, resulting in more carefully constructed and justified pricing. Finally, a variety of brands and products were collected to avoid the possible complication by any close relationship of a retailer with certain manufacturers, or by a manufacturer that dominated a particular category. Although this design may be vulnerable to the criticism that only popular items were collected and thus do not literally represent the whole market, popular products overwhelm other products with respect to the clicks generated and profit realized. These types of products thus have much more economic and managerial significance.

Although the default search results after clicking "See All Stores" are not ranked by price,
viewers could easily click "Sort by Price (Low to High)" to show the lowest price first. Prices for items that were currently out of stock were not included because customers would be unable to purchase such items. Prices of refurbished items were also carefully screened out from data collected because they would not be considered identical to new items due to defects, repairs, or other alterations. In addition, shipping and tax were added to prices to capture the exact cost to buyers\textsuperscript{12}.

When potential buyers view search results for a homogeneous product, other than prices, seller’s rating, number of reviews, and "trusted" sellers at Shopping.com\textsuperscript{13} are the only seller information they directly observe. Such information may signal the added value by quality service and therefore were also recorded.

**Estimation**

Unfortunately, prices of the same products by traditional sellers are difficult to find, preventing a complete test of asymmetric pricing across all the three types of sellers. Therefore, this paper focuses on the comparison between multichannel retailers and e-tailers. There are 10,755 prices from 127 sellers of 39 products in the data set. 23.62\% of the sellers are multichannel retailers and their prices constitute 32.36\% of all the prices. Table 2 reports a brief comparison between the multichannel retailers and e-tailers in the data set. The differences in rating and in proportion of "trusted" sellers between the two types of retailers are statistically significant. On average, the multichannel retailers have lower ratings than the e-tailers. However, relatively more multichannel retailers than e-tailers are "trusted" sellers.

**Table 2: Comparison between E-tailers and Multichannel Retailers**

\textsuperscript{12} Shipping and tax are calculated using Zip Code IN47403.
\textsuperscript{13} "Trusted" is a logo assigned to some sellers at Shopping.com and thus is used as a dummy variable in the regression.

"All Trusted Stores must meet the following requirements: Consistently receive positive customer ratings and reviews. Provide accurate price, shipping, and availability information. Provide basic store information including store address. Must note when products are used or refurbished. Must be listed on Shopping.com for at least one year. Honor prices listed on their site without excuses. Resolve customer service complaints quickly and fairly."

— Shopping.com
<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>29</td>
<td>3.672</td>
<td>0.163</td>
<td>82</td>
<td>4.098</td>
<td>0.074</td>
</tr>
<tr>
<td>Review</td>
<td>29</td>
<td>1488.241</td>
<td>823.822</td>
<td>82</td>
<td>1602.512</td>
<td>564.339</td>
</tr>
<tr>
<td>Trusted</td>
<td>30</td>
<td>0.633</td>
<td>0.089</td>
<td>97</td>
<td>0.402</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Note: 1 multichannel retailer and 15 e-tailers were not reviewed and thus have no ratings.

The key implications of the model with asymmetric proportions of switchers are 1) e-tailers price more aggressively than multichannel retailers and thus are expected to offer larger discounts to customers, and 2) multichannel retailers, forced to compete along their two frontiers simultaneously, mix prices over wider intervals than e-tailers, who focus only on the online channel. The following sections test these two hypotheses using the data set from Shopping.com. Potential mis-specifications are addressed and estimates are reported.

Do e-tailers offer better prices to customer?

Among the three types of sellers examined in this paper, e-tailers have the highest proportion of switchers in their customer base. Because of this, they are willing to offer the most competitive prices to win switchers. On the other hand, multichannel retailers are not able to match the aggressive pricing by e-tailers because it would impair their profits from the off-line channel. As a result, E-tailers are expected to offer lower prices than multichannel retailers.

Therefore, prices pooled across sellers and time are regressed on observable seller heterogeneities: multichannel, rating, number of reviews received, and "trusted" sellers. The specific function form under test is

\[
\text{Price} = \beta_0 + \beta_1 \text{Multichannel} + \beta_2 \text{Rating} + \beta_3 \log(\text{Review}) + \beta_4 \text{Trusted} + \varepsilon,
\]

where \text{Multichannel} is a dummy variable equal to 1 if the price is charged by a multichannel retailer.

