Tight Money, Real Interest Rates, and Inflation in Sub-Saharan Africa

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June 2001
Active markets for short-term government debt now exist in much of Sub-Saharan Africa (SSA). This positive development has been associated with some interesting and controversial experiments in monetary policy. In different periods, the central banks of Kenya, Zambia, Zimbabwe, The Gambia, Ghana and Nigeria have financed more of the fiscal deficit through bond sales on the assumption that slower money growth alone would reduce inflation. These attempts to exercise independent tight monetary policy have produced mixed and at times disastrous results (see Table 1). In some episodes, inflation and the real interest rate have decreased in the short/medium run. But when Kenya reduced monetization of the deficit in 1993, the real interest rate shot up to 15-40%, the shilling depreciated 85%, and the inflation rate jumped from 34% to 67%. Remarkably, the numbers were worse a bit further to the south; tight money triggered even larger increases in the real interest rate and more extreme inflationary pressures in Zambia in the same year.

Existing theory is of limited help in understanding the strange effects of tight money in SSA. The potentially relevant literature consists of Calvo’s work on repudiation risk and a small set of papers that analyze the consequences of slower money growth in highly stylized, closed-economy models. The central hypothesis in Calvo (1988) is that high real interest rates stem from fears of debt repudiation and that such fears may prove self-fulfilling even when policy is controlled by a benevolent government. On first acquaintance, this hypothesis seems attractive. It runs into problems, however, when confronted with the large, erratic fluctuations of the real interest rate seen in different tight-money episodes: maybe the real interest rate in Kenya fell sharply between July 1993 and November 1993 because the public judged the government to be more trustworthy at the latter date; but surely the more natural explanation is that the tight-money policy had been abandoned two months earlier. Moreover, the perception that high real interest rates have discouraged private investment and strongly influenced capital flows is wrong if the high rates reflect only high risk premia; since treasury bills are just junk bonds, their high yields should not disturb the capital account or adversely affect the incentives for private capital accumulation. This and the preceding point suggest that temporary, high real interest rates are rooted in the fundamentals of the adjustment process rather than abnormally large risk premia.

The other place to look for insights is in the developed country literature on “monetarist
arithmetic.” Sargent and Wallace (1981) argued in a seminal paper that tight money is an ill-advised policy: at best, it trades lower inflation in the short run for higher inflation in the long run; at worst, it leads to higher inflation in both the short and the long run. The explanation for this paradoxical conclusion is that tight money must eventually give way to loose money when expenditure cuts or tax increases are not introduced concurrently to lower the primary fiscal deficit. In the absence of supporting adjustments in expenditure or taxes, slower money growth leads to more bond sales, higher interest payments on government debt, and progressively larger fiscal deficits. If there is an upper limit to the amount of government bonds the public can absorb, slow money growth will be succeeded by rapid money growth and higher inflation in the future. The anticipation of higher inflation may then provoke a flight from real money balances that drives up inflation immediately after the announcement of a new tight-money policy. This result has come to be known as the tight-money paradox.

While monetarist arithmetic makes the key point that tight money may worsen the fiscal deficit and immediately exacerbate inflationary pressures, it fails to explain the volatile responses of other important macroeconomic variables. The models employed by Sargent and Wallace (1981), Liviatan (1984), and Drazen (1985) assume a closed economy and are structured so that the real interest rate is constant not only across steady states but also on the transition path. These are major limitations. Closed-economy models miss the potentially destabilizing feedback effects running from higher inflation to capital flight to depreciation of the real exchange rate and greater outlays to service the public sector foreign debt.\(^1\) The assumption of a constant real interest rate is badly at variance with the facts and rules out spectacular failures of the sort seen in Kenya and Zambia, where the macroeconomy spiraled out of control when high real interest rates, rapid debt accumulation and accelerating inflation proved mutually reinforcing.

In this paper I analyze the effects of tight monetary policy in an optimizing, currency substitution model of a small open economy that operates under an open capital account and a flexible exchange rate. The interactions in the model between the exchange rate, capital flows, the real interest rate, and the fiscal deficit generate rich dynamics that shed light on (i) why tight money (TM hereafter) has provoked stupendous increases in inflation and the
real interest rate in some episodes; (ii) whether TM is a foolish, unsustainable policy that always worsens the fiscal deficit and raises the inflation rate in the long and (iii) the real costs of TM policies that fail badly — i.e., the impact of temporary, very high real interest rates on capital accumulation, employment and real output. The paper’s most important results, previewed below, speak directly to these three issues:

- Open economy factors are critical when trying to understand the magnitude of the increase in inflation in failed TM episodes. In the closed economy models of Liviatan (1984) and Drazen (1984), inflation rises steadily over time and arrives at its higher long-run level without a jump. In the open economy, by contrast, inflation typically overshoots its steady-state level. Moreover, overshooting is often extreme: in many cases, inflation is 2-10 times higher than its steady-state level at the end of TM phase. When domestic and foreign currency are sufficiently close substitutes, this produces an ultra-strong form of the TM paradox: inflation is continuously higher and rises far above the level that prevails after larger fiscal deficits are fully monetized.

- When the elasticity of intertemporal substitution is low (< .30) and domestic and foreign currency are close substitutes, TM has catastrophic effects that recall the experiences of Kenya and Zambia in 1993: the real interest rate rises 10-40 percentage points, and inflation and the internal debt increase at explosive rates. The perverse dynamics are compatible with normal values for the elasticity of money demand with respect inflation and the interest rate.

- Monetarist arithmetic is not always unpleasant. On the transition path, extra seigniorage and fiscal gains from transitory decreases in the real interest rate and temporary appreciation of the real exchange rate may enable the government to permanently reduce inflation by paying down the internal debt. This outcome — pleasant monetarist arithmetic — occurs in a sizeable part of the relevant parameter space. It constitutes, therefore, a serious challenge to the conventional wisdom that TM cannot work unless it is coordinated with measures to reduce the primary fiscal deficit. Attempting to beat down inflation with TM alone is risky but not foolish.

- The form of the policy rule is important. TM is much less likely to succeed if the government commits only to monetize a smaller share of the deficit. Pleasant monetarist arithmetic then disappears or is confined to a small part of the parameter space that requires domestic and foreign currency to be much closer substitutes than when the policy rule fixes the growth rate of the money supply.

- TM does not affect real output in the long run when the real wage adjusts to clear the labor market. But if the policy drives the real interest rate up to the levels seen in the Kenyan and Zambian episodes, the path back to the initial level of output entails a deep and protracted recession.
The adjustment process exhibits hysteresis when the real wage is rigid in one or more sectors; TM then has permanent, and in some cases quite large, effects on employment, the capital stock and real output.

The paper is organized into nine sections. Section 1 is devoted to capsule summaries of the most interesting and important tight money episodes. Following this, I show in sections 2-6 that there is a reasonably good fit between the stylized facts in these episodes and the dynamics generated by a simple open-economy model that assumes real output is fixed in the tradables and nontradables sectors. Sections 7 and 8 extend the model to incorporate sector-specific capital accumulation and either temporary or permanent real wage rigidity. The final section contains concluding remarks.

1. Capsule Summaries of Selected Tight Money Episodes

Table 1 provides a broad overview of the outcomes in different TM episodes but is short on specifics. The ensuing summaries of the episodes in Kenya, Nigeria and Zambia provide more detailed accounts of TM policies and their impact on the major macroeconomic aggregates.

Kenya: March 1993-August 1993

Extensive open-market operations were used to implement very tight monetary policy between March and August 1993. Reserve money growth fell from 53% in 1992 to just 5.6% in the first half of 1993. (The decreases in the growth rates of M1 and M2 were similarly large.) Following the switch to tight money (TM), the nominal interest rate on 91-day treasury bills jumped from 20% in February to 37% in March, 69% in May, and 85% in June and July. The annual inflation rate rose from 33.7% in 1992 to 66.8% for January-August 1993, while depreciation of the shilling against the dollar increased from 29% to 83%. In the last four months of the year, after the TM policy was abandoned, the exchange rate stabilized, the annual inflation rate dropped to 33%, and the nominal interest rate on T-Bills declined to 39% (in December). The current account balance improved substantially, registering a $153mn. surplus (vs. a $97mn. deficit in 1992). The capital account also improved, but most of the improvement occurred after the TM policy ended. Liberalization of foreign exchange controls coincided with large capital inflows in the first quarter of the year, but then, as interest rates on T-Bills rose from 45% to 85%, capital outflows reduced...
the government’s foreign exchange reserves from K£1255mn. in April to K£528 in August. The policy reversal in late August was associated with large capital inflows that boosted foreign exchange reserves to K£1641 by the end of the year. Finally, high interest payments on the internal debt drove the fiscal deficit up from 1.9% to GDP in FY1991/92 to 6.5% in FY1992/93 and 5.9% in FY1993/94.


Kenya: January 1999-September 1999

Nominal reserve money decreased 6.8% in the first three quarters of 1999 following a slight drop of .4% in 1998. The nominal interest rate on 91-day T-Bills decreased from 11.1% in December 1998 to 9.3% in April 1999, and then rose to 13.4% in June and 16.7% in September. Inflation increased from 2.5% in 1998 to 7.2% for Jan.-Sept. 1999. (The period-average inflation rate increased much less, from 7% to 7.8%.) The current account deficit decreased by two percentage points of GDP, but the capital account worsened and official foreign exchange reserves decreased by $100mn. After appreciating slightly in 1998, the shilling depreciated 24.4% against the dollar from January to September.

Sources: Monthly Economic Review (Central Bank of Kenya) and International Financial Statistics.

Zambia: January 1993-August 1993

An auction of treasury bills was introduced in January 1993. Shortly afterward, the government decided to drastically reduce monetization of the fiscal deficit. Although inflation in 1992 was 165%, planned growth for M2 was only 22%. TM was accompanied by a loosening of exchange controls that effectively reduced the cost of holding illegal foreign currency. The exchange rate system was a managed float.

Interest rates and inflation soared. The nominal rate on 91-day T-Bills rose from 64% in December 1992 to 260% in March 1993 and 347% in July 1993. The real stock of internal debt increased 25% in the first quarter of 1993, and “real” interest rates of 50-100% prevailed
until mid-year.\(^3\) The annualized monthly inflation rate quickly surged above 200% and by June stood at 490%.\(^4\) (Inflation fell to zero in October 1993 after adoption of a cash budget forced severe expenditure cuts.) The real exchange rate appreciated strongly, but the impact on capital flows was unclear. According to some accounts, high interest rates attracted large capital inflows. Adam reports (1995, fn.6), however, that some current account transactions served as vehicles for extensive capital flight through mid-1993. The sharp decrease in central bank foreign exchange reserves from $355mn. at the end of 1992 to $170 n. in the second quarter of 1993 also raises doubts about the size and the sign of capital flows during the TM period.\(^5\)

Sources: Adam (1995) and internal IMF documents.

**Nigeria: 1989-1992**

Following the reflationary budget of 1988, the government switched abruptly to a tight monetary policy. Over the next three years, bond sales financed 65-80% of the fiscal deficit. In January 1989, the official and interbank markets were merged and the exchange rate determined by daily bids at a central bank auction. A parallel market persisted, however, because of unsatisfied demand for foreign exchange. (The central bank auction was restricted to authorized dealers.) Interest rates on government debt were fixed administratively until November 1989, at which time an auction-based market in T-Bills and certificates commenced.

Reserve money growth slowed from 42% in 1988 to 31% in 1989, inflation decreased slightly from 49% to 30%,\(^6\) and depreciation of the currency (the naira/dollar rate) increased from 29% to 43%. The current account shifted from a deficit of $181mn. to a surplus of $1.1bn., but the capital account worsened by over $500mn. Real interest rates were highly negative throughout the year.

In 1990, the capital account balance worsened by $1.7bn., but high oil prices brought on by the Gulf War produced a large overall payments surplus.\(^7\) Unsterilized foreign exchange reserves allowed the stock of high-powered money to grow 34%, while the rate of currency depreciation slowed to 18% and the inflation rate dropped to 3.5%. The continued reliance on bond sales to finance the fiscal deficit drove the real T-Bill and prime lending rates up to 11% and 18.5% respectively.
Central bank intervention secured a reduction in the rate of currency depreciation to 9.6% in 1991. A rising fiscal deficit, however, increased the growth rate of reserve money to 42%. The inflation rate jumped to 23% and speculation against the naira pushed the premium in the parallel market to 100% by the end of the year. Fiscal pressures and concern about high lending rates led to the reimposition of controls on interest rates.

Fiscal and balance of payments problems continued to worsen in 1992. The exchange rate system switched from being a heavily managed to a nearly clean float as the central bank withdrew from the foreign exchange market. Although the current account improved, large capital outflows caused the naira to depreciate 99% against the dollar. The inflation rate continued its upward march, rising from 23% in 1991 to 49% in 1992 and to 61% in 1993.


It is clear from these episodes that there are strong interactions between capital flows, the real exchange rate, inflation and real interest rates. The diversity of outcomes, however, is bewildering. Apart possibly from the Zambian case, there is no support for the textbook story that TM raises real interest rates and attracts capital inflows that appreciate the currency. All sorts of other combinations appear in the data; the only general guideline for theory is that TM should be analyzed in an open-economy setting.

2. A Simple Currency Substitution Model

The economy produces a nontraded good and a composite traded good. World prices equal unity, so the domestic price of the tradable good is set by the nominal exchange rate $e$. The capital account is open and the private sector holds three financial assets: domestic currency $M$, foreign currency $F$, and indexed treasury bonds $B$. The consumer price index is $P$, and $m \equiv M/P$, $b \equiv B/P$, and $v \equiv e/P$ denote real holdings of domestic currency, the real stock of bonds, and the real exchange rate. Real consumption expenditure $E$ is measured in units of the traded good and $\gamma$ is the consumption share of the nontraded good at current prices. $P$ is a geometric weighted average of the prices of the two goods; expressed in terms of $e$...
and the relative price of the nontraded good $P_n$,\(^9\)

$$P = eP_0^n, \quad 0 < \gamma < 1.$$ \hfill (1)

All economic activity in the private sector is undertaken by a representative agent who possesses an instantaneous utility function of the form $V(P_n, E) + \phi(m, vF)$. $V(\cdot)$ is a standard indirect utility function that measures utility from goods consumption, while $\phi(\cdot)$ reflects liquidity services generated by holdings of domestic and foreign currency. The private agent chooses $m, b, F$ and $E$ to maximize

$$U = \int_0^\infty [V(P_n, E) + \phi(m, vF)]e^{-\rho t}dt,$$ \hfill (2)

subject to the wealth constraint

$$A = m + b + vF$$ \hfill (3)

and the budget constraint

$$\dot{A} = v(Q_T + P_nQ_n) + g + rb + (\chi - \pi)vF - \pi m - vE(1 + z),$$ \hfill (4)

where $\rho$ is the time preference rate, $Q_i$ is output in sector $i$, $g$ is lump-sum transfers, $r$ is the real interest rate, $\chi \equiv \hat{e}/e$ is the rate of currency depreciation, $\pi \equiv \hat{P}/P$ is the inflation rate, and $z$ is a consumption-based value added tax. The output variables in (4) are constant. This assumption will be relaxed in section 7 when the model is extended to include capital accumulation.

The optimal choices for the three assets and consumption spending satisfy the first-order conditions

$$V_E = v\beta(1 + z),$$ \hfill (5)

$$\phi_m = \phi_f + \beta\chi,$$ \hfill (6)

$$\frac{\phi_f}{\beta} + \chi = r + \pi,$$ \hfill (7)

and the co-state equation

$$\dot{\beta} = \beta(\rho - r),$$ \hfill (8)
where $\beta$ is the multiplier attached to the budget constraint (4). Equation (5) is a variant of the familiar condition that the shadow price of wealth $\beta$ should equal the marginal utility of consumption $V_E$. The term $1 + z$ captures the consumption tax paid to the government, and $v$ enters because the deflator for consumption ($c$) differs from the deflator for wealth ($P$).

The other two first-order conditions are straightforward arbitrage conditions. The total return on foreign currency is $\chi$, the percentage depreciation of the currency, plus $\phi_f/\beta$, the value of additional liquidity services. Domestic currency yields only liquidity services $\phi_m$ and government bonds pay the nominal interest rate $r + \pi$. Equations (6) and (7) thus require equal returns on the three assets.

2.1 Tight Monetary Policy and Bond-Financed Fiscal Deficits

The exchange rate is determined entirely by market forces. Since the government does not buy or sell foreign currency, the change in the money supply equals the change in the domestic credit component of the monetary base. To economize on notation, I assume the central bank holds no foreign exchange reserves. This makes the stock of central bank credit to the government one and the same as the money supply.

In the initial steady-state equilibrium, seigniorage pays for the entire fiscal deficit, the real interest rate equals the time preference rate, and the real money supply and the real stock of bonds are constant. That is

$$\mu_o = \pi_o \quad \text{(Pre-TM phase)}$$

and

$$\pi_o m_o = g + \rho b_o + v_o X - z v_o E_o \quad \text{(pre-TM phase)}$$

where $\mu$ is nominal money growth and an o subscript indicates the initial value of a variable (omitted for variables that do not change). The term $X$ on the right side of the government budget constraint represents either revenue from a state export monopoly ($X < 0$) or interest payments on the public sector foreign debt. When $X$ equals zero, changes in the real exchange rate are fiscally neutral. ($z v_o E_o$ is nominal tax revenues deflated by the CPI because, to repeat, $E$ is measured in dollars.)
When the government switches to a TM policy, nominal money growth decreases to $\mu_1$ and bond sales adjust as needed to cover the rest of the fiscal deficit. Hence, during the TM phase, $m$ and $b$ evolve according to

\[ \dot{m} = (\mu_1 - \pi)m, \quad 0 \leq t < t_1 \quad \text{(TM phase)} \] (9)

\[ \dot{b} = g + rb + vX - \mu m - zvE, \quad 0 \leq t < t_1. \quad \text{(TM phase)} \] (10)

At time $t_1$ bond sales/purchases cease and the deficit is fully monetized. From $t_1$ onward, therefore, $\dot{b} = 0$ and

\[ \dot{m} = g + rb(t_1) + vX - zvE - \pi m, \quad t \geq t_1. \quad \text{(Post-TM phase)} \] (11)

Following the practice in the literature on monetarist arithmetic, I assume the switch from loose to tight money at $t = 0$ catches the public by surprise. The subsequent policy reversal at $t_1$, however, is perfectly anticipated; the public realizes from the outset that the TM regime is not sustainable.