In addition, 38 product dummies and 19 time dummies are used to control for unobservable product heterogeneities and time trend effects. The regression results are reported in Table 3.
Model 1 is ordinary least square estimation with robust standard errors clustered by seller. The *Multichannel* dummy is significantly positive. On average, the multichannel retailers charge $28 more than the e-tailers for identical items. Although observable seller heterogeneities of channel use, rating, number of reviews, and "trusted" sellers are controlled in Model 1, unobservable seller heterogeneities may exist and have significant effect on pricing. Therefore, to take precaution against these unobservable seller heterogeneities, Model 2 estimates the data using generalized least square random-effects (mixed) estimator and Model 3 uses maximum likelihood random-effects estimator. In addition, to correct the potential serial correlation in the error term, Model 4 uses generalized least square random-effects estimator when the disturbance term is first-order autoregressive. Finally, Model 5 estimates the data with feasible generalized least square estimator in the presence of first-order autocorrelation within panels and cross-sectional correlation and heteroskedasticity across panels. The estimate of *Multichannel* is robust to all the above specifications; the multichannel retailers are expected to charge $28 more than e-tailers.

In addition, the number of reviews and "trusted’ dummy tend to be significant after heteroskedasticity is addressed. Surprisingly, rating of seller has no noticeable effect on the prices the seller charges.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(6.584)***</td>
<td>(3.595)***</td>
<td>(3.540)***</td>
<td>(3.450)***</td>
<td>(0.552)***</td>
</tr>
<tr>
<td><em>Rating</em></td>
<td>-2.25336</td>
<td>-1.248</td>
<td>-1.253</td>
<td>-0.930</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>(4.565)</td>
<td>(3.034)</td>
<td>(2.988)</td>
<td>(2.915)</td>
<td>(0.472)</td>
</tr>
<tr>
<td>Log(review)</td>
<td>-2.725</td>
<td>-0.877</td>
<td>-0.879</td>
<td>-0.985</td>
<td>-2.73</td>
</tr>
<tr>
<td></td>
<td>(1.367)**</td>
<td>(0.977)</td>
<td>(0.962)</td>
<td>(0.940)</td>
<td>(0.154)***</td>
</tr>
<tr>
<td><em>Trusted</em></td>
<td>13.382</td>
<td>2.625</td>
<td>2.64</td>
<td>2.195</td>
<td>8.011</td>
</tr>
<tr>
<td></td>
<td>(6.279)**</td>
<td>(4.363)</td>
<td>(4.297)</td>
<td>(4.200)</td>
<td>(0.651)***</td>
</tr>
<tr>
<td>Observations</td>
<td>10516</td>
<td>10516</td>
<td>10516</td>
<td>10516</td>
<td>10430(^{14})</td>
</tr>
</tbody>
</table>

*Note: t statistics in parentheses. * significant at 10%, ** significant at 5%, *** significant at |

\(^{14}\) 86 observations are dropped because only 1 observation is in the group.
Do multichannel retailers have wider pricing intervals?

Although there is strong evidence that multichannel retailers are charging higher prices than e-tailers, this finding is also consistent with the predictions in the literature on differentiated products. In that literature, multichannel retailers are believed offering better services and thus have higher prices in the resulted equilibrium. What distinguishes this paper from models of differentiated product is this paper predicts multichannel retailers price over wider intervals than e-tailers. When customers pay multichannel retailers more for their quality service, it only increases the price levels by multichannel retailers but has no effect on price dispersion. That is, prices by the multichannel retailers and those by e-tailers should match each other after being de-meaned. However, the results obtained from the analysis in this paper suggest that multichannel retailers have to price over wider intervals because they are caught in the crossfire between e-tailers and traditional retailers.

First note that the mixed strategy equilibrium derived from the one-shot game can be easily extended to repeated games if there is no significant change in the market structure. In the repeated game, the different types of sellers will just repeat their pricing strategies as in Proposition 5 because their decisions do not change the market structure in the next period.

Since the data period is as short as two and half months, no significant changes in reservation prices and costs are expected. As illustrated in Figure 2, the traffic at Shopping.com did fluctuate noticeably during the period. However, there was no trend and the theoretical model allows for change in the size of online switchers as long as the proportion of switchers and the relative size of customers online and off-line are not changed.

Since the market structure tends to be stable during the period, the sellers repeat the same mixed pricing strategies over time. Therefore, prices of the same product by the same seller are pooled across time to generate the distribution of the mixed strategy pricing. The empirical range of the prices by a seller is treated as the interval over which it mixes prices and used to test if multichannel retailers have wider ranges than e-tailers.

By Proposition 5, pricing range of the multichannel retailer \( R_m = V - p = \frac{(l_f+s_f)s_f+l_fs_n}{(l_f+s_f)^2+l_f(l_n+s_n)} V \) and range of the e-tailer is \( R_e = p^* - p = \frac{l_fs_n}{(l_f+s_f)^2+l_f(l_n+s_n)} V \). Therefore, it follows \( \ln(R_m) - \ln(R_e) = \).
(l_{f+s}^{f} l_{f+s}^{s}) > 0. Also, to control for the potential effects of observable seller heterogeneities on pricing range, the empirical range is regressed on channel use, rating, number of reviews received, and "trusted" sellers\textsuperscript{15}:

\[
\text{Log}(\text{Range}) = \beta_0 + \beta_1 \text{Multichannel} + \beta_2 \text{Rating} + \beta_3 \text{Log(Review)} + \beta_4 \text{Trusted} + \varepsilon,
\]

Again, 38 product dummies are used to control for unobservable product heterogeneities and the Huber-White “sandwich estimator” is used to estimate the robust standard errors in this and all the following estimations. The estimate of Multichannel is positive and significant at 10\% level, implying there is significant evidence that multichannel retailers are using wider pricing ranges than e-tailers. Rating and Trusted have negative effects on logarithm of the empirical range but are statistically insignificant. Log(Review) has significant positive effect on Log(Range).

One potential concern of the above estimation is that the Multichannel dummy tends to be endogenous. Recall that Multichannel represents the difference in the logarithm of range between

\textsuperscript{15}After pooling across time, we end up with a cross-section data set, and thus cannot use random effect models or autoregressive models. Also, time dummies are not relevant in the new data set.
multichannel retailers and e-tailers. Theoretical prediction suggests the difference is a function of fractions of switchers and the relative size of the online channel to the off-line channel. These factors are left into error term in the regression and thus may be correlated with the Multichannel dummy.

However, note that these factors tend to be constant across time within each product and thus should be efficiently controlled by the product dummies. To test if such factors have been fully controlled by the product dummies, a proxy of such factors – product popularity rank at Shopping.com – was collected during the data period. The ranking was performed within the product category and thus is not comparable across products from different categories. To address this problem, the rank is divided by the total number of products in the category to derive a relative rank. Also note that inconsistency in relative ranks between categories can be somehow controlled by the product dummies included in regression\(^{16}\). Model 2 includes the average relative rank of the product in the regression. The average relative rank is not significant and the estimate of Multichannel is barely changed but still significant, implying there is no endogeneity problem.

Another concern is the efficiency loss when data are pooled over time. Seller pricing is not balanced in the data; some sellers priced more often than others during the data period. As a result, the empirical ranges of those who priced more often tend to be more accurate. However, in Model 1 and 2, all sellers are treated evenly; an empirical range estimated from 20 prices does not contribute to the regression more than one estimated from only 2 prices. This kind of weighting reduces the efficiency of the estimation based on the empirical ranges and also may exaggerate outliers’ effects and thus potentially bias the estimates.

To address this problem, sellers’ contribution to regression are weighted by how often they priced during the period. Model 3 reports the results after such weighting without controlling for the relative rank and Model 4 reports the results with controlling for the relative rank. The estimates of Multichannel are somewhat larger and more significant.

Table 4: Regression of Log(Range)

\(^{16}\)For example, product dummies can fully control the inconsistency if the relative rank in one category equals the relative rank in another category plus a category-specific constant.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multichannel</td>
<td>0.2032</td>
<td>0.2005</td>
<td>0.2115</td>
<td>0.2110</td>
</tr>
<tr>
<td></td>
<td>(0.1190)*</td>
<td>(0.1194)*</td>
<td>(0.1132)*</td>
<td>(0.1133)*</td>
</tr>
<tr>
<td>Rating</td>
<td>-0.1139</td>
<td>-0.1170</td>
<td>-0.0802</td>
<td>-0.0808</td>
</tr>
<tr>
<td></td>
<td>(0.1057)</td>
<td>(0.1055)</td>
<td>(0.1013)</td>
<td>(0.1013)</td>
</tr>
<tr>
<td>Log(review)</td>
<td>0.0811</td>
<td>0.0808</td>
<td>0.1198</td>
<td>0.1198</td>
</tr>
<tr>
<td></td>
<td>(0.0371)**</td>
<td>(0.0372)**</td>
<td>(0.0367)***</td>
<td>(0.0367)***</td>
</tr>
<tr>
<td>Trusted</td>
<td>-0.1537</td>
<td>-0.1509</td>
<td>-0.3728</td>
<td>-0.3724</td>
</tr>
<tr>
<td></td>
<td>(0.1453)</td>
<td>(0.1455)</td>
<td>(0.1453)**</td>
<td>(0.1454)**</td>
</tr>
<tr>
<td>Rank</td>
<td>10.9553</td>
<td>1.9560</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(24.7875)</td>
<td></td>
<td>(10.4929)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>631</td>
<td>631</td>
<td>631</td>
<td>631</td>
</tr>
</tbody>
</table>

*Note: t statistics in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%.