2.2 Capital Flows, the Current Account and the Market-Clearing Condition in the Nontradables Sector

The price of the nontraded good adjusts continuously to equate demand and supply. Since consumer purchases $D_n$ are the only source of demand, the market clears when

\[ D_n(P_n, E) = Q_n. \] (12)

Finally, the model is closed with the aid of Walras’ Law. Adding the budget constraints of the private sector and the government yields

\[ \hat{F} = P_n Q_n + Q_T - E - X, \] (13)

which says that capital outflows are paid for dollar-for-dollar by current account surpluses.

2.3 Issues Relating to the Specification of the Model

Before proceeding further, I would like to briefly discuss two aspects of the model. The first pertains to the exchange rate system. I have assumed that the government lets the market determine the exchange rate. This is certainly the right assumption for the Gambian
episode of 1986-88. It may also be approximately correct for Kenya circa 1993-94: although the monetary authorities operated a managed float, the exchange rate was allowed to depreciate/appreciate rapidly when capital flows put pressure on the balance of payments.\textsuperscript{10}

In other TM episodes, the government intervened more strongly in foreign exchange markets and the exchange rate system was closer to a crawling peg or a dirty float. I ignore such cases in the present paper because they represent a different policy experiment. In a crawling peg or manged float, TM is paired with some rule for adjustment of the exchange rate; this brings into play the entire literature on speculative attacks and exchange-rate-based stabilization programs. The assumption of a clean float is essential therefore to a conceptually pure analysis aimed at isolating the effects of TM from the effects of other policies. The disadvantage of taking the theory-pure approach is that it is hard to account for some aspects of the empirical record without analyzing the case where slower money growth is combined with temporary manipulation of the exchange rate. I will return to this point later in section 6.3.

The other feature of the model which merits a few words is the assumption that the foreign asset is foreign currency (i.e., some asset that yields liquidity services) rather than a foreign bond. This assumption is needed to escape the interest parity condition. In an optimizing, perfect-foresight model with an open capital account, domestic and foreign bonds are perfect substitutes, and consequently the domestic real interest rate differs from the foreign rate only by the percentage depreciation of the real exchange rate. But this does not sit well with the empirical evidence. There are too many examples where the real interest rate on government debt and the real exchange rate move in opposite directions. And even when the two variables move in the same direction, anticipated depreciation of the real exchange rate cannot explain the wild gyrations of the real interest rate in many TM episodes.

3. A Brief Sketch of the Solution Procedure

The analysis of TM has the unusual twist that the solution for the steady-state equilibrium is not independent of the solution for the transition path. The two solutions have to be derived jointly because the long-run effects on inflation and real holdings of domestic and
foreign currency depend on the amount of debt the economy inherits at the end of the TM phase, and vice versa. Since the algebra is dense and unpleasant, I skip over most of the details and simply outline the logic of the solution procedure. The full mathematical solution is relegated to the appendix.

The first step in the solution procedure is to eliminate the unobservable shadow price $\beta$ from the dynamic system. This can be accomplished by differentiating (5) with respect to time and substituting for $\dot{\beta}$ from (8). Straightforward manipulations then give (see the appendix)

$$\frac{\dot{E}}{E} - \gamma \frac{\dot{P}_n}{P_n} = \tau (r - \rho)$$

under the assumption of homothetic preferences. Observe that $P_n$ depends only on total consumption spending $E$. From (12),

$$\dot{P}_n = k \dot{E},$$

where $k \equiv (\eta + \gamma)^{-1}$, $\eta$ is the compensated own-price elasticity of demand, and a circumflex indicates a percentage change (i.e., $\dot{x} = dx/x$). Interpret the differentials in (15) as time derivatives and substitute for $\dot{P}_n/P_n$:

$$(1 - \gamma k) \dot{E} = \tau E (r - \rho).$$

During the TM phase the dynamics are governed by (9), (10), (13) and (16), a 4x4 system involving $m, b, F$ and $E$. To solve the system, we need to relate $\chi, \pi$ and $r$ to these variables. Working toward this end, write (6) and (7) as

$$\phi_m(m, vF) = \frac{(r + \pi) V_E(P_n, E)}{v(1 + z)},$$

$$\phi_f(m, vF) = \frac{(r + \pi - \chi) V_E(P_n, E)}{v(1 + z)},$$

and note from (1), (15) and (16) that

$$\pi = \frac{\gamma k}{1 - \gamma k} (r - \rho) + \chi.$$
Equations (6'), (7') and (17) can be solved for $\pi$, $\chi$ and $r$ as a function of $m$, $E$ and $F$:

$$
\pi = C^1(m, E, F), \quad r = C^2(m, E, F), \quad \chi = C^3(m, E, F). \tag{18a-18c}
$$

Equations (9), (10), (13) and (16), together with the above solutions in (18a)-(18c), then define a self-contained system of four differential equations in $m$, $E$, $F$ and $b$.

When bond-financing ends at time $t_1$ the system defined by (11), (13) and (16) takes over. To rule out perverse comparative statics results and ensure well-behaved dynamics, I assume the economy operates on the upward-sloping portion of the seigniorage Laffer curve where the elasticity of money demand with respect to inflation is less than unity.\footnote{Under this restriction, the system is saddlepoint stable.} From $t = 0$ up to time $t_1$, the economy follows a nonconvergent path of the system associated with the TM regime [the 4x4 system defined by (9), (10), (13) and (16)]. This path connects with the saddle path of the full-monetization regime that prevails after $t_1$. The general solution for the nonconvergent path in the period $[0, t_1]$ involves four constants; the saddlepoint solution for $t \geq t_1$ adds another constant. Six boundary conditions determine these five constants and the level of government debt at the end of the TM phase. The values of the predetermined variables $F$ and $b$ at $t = 0$ and $t = t_1$ provide four boundary conditions. The remaining two conditions can be derived from the fact that optimizing behavior requires the paths of $m$ and $E$ to be continuous after their initial jumps at $t = 0$. This implies that $m(t^-_1) = m(t^+_1)$ and $E(t^-_1) = E(t^+_1)$ — the solutions at the end of the TM regime must be the same as the solutions at the start of the full-monetization regime.

Once the solutions for $m(t)$, $F(t)$ and $E(t)$ are in hand, the paths for $r$, $\chi$ and $\pi$ can be retrieved via (18a)-(18c). At the new long-run equilibrium ($\bar{m}, \bar{E}, \bar{\pi}$, etc.), currency depreciation equals the inflation rate and real expenditure, the relative price of the nontraded good, the real interest rate, and the real exchange rate equal their initial values:

$$
\bar{E} = E_0, \quad r = \rho, \quad \bar{P}_n = P_{no}, \quad \bar{\pi} = \bar{\chi}, \quad \bar{v} = v_0.
$$

To prepare the model for calibration, I assume $\phi(m, vF)$ is a linearly homogeneous CES
function. In this case, the steady-state solutions for \( m, \pi \) and \( F \) are:

\[
\begin{align*}
\bar{m} - m_o &= -\frac{\rho \epsilon}{\pi(1 - \epsilon)}[b(t_1) - b_o], \\
\bar{\pi} - \pi_o &= \frac{\rho}{m(1 - \epsilon)}[b(t_1) - b_o], \\
\bar{F} - F_o &= \frac{(\sigma - \tau)\theta_m \rho F}{(\rho + \pi)m(1 - \epsilon)}[b(t_1) - b_o],
\end{align*}
\]

where \( \sigma \) is the elasticity of substitution between \( m \) and \( vF \); \( \theta_m \) and \( \theta_f \) are the shares of liquidity services provided by domestic and foreign currency and \( \epsilon = (\tau \theta_m + \sigma \theta_f)\pi / (\rho + \pi) < 1 \) is the elasticity of money demand with respect to inflation. The solutions in (19) and (20) indicate that inflation rises and real money balances fall if TM leads to the accumulation of more debt. Equation (21) says that higher inflation causes flight from domestic to foreign currency provided it is easier to substitute between the two currencies that to substitute intertemporally in consumption (i.e., \( \sigma > \tau \)).

4. Calibration of the Model

Even in this simple model, the dynamics are complicated enough to place analytical results out of reach. It is necessary, therefore, to calibrate the model and rely on numerical methods. In simulations for the Kenyan episode in 1993 and the Nigerian episode of 1989-1991, I set

**Kenya**: \( \eta = .25, \gamma = .50, \pi_o = .30, \rho = .10, X/GDP = .036, \)
\( z = .20, m/GDP = .08, F/GDP = .08, b/GDP = .31, \)
\( t_1 = .5,1, \tau = .10,.25, \sigma = 2 - 6, \)

**Nigeria**: \( \eta = .25, \gamma = .33, \pi_o = .45, \rho = .10, X/GDP = -.09, \)
\( z = .04, m/GDP = .08, F/GDP = .16, b/GDP = .13, \)
\( t_1 = 2,3, \tau = .10,.25, \sigma = 2 - 6. \)

The choice of parameter values was guided by a mixture of hard data and diverse empirical estimates:

- *Compensated elasticity of demand for the nontraded good* (\( \eta \)). The value for \( \eta \) agrees with the finding in empirical studies that compensated elasticities of demand tend to be small.
at high levels of aggregation.

- Consumption share of the nontraded good ($\gamma$). The value of $\gamma$ is .50 in the two Kenyan episodes. This is consistent with the share of non-government services in GDP (.50 in the 1992 National Income Accounts) and the allocation of weights in the country’s CPI.$^{13}$ In Nigeria, non-government services accounted for only 24.1% of GDP in 1988 (Moser, Rogers and van Til, 1997). But high tariffs and import restrictions protected most of the manufacturing sector from external competition.$^{14}$ The assigned value of 33% assumes that industrial output was effectively nontradable.$^{15}$

- Initial inflation rate ($\pi_0$). The period-average and end-of-period inflation rates differ by a nontrivial amount in the year preceding the TM episodes. I set the initial inflation rate equal to the average of the two.

- Time preference rate ($\rho$). For $\rho$, I chose the high value .10 because the only role the time preference rate plays in the model is to fix the real interest rate on government debt across steady states.$^{16}$

- Net sales/purchases of foreign exchange by the public sector ($X/GDP$). In theory, $X$ should be determined by decomposing all government expenditure and revenue into tradable and nontradable components. Lacking the data to do this, I set $X$ equal to interest payments on the foreign debt in the Kenyan episode and to government revenue from oil less interest payments on the foreign debt in Nigeria. [Sources: Ajayi (1997, Table 7) and Kenya Monthly Economic Review for Kenya; Moser, Rogers and van Til (1997, Tables 5 and 6) for Nigeria.]

- VAT tax ($z$). The value of $z$ (20%) is slightly higher than the standard VAT (18%) that prevailed in Kenya in 1992-93 (Central Bank of Kenya, 1993).$^{17}$ In the Nigerian case, $z$ is much smaller (4%) because revenue from indirect taxes was only 3.5-4.1% of aggregate consumption in 1987-88 (Economic and Financial Review 1990).

- Ratio of high-powered money to GDP ($m/GDP$). The sum of currency held by the public plus reserves of commercial banks was 7.9-8.1% of GDP in Nigeria from 1986-1988 and in Kenya from 1988-1991 (International Financial Statistics).$^{18}$

- Ratio of foreign currency to GDP ($vF/GDP$). Dollarization ratios (foreign currency deposits/M2) generally lie between 20% and 60% in Latin American countries experiencing moderate inflation (Savastano, 1996). But data from several sources suggest that holdings of foreign currency assets may be much larger in SSA. Cross-border deposits of the non-bank public were 7.2% of GDP in Nigeria in 1988 and an astounding 31.4% in Kenya in 1992 (International Financial Statistics). Ajayi (1997) estimates that in 1991 the stock of flight capital was 21-70% of GNP in Kenya and 103% of GNP in Nigeria (third highest figure in the sample of eighteen countries). Collier, Hoefller and Pattillo’s (1999) estimate for Nigeria is also huge (61% of a broad measure of private wealth that includes the value of the capital stock); puzzlingly, however, they do not detect any evidence of capital flight for Kenya.$^{19}$

While corruption, tax evasion and political instability may explain much of the capital flight, these huge figures argue that the stock of foreign currency assets responsive to
economic variables is large in Kenya and probably very large in Nigeria. Accordingly, the ratio of foreign to domestic currency is one in the Kenyan 1993 episode \((vF/GDP = .08)\) and two in the Nigerian episode \((vF/GDP = .16)\).20

- **Ratio of government debt to GDP \((b/GDP)\).** Central government debt held by the private sector was 13% of GDP in Nigeria at the end of 1988 [Economic and Financial Review (Central Bank of Nigeria)]. The internal debt in Kenya stood at 22% of GDP on June 30, 1992; a year later, the figure had risen to 31%. For technical reasons, simulations based on the latter value are more likely to provide an accurate approximation to the dynamics in the 1993 episode.22

- **Length of the TM regime \((t_1)\).** The Kenyan episode lasted six months. I also carry out runs for \(t_1 = 1\) to test the sensitivity of the results to longer-lived TM regimes.

The right value of \(t_1\) in the Nigerian episode depends on the specification of the policy rule. The IMF team headed by Moser, Rogers and van Til (1997) treats 1990 as the last year of the adjustment program. Since money growth was approximately constant in 1989 and 1990, this is consistent with \(t_1 = 2\) under a fixed money growth rule. But policy makers seemed to follow a rule of monetizing a smaller share of the fiscal deficit from 1989 until 1992: seigniorage financed 69% of the fiscal deficit in 1988, 28% in 1989, 29% in 1990, 34% in 1991, and 51% in 1992. These numbers and the account in Garba (1996) make a solid case that the TM episode lasted three years. I set \(t_1\) equal to three therefore in the simulations that assume money growth was determined by a deficit-share rule.

- **Elasticity of intertemporal substitution \((\tau)\).** Needless to say, there is considerable uncertainty about the true value of \(\tau\) in Kenya and Nigeria. Most estimates place the elasticity of intertemporal substitution somewhere between zero and .5 in LDCs.23 A value of .25 is in line with these estimates and the mean estimate for the poorest countries in Ogaki, Ostry and Reinhart (1996). This value, however, could be too high. Numerous estimates suggest that \(\tau\) might be only .10-.30 in the United States (Poterba and Singleton, 1987; Hall, 1988; Patterson and Pesaran, 1992; Dutkowsky and Foote, 1992; Deaton, 1992, p.73). In each episode, therefore, I compare outcomes for \(\tau = .25\) and \(\tau = .10\).

- **Elasticity of substitution between domestic and foreign currency \((\sigma)\).** There are no reliable estimates of \(\sigma\) for Kenya, Nigeria or most African countries. For Latin America, the estimates range from 1.5 to 8 (Ramirez-Rojas, 1985; Marquez, 1987; Giovannini and Turtleboom, 1992; Kamin and Ericsson, 1993). There is ample room for disagreement, however, about how much specification error, noisy data and other problems distort the estimates; that said, they accord with the general sentiment in the literature that currency substitution is strong and highly responsive to changes in expected inflation/depreciation (Camacho and Gonzalez-Vega, 1985; Mizen and Pentecost, 1996). In deference to this view and the range of empirical estimates, I allow \(\sigma\) to vary from 2 to 6.
5. Numerical Solutions

The numerical solutions for the Kenyan and Nigerian episodes are presented in sections 5.1 and 5.3. In the Kenyan case, credibility may be the difference between success and failure in a sizeable part of the parameter space. This point is developed in section 5.2.

5.1 Kenya 1993

Although the model is too complicated to solve analytically, it is not an impenetrable black box. It is fairly easy to trace the interactions that link asset demands to the paths of inflation, the real interest rate, and the fiscal deficit. The single most important variable is the capital account. When TM fails, inflation and the demand for foreign currency increase in the long run. At some point, therefore, the expectation of higher future inflation provokes capital flight. This is not incompatible with temporary capital inflows in response to a temporary decrease in inflation. But the pull of the long-run fundamentals usually overwhelms the effect of transitory, lower inflation. Consequently, from the very outset, two factors tend to increase the fiscal deficit. First, revenues collected from the VAT tax decline because in general equilibrium capital outflows are financed by current account surpluses and lower consumption spending. Second, since a cut in consumption spending raises the real exchange rate, capital flight indirectly increases real interest payments on the public sector’s foreign debt.

The adverse effects of lower tax revenues and higher external debt service will be compounded by an immediate increase in the real interest rate on internal debt if consumption rises monotonically after its initial downward jump. To see this, return to the Euler condition in (16) and put \( r \) by itself on the left side:

\[
r = \rho + \frac{1 - \gamma k \dot{E}}{\tau} \frac{\dot{E}}{E}. \tag{16'}
\]

As usual, the real interest rate exceeds \( \rho \) when \( E \) is rising and the marginal utility of consumption is falling. This relationship is depicted in Figures 1a and 1b. In Figure 1a, the initial downward jump in \( E \) is followed by further decreases that push \( r \) below \( \rho \). This phase comes to an end before \( t_1 \), so in the later stages of the TM regime the real interest rate climbs above its previous level. In Figure 1b, by contrast, consumption rises monotonically
and the real interest rate is higher throughout the TM regime. In both cases, when the policy collapses, consumption continues to increase but at a slower rate. The return to fully monetized fiscal deficits is accompanied therefore by steady decreases in the real interest rate. I should acknowledge here that I have not found a mathematical proof that the paths of $E$ and $r$ always conform to either Figure 1a or Figure 1b. The justification for drawing the figures is what Judd calls a “virtual proof”: other paths may exist, but they never materialized in the simulations generated for this section or in wide-ranging sensitivity tests.

The magnitudes of the various adverse fiscal effects depend principally on how much inflation increases across steady states and how strongly this raises the demand for foreign currency. Both the long-run outcome and the dynamics are highly sensitive therefore to $\sigma$, the elasticity of substitution between domestic and foreign currency. Obviously, any given increase in inflation induces more capital flight when $\sigma$ is large. Moreover, the larger is $\sigma$ the flatter is the slope of the seigniorage Laffer curve and the more inflation increases in the long run when the fiscal deficit rises. Thus, as $\sigma$ assumes higher values, the macroeconomic dynamics become more volatile and there is a greater risk that tight monetary policy will trigger explosive increases in the real interest rate, the inflation rate, and the fiscal deficit.

But this is not the whole story. Lower inflation is also a potential steady-state outcome.^{24} If the private sector expects TM to succeed, the fiscal effects change sign: on impact, expectations of lower inflation induce capital inflows and an upward jump in consumption spending; while consumption declines towards its original level, tax revenues are higher, and the real interest rate, the real exchange rate and external debt service are lower. Since the entire fiscal windfall is dedicated to reducing the internal debt, the saving in interest payments may lower the fiscal deficit enough to validate expectations of permanently lower inflation. But the parameter values have to be right for events to play out in this way; more specifically, the elasticity of substitution between domestic and foreign currency must be large so that the transitory fiscal gains and the paydown of the debt suffice to offset the loss in seigniorage. If this critical elasticity is large, but not quite large enough, the switch to tight money leads instead to rising inflation and extremely high real interest rates.