Therefore, the data collected from Shopping.com provide empirical support for the prediction that multichannel retailers charge higher prices and mix over wider intervals than e-tailers. This finding is consistent with the model with multiple channels in this paper and cannot be fully explained by theories of product differentiation.

**Conclusion**

This paper jointly studies two of the most important asymmetries in a homogeneous product market with two channels: the asymmetric channel use by sellers and the asymmetric fractions of customers who actively search and compare prices. The two asymmetries generate significantly different pricing strategies among the three types of sellers in the market.

Specifically, both traditional retailers and e-tailers mix prices, although traditional retailers charge higher prices than e-tailers because relatively fewer off-line customers search prices. Forced to balance their two channels, multichannel retailers strategically mix prices to compete with both traditional retailers and e-tailers simultaneously. In addition, multichannel retailers partially give up online price searchers to e-tailers and price less aggressively to not only remain competitive
along the two channels but also earn more profit. Traditional retailers, cornered by multichannel retailers, must depend heavily on their loyal customers to maintain their profits, and thus promote the least often and offer the smallest discounts.

Therefore, both price level and price dispersion are asymmetric in the online and off-line channels. The channel with relatively more searchers has lower prices but does not necessarily have less price dispersion.

Compared to extant literature on symmetric and asymmetric oligopolies in a homogeneous product market, this paper is unique in the sense that it captures the fact that most homogeneous product markets function in both online channel and off-line channel, and both buyers and sellers are asymmetric across the two channels. Compared to extant literature that models identical products from different types of sellers as differentiated products, this paper is distinctive because it allows a fraction of customers to treat identical products from different types of sellers as perfect substitutes.

The prediction of this paper is consistent with empirical observations. Data on 39 most popular consumer electronics products collected from a leading price comparison site in the U.S. support the conclusion that multichannel retailers who operate online and off-line at the same time charge higher prices and have wider pricing ranges than online-only e-tailers.

Appendix

Proof of Proposition 5

First, show the above equilibrium is the only solution to the asymmetric game:

By Proposition 4, \((p_t, \bar{p}_t) \cap (p_e, \bar{p}_e) = \emptyset\). And because \((p_t, \bar{p}_t) \cup (p_e, \bar{p}_e) = (p_m, \bar{p}_m)\), it is implied that \(p_t = \bar{p}_e\) or \(p_e = \bar{p}_t\). In addition, because \(d_t > d_m > d_e\), seller \(e\) and \(m\) may price more aggressively than seller \(t\). Therefore, \(p_e = p_m\) and thus \(p_e = \bar{p}_e\). Let \(p_t = \bar{p}_e = p^*\), then in the equilibrium \(\pi_e = p(l_e + s_e), \pi_m = p(l_f + s_f + l_m + s_m)\) and thus \(\pi_e = \frac{l_e + s_e}{l_f + s_f + l_m + s_m} \pi_m\).

Over interval \([p^*, V]\), \(\Pi_e(p^*) = l_f V\) implies \(l_f V = p^* \left( l_f - \frac{s_f}{s_n} l_n \right) + \frac{s_f}{s_n} \pi_e\). Plug in \(p^* = \frac{\pi_m}{l_f + s_f + l_m}\)

17 Note that \(p_t = \bar{p}_e\) cannot be true. Otherwise, seller \(e\)'s expected profit in equilibrium is \(l_e V\), which is not optimal because it can generate more profit by pricing anywhere between \(d_e\) and \(d_m\).
and \( \pi_e = \frac{t_n + s_n}{l_f + s_f + t_n + s_n} \pi_m \), solve \( \pi_m = \frac{\left( l_f + s_f + t_n + s_n \right) \left( l_f + s_f + t_n \right) l_f V}{\left( l_f + s_f \right)^2 + l_f \left( t_n + s_n \right)} \). It follows \( p = \frac{\left( l_f + s_f + t_n \right) l_f V}{\left( l_f + s_f \right)^2 + l_f \left( t_n + s_n \right)} \), and all other results follow.