Turning now to the results, in Tables 2a and 2b the numbers under the columns headed
by $\pi(0), \pi(t_1), r(0)$ and $r(t_1)$ are the elasticities of inflation and the semi-elasticities of the real interest rate (i.e., $\frac{dr}{d\mu/a}$) at $t = 0$ and $t = t_1$, while those in the columns for $b(t_1)$ and $\pi_{ss}$ refer to the elasticity of debt at $t = t_1$ and the elasticity of steady-state inflation. All of the elasticities are calculated with respect to $\mu$ [money growth during the period $(0,t_1)$] and multiplied by -1 so that they take the same sign as the change in the variable. The table also shows how $\epsilon$, the elasticity of (domestic) money demand with respect to inflation, varies with $\sigma$. I report the value of $\epsilon$ to emphasize that normal values for this elasticity are compatible with quite large values of $\sigma$. The results would not provide a convincing explanation of the stylized facts if they required values of $\epsilon$ greater than .90 — if they exploited the extreme sensitivity of inflation to small changes in the fiscal deficit at equilibria close to the top of the seigniorage Laffer curve.

The most striking and robust result in the tables is that the elasticities for inflation at the end of the TM phase are very large in absolute terms and 2-10 times larger than the elasticities for steady-state inflation. This reflects the impact of TM on the real interest rate, real expenditure, and the real exchange rate. As explained earlier in connection with Figures 1a and 1b, at the end of the TM phase, revenue from the consumption tax is always lower and external debt service and the real interest rate are always higher. All of these variables are unchanged across steady states; consequently, the fiscal deficit and nominal money growth always exceed their long-run levels just after the policy reversal at $t_1$. Since the private sector anticipates this development, inflation overshoots its steady-state level while the TM policy is in force. The degree of overshooting is surprising and might be judged implausible were it not for the empirical record: TM was massively inflationary in Kenya and Zambia in 1993; and when the policy was abandoned, the inflation rate declined rapidly.

It is less clear whether the model successfully explains the impact of TM on the real interest rate. The solution grids contain some large semi-elasticities for $r(t_1)$, but they are limited to a narrow range of values for $\sigma$. In the case where $t_1 = .5$, for example, large increases occur only when $\sigma$ lies between 3.75 and 4.2 in Table 2a and between 3 and 3.65 in Table 2b. At first glance, therefore, the results suggest that the high real interest rates seen in the Kenyan and Zambian episodes are an unusual phenomenon confined to a small
part of the parameter space. This conclusion is specific, however, to the fixed money growth rule; it does not apply to the deficit-share rule analyzed in section 6. In addition, it ignores the possibility that high real interest rates themselves provoke the policy reversal — that for $\tau = .25$ the policy reversal occurs at $t_1 = 1$ when $\sigma = 3.5$, at $t_1 = .5$ when $\sigma = 4$, etc. On this interpretation, the parameter space consistent with high real interest rates expands to 3-4.7 in Table 2a (2.75-4.15 in Table 2b), the union of relevant parameter spaces for $t_1 = .25 - 4$.

5.2 Credibility, Outcome-Based Policy Rules and Multiple Equilibria

The penultimate row in each panel shows the critical value of $\sigma$ that divides the region where TM fails from the region where it succeeds. Since time is needed for cumulative fiscal gains on the transition path to paydown the debt enough to make TM sustainable, the borderline value of $\sigma$ decreases with $t_1$. This has important implications for policy, the most obvious being that the efficacy of TM may depend solely on credibility. To illustrate, suppose that $\sigma$ equals 3.5 in Figure 2. Pleasant monetarist arithmetic (PMA) is then the unique equilibrium if the government can precommit to slower money growth for at least 1.64 years. Success does not require perfect credibility, just enough to induce the private sector to coordinate on the PMA equilibrium.

This sounds too easy, and it is. The precommitment solution demands that the government abide by the TM policy irrespective of its consequences. The consequences, however, may be awful. Consider what happens when the reduction in money growth is 50% and, despite government pronouncements to the contrary, the public expects rapid money growth to resume after one year. In this case, the first year of the TM regime sees the real interest rise from 10% to 18% and the inflation rate soar from 30% to 84% (see Table 2a). Is there really any doubt that the commitment to TM would then go by the wayside? Policy makers are practical men, not theorists; when faced with brute facts which argue that a policy has failed, they are apt to side with the facts. Outcome-based policy rules are thus inherently fragile; by opening the door to multiple equilibria, they allow private sector beliefs alone to determine whether TM succeeds or fails.
5.3 Nigeria 1989-1991

Since the government was a large net seller of foreign exchange and its internal debt was only 13% of GDP in 1988, Nigeria enjoyed a fair degree of natural protection against explosive, unstable debt dynamics. This is reflected in the data. The inflation rate decreased in 1989 and 1990 (albeit with help from good harvests and tighter fiscal policy in the second year); it rose rapidly in 1991 and 1992 but on a trajectory consistent with undershooting of the new steady-state inflation rate. Real interest rates were low or negative until mid-1990, and the real rate on treasury bills peaked at 11% before controls were reimposed on bank rates in January 1991. TM failed in Nigeria, but the adverse macroeconomic effects materialized more slowly and were far less extreme than in Zambia and Kenya.

The numbers in Table 3 are broadly consistent with the outcome in the Nigerian TM episode. Inflation always decreases on impact, and undershooting occurs in 9 of the 10 cases, with \( \pi_{ss} \) being considerably larger than \( \pi(t_1) \) when \( \sigma = 3.5 - 4.2 \). The real interest rate decreases initially but is 1-6 points higher by the end of the TM episode.

While many parameter values differ in the simulations for Kenya and Nigeria, the one that matters most is \( X \), net sales of foreign exchange by the public sector. Inflation undershoots its long-run level in Table 3 because, unlike in the Kenyan case, depreciation of the real exchange rate greatly reduces the fiscal deficit. Lower revenue from consumption taxes and higher real interest rates near the end of the TM regime pull in the opposite direction, but these effects are comparatively weak when the internal debt is small (13% of GDP) and the consumption tax low (4%). Hence the fiscal deficit is usually below its steady-state level at time \( t_1 \). After the policy reversal, the real exchange rate appreciates until it regains its original level (\( \nu \) is unchanged across steady states). The deficit then increases steadily as the revenue losses from appreciation swamp the gains from falling real interest rates and collection of more consumption taxes (\( \hat{E} > 0 \) for \( t > t_1 \)). Since nominal money growth rises monotonically in the post-TM phase, so also does the inflation rate.

The fiscal impact of real appreciation explains another important aspect of the solutions, namely that there is little hope of achieving PMA. In the simulations for Kenya, PMA is a potential equilibrium when \( \epsilon = .6 - .99 \) because real appreciation, higher consumption spending, and lower real interest rates all work powerfully to reduce the fiscal deficit on the
transition path. But in the Nigerian case the gains conferred by lower real interest rates and higher consumption taxes have to overcome the losses caused by real appreciation. This is not easily done when the government’s net sales of foreign exchange amount to 9% of GDP. As a result, failure claims almost the entire parameter space — PMA is confined to a small segment of the seigniorage Laffer curve where $0.95 < \epsilon < 1$.

6. An Alternative Specification of the TM Rule

I have followed Sargent and Wallace in assuming that the government exercises perfect control over the growth rate of high-powered money during the TM regime. This may be unrealistic. From 1989-1991, the Nigerian government seemed to follow the rule that seigniorage was to finance no more than 35% of the fiscal deficit. In other cases, the intention to follow a fixed money growth rule gave way, ex post, to a deficit-share rule. Zambia lost control of the money supply almost immediately in its attempts to enforce tight monetary policy in both 1986 and 1993. In the 1993 episode, for example, the government declared that the growth rate of the money supply would be 22%. But when real interest payments on government debt increased 285% in the first quarter of the year, the 22% figure proved impractical. Actual money growth for the year turned out to be 87%; in retrospect, it seems clear that the commitment to TM was only a commitment to monetize a smaller share of the deficit.

Adopting this alternative specification, let $\alpha$ denote the fraction of the fiscal deficit financed by printing money. During the TM phase, the nominal money supply ($M$) and the nominal stock of bonds ($B$) increase by

$$\frac{\dot{M}}{P} = \alpha [g + (r + \pi)b + v(X - zE)],$$

$$\frac{\dot{B}}{P} = (1 - \alpha) [g + (r + \pi)b + v(X - zE)],$$

which imply

$$\dot{m} = \alpha [g + rb + v(X - zE)] - \pi (m - \alpha b), \quad (22)$$

$$\dot{b} = (1 - \alpha) [g + rb + v(X - zE)] - \alpha \pi b. \quad (23)$$
At the initial steady state, $\dot{M}/M = \dot{B}/B = \pi$ and

$$\alpha = \frac{m}{m+b}.$$

Tight monetary policy takes form of an announced reduction in $\alpha$. The model is solved in the same way as before, but with (22)-(23) replacing (9)-(10). (See the appendix for additional details.)

6.1 Kenya 1993

The underlying policy rule may have changed during Kenya’s 1993 TM episode. Policy makers started out with the intention of adhering to a fixed money growth rule. But as rising interest payments on the internal debt pushed the fiscal deficit above 6% of GDP (vs. 1.9% in FY 1991/92) it became increasingly difficult to control money growth (Roe and Sowa, pp.257-258). It is desireable therefore to know how the earlier results for the Kenyan 1993 episode change when TM is implemented through a deficit-share rule instead of a fixed money growth rule. The outcome for the deficit-share rule may also shed light on the source of high real interest rates in Zambia 1993.²⁸

The deficit-share rule weakens but does not sever the link between variations in the fiscal deficit and money growth. Consequently, in those parts of the parameter space where a rising fiscal deficit causes TM to fail, the stock of government bonds grows more slowly than under a fixed money growth rule. And since the price dynamics are sensitive to the path of the internal debt, the increase in inflation is often less not only across steady states but also on the transition path. Compare the results in Tables 2a-2b with those in Tables 4a-4b for $\sigma = 2 - 4$. In all but two cases, the numbers for $b(t_1), \pi_{ss}$ and $\pi(t_1)$ are lower under the deficit-share rule; the impact effect on $\pi$ is also smaller whenever the tight-money paradox obtains (i.e., whenever $\dot{\pi}(0) > 0$).²⁹ More importantly, the difference in the numbers is often quantitatively large. In the case where $\sigma = 3.5$, $\tau = .25$ and $t_1 = 1$, for example, the elasticities for $\pi(t_1), \pi_{ss}, r(t_1)$ and $b(t_1)$ are four times larger in Table 2a than in Table 4a. The deficit-share rule is less harmful in this case because it allows for a partial, endogenous retreat from bad policy during the period $(0, t_1)$.

Although the comparison generally favors the deficit-share rule for $\sigma < 4$, the overall
ranking of the two rules is uncertain. The deficit-share rule has a major shortcoming. Observe that the tight-money paradox combines with very high real interest rates when $\sigma = 3 - 5.5$ in Tables 4a-4b. The zone of ugly macroeconomic failure is thus much larger for the deficit-share rule and covers most of the region where a fixed money growth rule generates pleasant monetarist arithmetic. In a sizeable part of the parameter space, the deficit-share rule leads to failure when success was there for the taking with the right policy rule.


Direct comparisons of the solutions in Tables 3 and 5 are not quite kosher since the TM regime lasts one year longer in the simulations for the deficit-share rule. It is evident from inspection of the two tables, however, that the choice of policy rule does not matter much in the Nigerian case. Inflation and the real interest rate decrease more in the short run in Table 5 (especially when $\tau = .10$). Otherwise the results follow the same pattern as in Table 3. Under both policy rules, the real interest rate rises sharply toward the end of the TM regime and inflation undershoots its steady-state level on the transition path.

6.3 Policy Rules, Capital Flows, and the Puzzle of Transitory, High Real Interest Rates

The distinction between the two TM rules is relevant to the high real interest rate puzzle. Under a fixed money growth rule, very high real interest rates are “unusual” in the sense that they are confined to a small part of the parameter space: $\sigma = 3.75 - 4.2$ for $\tau = .25$, and $\sigma = 3 - 3.65$ for $\tau = .10$. The parameter space associated with high real interest rates is somewhat larger when $\tau = .25$ and the government follows a deficit-share rule, but explanation of the puzzle requires values of $\sigma$ in the 4.5-5.5 range. Such values may be implausibly large, evidence of easy currency substitution notwithstanding.

Table 4b advances another, perhaps more convincing explanation for transitory, high real interest rates. Estimates of the intertemporal elasticity of substitution suggest that $\tau$ might be as low as .10-.15 in SSA. If this is actually the case, then high real interest rates are consistent with a wide range of plausible values for $\sigma$. For $\tau = .10$ and $t_1 = .5 - 1$, the zone of high real interest rates in Table 4b is $\sigma = 3 - 5.8$. The zone is smaller at higher
values of $t_1$, but TM still gives rise to very high real rates when $t_1 = 2 - 3$ and $\sigma = 3 - 5$. Observe also that the elasticities for $b(t_1)$ are extremely small. This is relevant to Zambia's experience. The real internal debt increased very little in Zambia in 1993. Nevertheless, it would be wrong to conclude that something other than TM must have been the principal cause of high inflation and high real interest rates.

While the zone of high real interest rates differs for the two policy rules, in both high rates are associated with capital outflows and depreciation of the real exchange rate. It is hard to judge whether this or the alternative combination of high real interest rates, capital inflows, and appreciation of the real exchange rate fits the stylized facts better. In the Nigerian episode, capital outflows were associated with continuous, strong depreciation of either the official or the parallel exchange rate and with high real interest rates after the second quarter of 1990. According to some accounts [e.g., Roe and Sowa (1995), p.242], Kenya’s high interest rates attracted large capital inflows in 1993. A closer examination of the data, however, reveals sizeable net capital outflows during the TM period, with the outflows being very large in the later stages when nominal and real interest rates were at their peak levels. And regardless of what happened earlier, Kenya’s latest experiment with TM seems to have led to higher interest rates, substantial depreciation of the real exchange rate, and capital outflows. Zambia 1993 is certainly a case where the real exchange rate appreciated strongly. But the exchange rate system was far from a pure float in 1993; moreover, the impact on the capital account is hard to gauge because the current account figures may conceal substantial capital flight and because some of the observed gross inflows were probably a response to the loosening of exchange controls and the extreme liquidity shortage the economy faced at the end of 1992.

Summing up, the facts are thoroughly confusing; almost every conceivable combination of changes in the real interest rate, the real exchange rate and capital flows can be found — or arguably might exist — in the empirical record. I suspect that a satisfactory account of the full range of outcomes seen in different TM episodes requires both better data and further theoretical analysis that allows for different initial conditions and alternative rules for management of the exchange rate (e.g., different types of managed floats).
7. Capital Accumulation, Real Wage Rigidity, and Real Output: How Large are the Real Costs of Transitory, High Real Interest Rates?

The assumption of fixed real output is useful in highlighting the forces that drive the dynamics for inflation, the real interest rate and the fiscal deficit. But it also limits the set of issues the model can engage. Accordingly, in this section I extend the model to allow for sector-specific capital accumulation and temporary real wage rigidity. The more complete model can address the concern in Kenya, Zambia, Zimbabwe and Nigeria that high real interest rates have depressed private investment and reduced employment growth.

Let $K_i, I_i, L_i, \delta$ and $P_k$ denote the capital stock, investment, employment, the depreciation rate, and the supply price of capital in dollars. The private agent’s optimization problem can then be formulated as

$$\text{Max}_{\{E, m, b, F, I_n, I_T\}} \int_0^\infty [V(P_n, E) + \phi(m, vF)]e^{-\rho t}dt, \quad (24)$$

subject to

$$A = m + b + vF, \quad (25)$$
$$\dot{A} = vR(P_n, K_n, K_T, L_n + L_T) + g + rb + (\chi - \pi)vF - \pi m - vE(1 + z)$$
$$\quad - vP_k[I_T + \psi_T(I_T - \delta K_T) + I_n + \psi_n(I_n - \delta K_n)], \quad (26)$$
$$\dot{K}_n = I_n - \delta K_n, \quad (27)$$
$$\dot{K}_T = I_T - \delta K_T. \quad (28)$$

$K_n$ and $K_T$ do not appear in the wealth constraint (25) because there is no equities market in which money can be swapped for claims to the capital stock. In the budget constraint (26), $R(\cdot)$ is a revenue function that measures GDP (in units of the traded good) and $\psi_i(\cdot)$ are adjustment cost functions. The revenue function has the usual properties: $\partial R/\partial P_n = Q_n$, $\partial R/\partial K_i = r_i$, and $\partial R/\partial (L_T + L_n) = w/v$, where $r_i$ is the capital rental in sector $i$ and $w$ is the real consumption wage (i.e., the nominal wage deflated by the CPI). Adjustment costs are convex but positive only when net investment changes the capital stock: $\psi(0) = \psi'(0) = 0$, $\psi' \geq 0$ as $I_i \geq \delta K_i$, and $\psi'' > 0$. 26
Factories are built by combining $\kappa_1$ units of imported machinery with $\kappa_2$ units of a nontraded input (construction) in fixed proportions. The supply price of capital is thus

$$P_k = \kappa_1 + \kappa_2 P_n.$$  \hspace{1cm} (29)

The real wage is predetermined by existing contracts which call for full indexation of the nominal wage to the CPI. Over the medium term, however, wage settlements respond to excess demand/supply in the labor market as measured by deviations of the unemployment rate from some critical rate $u^*$:

$$\frac{\dot{w}}{w} = c[u^* - 1 + L_n(wP_n^{\gamma-1}, K_n) + L_T(wP_T^{\gamma}, K_T)], \quad c > 0,$$  \hspace{1cm} (30)

The supply of labor is fixed at unity and $L_i(\cdot)$ relates labor demand to the product wage and the capital stock. Outside the brackets, the parameter $c$ measures the responsiveness of the real wage to the state of the labor market.

The capital rentals $r_i$ are linked to wages and prices via zero-profit conditions. Assuming constant returns technology,

$$P_n = C^m(wP_n^{\gamma-1}, r_n), \hspace{1cm} \text{(31)}$$

$$1 = C^T(wP_n^{\gamma}, r_T), \hspace{1cm} \text{(32)}$$

where $C^i$ is the unit cost function in sector $i$.