Then, check the validity of the above cumulative distribution functions as pricing strategies in equilibrium:

a). Seller \( t \)'s profit

When pricing at \( p \in [0, p) \), seller \( t \)'s expected profit

\[
\Pi_t(p) = p(l_f + s_f) < p(l_f + s_f) = \frac{(l_f + s_f)^2 + l_n(l_f + s_f)l_f V}{\left( l_f + s_f \right)^2 + l_f \left( t_n + s_n \right)} < \pi_t \text{ when } \frac{s_n}{l_n} > \frac{s_f}{l_f}.
\]

When pricing at \( p \in [p, p^*) \),

\[
\Pi_t(p) = p \left( l_f - \frac{s_f l_n}{s_n} \right) + \frac{s_f l_n + s_n - s_f l_n}{s_n \left( l_f + s_f \right)^2 + l_f \left( l_n + s_n \right)} \left( l_f + s_f + l_n \right) l_f V = p^* \left( l_f - \frac{s_f l_n}{s_n} \right) + \frac{s_f l_n + s_n - s_f l_n}{s_n \left( l_f + s_f \right)^2 + l_f \left( l_n + s_n \right)} \left( l_f + s_f + l_n \right) l_f V = \pi_t
\]

When pricing at \( p \in [p^*, V] \), \( \Pi_t(p) = pl_f + p s_f \left( l_f \left( \frac{V}{p} - 1 \right) \right) = \pi_t \).

When pricing at \( p \in (V, \infty) \), \( \Pi_t(p) = 0 \) because no customer buys above the reservation price.

Therefore, seller \( t \)'s expected profit is constant over interval \([p^*, V]\), and is reduced when it prices out of this interval.

b). Seller \( e \)'s profit

When pricing at \( p \in [0, p) \), seller \( e \)'s expected profit

\( \Pi_e(p) = pl_n + s_n \langle p, l_n + s_n \rangle = \pi_e \).

When pricing at \( p \in [p, p^*) \), \( \Pi_e(p) = pl_n + s_n \left\{ \left[ \frac{\left( l_n + s_n \right) \left( l_f + s_f + l_n \right) l_f V}{s_n \left( l_f + s_f \right)^2 + l_f \left( l_n + s_n \right)} \right] p^{-1} - \frac{l_n}{s_n} \right\} = \pi_e \).
When pricing at $p \in (p^*, V]$,

$$
\Pi_e(p) = p \left( l_n - \frac{l_f s_n}{s_f} \right) + \frac{s_n l_f V}{s_f} < p^* \left( l_n - \frac{l_f s_n}{s_f} \right) + \frac{s_n l_f V}{s_f} = \pi_e
$$

When pricing at $p \in (V, \infty)$, $\Pi_e(p) = 0$ because no customer buys above the reservation price. Therefore, seller $e$’s expected profit is constant over interval $[p, p^*]$, and is reduced when it prices out of this interval.

**c). Seller $m$’s profit**

When pricing at $p \in [0, p)$, seller $m$’s expected profit

$$
\Pi_m(p) = p(l_f + s_f + l_n + s_n) < p(l_n + s_n) = \pi_m.
$$

When pricing at $p \in [p, p^*)$, $\Pi_m(p) = p(l_f + s_f + l_n) + ps_n (1 - F_e(p)) = \pi_m$.

When pricing at $p \in (p^*, V]$, $\Pi_m(p) = p(l_f + l_n) + ps_f (1 - F_t(p)) = \pi_m$.

When pricing at $p \in (V, \infty)$, $\Pi_m(p) = 0$ because no customer buys above the reservation price. Therefore, seller $e$’s expected profit is constant over interval $[p, V]$, and is reduced when it prices out of this interval.

At last, note that although there is a mass point at the reservation price in the distribution of pricing by seller $t$, no tie actually happens because the probability for seller $m$ to price at $V$ is zero. Also observe that $F_m(p)$ is continuous at $p^*$. 

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<thead>
<tr>
<th>ID</th>
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<tbody>
<tr>
<td>1</td>
<td>Apple iPod (30G, MA146LL/A)</td>
<td>Apple</td>
<td>Media Player</td>
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<td>DataTraveler I 1G USB2.0</td>
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<td>17</td>
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<td>Lumix DMC-TZ1 Digital Camera</td>
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<td>VX2025wm 20.1 inch LCD</td>
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</table>
References


[19] Pan, Xing, Brian Ratchford, and Venkatesh Shankar (2002), "Can Price Dispersion in Online Markets Be Explained by Differences in E-tailer Service Quality?" *Journal of the Academy of Marketing Science*


[22] Pan, Xing, Brian Ratchford, and Venkatesh Shankar (2004), "Price Dispersion on the Internet: a Review and Directions for Future Research," *Journal of Interactive Marketing*