Two other changes complete the model. Private investment now enters into the expression for the current account surplus

$$\dot{F} = R(P_n, K_n, K_T, L_n + L_T) - E - P_k[I_T + \psi_T(I_T - \delta K_T) + I_n + \psi_n(I_n - \delta K_n)] - X,$$  \hspace{1cm} (33)

and (12), the market-clearing condition in the nontradables sector, now includes the demand for capital inputs on the left side and the general equilibrium supply function $Q_n = R_n(\cdot) \equiv \partial R/\partial P_n$ on the right side:

$$D^a(P_n, E) + \kappa_2[I_T + \psi_T(I_T - \delta K_T) + I_n + \psi_n(I_n - \delta K_n)] = R_n(P_n, K_n, K_T, L_n + L_T).$$  \hspace{1cm} (34)
7.1 Modifications to the Solution Procedure

The inclusion of sector-specific capital accumulation and gradual real wage adjustment more than doubles the dimensions of the dynamic system. The solution procedure, however, is essentially the same as in section 3.

The solution to the private agent’s optimization problem satisfies the first-order conditions in (5)-(8) and additional conditions associated with the optimal choices for $I_T$ and $I_n$. These are

\[
\beta v P_k [1 + \psi_n' (I_n - \delta K_n)] = \beta_2, \tag{35}
\]

\[
\beta v P_k [1 + \psi_T' (I_T - \delta K_T)] = \beta_3, \tag{36}
\]

and the co-state equations

\[
\dot{\beta}_2 = \{[\rho(1 + \psi_n') + \delta]P_k - r_n\} \beta v, \tag{37}
\]

\[
\dot{\beta}_3 = \{[\rho(1 + \psi_T') + \delta]P_k - r_T\} \beta v, \tag{38}
\]

where $\beta, \beta_2$ and $\beta_3$ are the multipliers paired with the constraints (41)-(43). As before,

\[
\frac{\dot{E}}{E} - \gamma \frac{\dot{P}_n}{P_n} = \tau(r - \rho), \tag{14'}
\]

while (8), (42)-(44), (50a)-(50b), and (51a)-(51b) yield

\[
(1 + \psi_n')[\rho - r + (\alpha - \gamma)\dot{P}_n/P_n] + \psi_n''[\dot{I}_n - \delta(I_n - \delta K_n)] = \rho(1 + \psi_n') + \delta - \frac{r_n}{P_k}, \tag{39}
\]

\[
(1 + \psi_T')[\rho - r + (\alpha - \gamma)\dot{P}_n/P_n] + \psi_T''[\dot{I}_T - \delta(I_T - \delta K_T)] = \rho(1 + \psi_T') + \delta - \frac{r_T}{P_k}, \tag{40}
\]

where $\alpha \equiv P_n \kappa_2/P_k$, the cost share of the nontraded component in the production of capital goods. After solving the quasi-static variant of the model for the endogenous variables $P_n, r_n, r_T, P_k$, one can substitute for $\dot{P}_n/P_n$ in (14'), (39) and (40). Equations (9), (10), (14'), (27), (28), (30), (33), (39) and (40), together with the solutions for $G^1(\cdot)$-$G^7(\cdot)$, then define a self-
contained system of nine differential equations in $I_i, K_i, w, E, m, b$ and $F$. The economy traverses a nonconvergent path of this system during the TM phase. At time $t_1$, the path connects with the saddle path of the system defined by (11), (14'), (27), (28), (30), (33), (39) and (40). Fourteen boundary conditions are needed to pin down the nine constants in the system for the period $(0, t_1)$, the four constants in the saddlepoint solution for $t > t_1$, and $b(t_1) - b_0$. The positions of the predetermined variables $K_i, w, b$ and $F$ at $t = 0$ and $t = t_1$ supply ten boundary conditions. The other four come from restrictions on the paths of the jump variables: optimizing behavior and perfect foresight imply that $I_i, E$ and $m$ do not jump at $t_1$; hence, at the juncture of the two policy regimes, the solutions for these variables must be the same in the two systems [i.e., $I_n(t^-_1) = I_n(t^+_1)$, etc.].

7.2 Calibration of the Model

To calibrate the model, I assigned the following values to the cost shares of labor ($\theta^i_L$), the elasticity of substitution between capital and labor ($\sigma_i$), the cost share of nontraded components in the production of capital goods ($\alpha$), the speed-of-adjustment parameter in the labor market ($c$), the depreciation rate ($\delta$), and the q-elasticity of investment spending ($\Omega$):

$$\sigma_n = \sigma_T = .75, \quad \alpha = .43 \text{ (Kenya), .35 (Nigeria), } \delta = .05,$$

$$\theta^n_L = .37 \text{ (Kenya), .28 (Nigeria), } \theta^T_L = .37, \quad c = 1, \quad \Omega = 1.$$  

The values for $\sigma_n$ and $\sigma_T$ agree with estimates of capital-labor substitutability in developing countries, while $\alpha$ equals one minus the average share of imported machinery in GFCF in the two years preceding the TM episode. According to the National Income Accounts, the cost share of labor was approximately 37% in both manufacturing and services + construction in Kenya in 1991-92. In Nigeria, labor’s share in 1988 was 30% in manufacturing and 27.5% in services + construction. Unfortunately, there is no reliable data on factor intensities in agriculture. Since data for the agricultural sector in other LDCs typically places labor’s share between .30 and .45 (Buffie, 2001b, chapter 5), I decided to set $\theta_T^L$ at 37% in both countries.

Empirical studies are of little help in setting $c$, the parameter that measures the degree of
resistance to real wage cuts (in the formal sector). To guide the choice for this parameter, I calculated how rapidly the real wage in (30) approaches its market-clearing level when the capital stocks and the relative price of the nontraded good are held constant. For \( c = 1 \), the speed of adjustment seems realistic: in the absence of further shocks, it takes roughly 1.5 years for the labor market to clear.\(^{36}\)

Finally, the \( q \)-elasticity of investment spending, \( \Omega \), indirectly defines the adjustment cost functions \( \psi_i(I_i - \delta K_i) \). Explicit adjustment cost functions are not needed to characterize the steady-state equilibrium because \( \psi_i = \psi_i' = 0 \) at \( I_i = \delta K_i \). The second derivatives \( \psi_i'' \), however, enter into the solution for the transition path. To tie down these derivatives, return to (35)-(36) and observe that \( \beta_j/\beta v = \beta_j(1 + z)/V_E \) \((j = 2, 3)\) is the shadow price of \( K_i \) measured in domestic currency. Thus, \( \beta_j/\beta v P_k \) is effectively Tobin’s \( q \), the ratio of the demand price to the supply price of capital. Adopting this notation, we have from (35)-(36) that

\[
\psi_i'' \delta \hat{I}_i/\hat{q}_i = 1, \quad i = n, T,
\]
evaluated at a stationary equilibrium. Define \( \Omega_i \equiv \hat{I}_i/\hat{q}_i \). Under the neutral assumption that the \( q \)-elasticity of investment spending is the same in the two sectors,

\[
\psi''_n = \psi''_T = \frac{1}{\delta \Omega}.
\]

The convexity of the adjustment cost functions is determined therefore by the depreciation rate and the \( q \)-elasticity of investment spending.

I fixed \( \delta \) at 5% and \( \Omega \) at unity. The value for \( \Omega \) is based mainly on estimates for developed countries. These range from 0.2 to 2.3, with the majority being less than unity.\(^{37,38}\) In choosing a value above the mean, I gave weight to the results in Shafik (1990), where the elasticity of investment with respect to the supply price of capital is estimated to be around two for Egypt, and to recent theoretical and empirical work that suggests previous studies have substantially underestimated \( \Omega \) in developed countries (Barnett and Sakellaris, 1998).

### 7.3 Numerical Solutions for Kenya 1993

Table 6 presents solutions for the Kenyan variant of the model. Since the objective is to determine whether the real output losses might be sizeable, the table shows only the
solutions for which the semi-elasticity of the real interest rate at $t_1$ exceeds .04; when the semi-elasticity is smaller than this, the real effects are inconsequential and the results are virtually the same as in the case where real output is constant.

A quick comparison of Tables 2a-2b and Table 6 reveals many common features. The solutions for $\tau = .25$ are remarkably close; in both Table 2a and Table 6, inflation strongly overshoots its steady-state level during the TM phase and the real interest rate rises sharply when $\sigma$ lies between 3.25 and 4. The interaction between the real and monetary dynamics makes more of a difference to the results when $\tau = .10$, the high real interest rate zone shifting from 2.75-3.5 in Table 2b to 3.5-4.15 in Table 6. But the main point survives: extremely high real interest rates and the ultra-strong form of the tight-money paradox are not artifacts of models that ignore the impact of TM on capital accumulation and real output.

Turning to the central issue, are real output losses large when the real interest rate rises 5-30 points? The answer in the base case is a qualified no. The real interest rate falls rapidly after the TM policy is abandoned. This, in conjunction with convex adjustment costs and procyclical real wages, serves to moderate the downturn on the transition path that brings the economy back to its original real equilibrium (for $K_i$, $L_i$ and $Q_i$, not $m$ or $F$). Figures 3a-3b show the paths of $r, w, GDP, I$ and $K$ ($I = I_n + I_T$ and $K = K_n + K_T$) in the case where $t_1 = .5, \tau = .25$ and $\sigma = 4$. Although the semi-elasticity for the real interest rate vaults to .32 and investment contracts sharply in the short run, the decreases in the capital stock, real output and the real wage are small, on the order of 1-2% at the trough of the cycle when the reduction in money growth is 75%. Notice also, however, that recovery to the original real equilibrium is very slow. This qualifies the conclusion that the real losses are “small”: six months of TM may throw away 1-1.5% of GDP and cost workers 2% of their real wages for 20+ years.


The solutions in Table 7 are qualitatively similar to those in Table 3 where real output is fixed. In both tables, $\pi$ and $r$ are lower in the short run but higher at the end of the TM regime. On the other hand, the quantitative effects are not always close. In several cases, inflation and the real interest rate increase much more in the medium and long run when
the impact on capital accumulation and employment are taken into account (e.g., \( \tau = .10 \) and \( \sigma = 3.75 \)).

The other comparison of interest is between Tables 6 and 7. Given that the real interest rate decreases much more in the short run and is lower for most of the TM phase (the semi-elasticity for \( r \) becomes positive only when \( t > 1.5 \)), are investment and real output likely to increase in the short run or at least decrease less than in the Kenyan episode? Surprisingly this is not the case. Quite the contrary: the reductions in investment, real output, and the real wage are much larger because the TM policy lasts much longer. The case \( \tau = .10 \) and \( \sigma = 3.75 \), depicted in Figures 4a-4b, makes the point forcefully. Although the real interest rate decreases up to \( t = 1.1 \) and is lower until \( t = 1.8 \), investment declines *throughout* the TM phase. The strong positive correlation between \( I \) and \( r \) in the first 1.8 years reflects both substitution away from capital toward foreign currency and private sector anticipation of sharply higher real interest rates in the future. And because investment is depressed for a much longer period of time, the reductions in GDP, the capital stock, and the real wage are much larger than in Figure 3b, where the TM regime lasts only six months.

The deep, prolonged recession has important implications not only for welfare but also for the price dynamics. In contrast to the Kenyan and Zambian episodes, inflation continued to increase in Nigeria after the TM policy was abandoned. Consistent with this, the simulations for the deficit-share rule in Table 5 show \( \pi(t_1) < \pi_{ss} \) in nine out of ten cases. The reverse pattern holds in Table 7 but the results may still be consistent with the data. For \( \tau = .10 \), inflation keeps rising for 2-10 years after the TM policy ends. In Figure 4c, for example, the elasticity for \( \pi \) increases from 2.35 at \( t = 2 \) to 3.03 at \( t = 12 \), and then declines *very* slowly toward its steady-state level. (After 20 years, the elasticity has fallen only to 2.95.) For a couple of years, higher inflation can be explained by higher fiscal deficits. But after year four the deficit decreases steadily, and by year eight it lies below the level that prevailed at year two. It seems odd therefore that inflation increases so much in the decade following the policy reversal. The resolution of the puzzle is that capital decumulation strongly exacerbates inflationary pressures during the first fifteen years of adjustment by decreasing real income and real money demand. Consequently, inflation rises stubbornly until the recessionary cycle approaches its trough in year twelve. Arguably, this path matches the
facts as well as the paths in Table 3 — all we know from the data is that inflation was below its peak level at the end of the TM regime, not that it was below its steady-state level.

8. Real Wage Rigidity and Hysteresis

The preceding analysis assumes the real wage is fully flexible over a medium run of 1.5-2 years. While this is a reasonable view for some countries, it is not the only view worth examining. Nothing in the vast literature on optimizing union and insider-outsider models supports the notion that workers will gradually accede to real wage cuts when faced with a leftward shift of the labor demand schedule: insiders may not care about disenfranchised outsiders (the unemployed); similarly, in unions governed by seniority or the median-voter rule, most of the membership/leadership may oppose wage concessions because they know that their jobs are not threatened. In LDCs, it is not simply a matter of firms vs. workers as the government often participates in the bargaining process. But this may not alter the outcome appreciably — politically weak governments are seldom effective advocates for the the unemployed. It is important therefore to investigate the consequences of TM when the real wage is perfectly rigid.

Since the original values of \( w, K_i, L_i \) and \( Q_i \) are still steady-state equilibrium values, it is natural to conjecture that the degree of real wage rigidity affects only the dynamics, not the long-run outcome. But consider the nature of the equilibrium when \( K_i \) and \( L_i \) decrease proportionately. Under constant returns to scale and homothetic preferences, 10% decreases in \( K_i \) and \( L_i \) cause 10% decreases in sectoral output, national income, and the demand for each good. Since demand falls in step with supply, the equilibrium relative price of the nontraded good does not change. This implies that product wages do not change, and hence that sectoral employment actually does fall by the same percentage amount as the sectoral capital stocks. All that remains, therefore, is to verify that the capital stocks are at their long-run equilibrium levels. This requires \( r_n/P_k = r_T/P_k = \rho + \delta \), which clearly holds because \( P_k \) is unchanged \( (P_k = \kappa_1 + \kappa_2 P_n) \) and proportionate decreases in \( K_i \) and \( L_i \) do not disturb the marginal product of capital. Decreases in investment spending induced by transitory increases in the real interest rate thus lead to permanent, proportionate decreases in the capital stock, employment, and real output. In other words, the adjustment process
exhibits hysteresis: although the original real equilibrium is still a stationary equilibrium (for variables other than \( m \) and \( F \)), the economy never returns to it.

The magnitude of the permanent output loss depends on the impact on the real interest rate and the point during the adjustment process at which the economy locks into the new stationary equilibrium. (Does lock-in occur near the trough of the cycle or after partial recovery from the trough?) For short-lived TM episodes that produce high real interest rates, the loss is usually around 1% of GDP. In a minority of cases, however, the losses are much larger. One such case is shown in Figure 5.

Hysteresis also has significant effects on the dynamics for prices, the real interest rate, and domestic debt. All of the numbers for \( \pi, r \) and \( b \) in Table 8 are larger than their counterparts in Tables 6. The reason is that the fundamentals worsen more in Table 8. After the adjustment process is complete, lower real output is associated with lower real money balances and less tax revenue (zE falls). Both factors increase the steady-state inflation rate and the long-run demand for foreign currency. And since the private sector is forward-looking, capital flight is greater and inflation and the real interest rate are higher on the transition path as well.

9. Concluding Remarks

In this paper I have analyzed tight monetary policy in various models of small open economies in SSA. The results dispute the conventional wisdom that TM must fail if there is no change in the fiscal regime. The conventional view errs in concluding that lower seigniorage across steady states implies unstable growth of government debt. In focusing on steady states, it overlooks the possibility that transitory decreases in the real interest rate, transitory appreciation of the real exchange rate, transitory gains in seigniorage, and transitory increases in tax revenues may enable the government to reduce the internal debt to a level compatible with permanently lower money growth and hence permanently lower inflation. This outcome, dubbed Pleasant Monetarist Arithmetic (PMA), does not require an unusually high value for the elasticity of money demand with respect to inflation. The transitory fiscal gains may be large enough to engender PMA when the elasticity is as low as .5 or .6.\textsuperscript{41} Credibility and the form of the policy rule, however, are quite important. It is essential that
the government’s commitment to slower money growth be unconditional. Multiple equilibria rear their ugly heads when the public knows that sharp increases in inflation and/or the real interest rate will provoke a policy reversal: because policy is outcome-dependent, both expectation of success and expectation of failure are likely to be self-fulfilling equilibrium outcomes. The chance of getting onto an equilibrium path that delivers PMA is also much lower if the government commits only to monetize a smaller share of the deficit. The problem, once again, is that the policy rule does not embody a firm commitment to slower money growth. It says, implicitly, that money growth will be slower only if the fiscal deficit does not increase too much.

None of this denies that TM is a dangerous, risky policy and that governments would be well advised to attack inflation by coordinating slower money growth with expenditure cuts and tax increases. The relevant parameter space includes horrendous macroeconomic failure as well as PMA. When the intertemporal elasticity of substitution is low and domestic and foreign currency are close substitutes, the real interest rate may shoot up to 20-40% while inflation soars and the fiscal deficit spirals out of control. On the real side, small but highly persistent output losses from capital decumulation and rising unemployment add to the misery. Although the real interest rate falls rapidly after the TM policy is reversed, recovery to the initial level of GDP is exceedingly slow and real output losses of 1-4% may endure 10-30 years. Moreover, this slow pace of recovery presumes a moderate degree of real wage flexibility (in the formal sector). If workers refuse real wage cuts, the economy never recovers from the shock of temporary high real interest rates — the decreases in the capital stock, GDP and employment are permanent.
Appendix: Solving for the Transition Path

An outline of the solution procedure was given in section 3 of the main text. The appendix provides the full mathematical solution.

The Dynamic System During the TM Phase

Start by differentiating (5) with respect to time and substituting for \( \dot{\beta} \) from (8). Let \( V_{EP} \equiv \partial V_{E}/\partial P_{n} \) and define \( \tau \equiv -V_{E}/V_{EE}E \) to be the elasticity of intertemporal substitution. The algebra then gives

\[
\left( \frac{V_{EP}}{V_{EE}D_{n}} - \tau \right) \gamma \frac{\dot{P}_{n}}{P_{n}} + \frac{\dot{E}}{E} = \tau(r - \rho), \tag{A1}
\]
as \( \pi - \chi = \gamma \dot{P}_{n}/P_{n} \) [see (1)]. The term involving the second partial derivatives \( V_{ij} \) is a nuisance. To replace it with something more comprehensible, assume homothetic preferences and note from Roy’s identity \( D_{n} = -V_{P}/V_{E} \) that

\[
\frac{V_{EP}}{V_{EE}D_{n}} = \tau - 1.
\]

We also have from (12)

\[
\dot{P}_{n} = k\dot{E}, \tag{A2}
\]
where \( k \equiv (\eta + \gamma)^{-1}, \eta \) is the compensated own-price elasticity of demand, and a circumflex indicates a percentage change (i.e., \( \dot{x} = dx/x \)). Interpret the differentials in (A2) as time derivatives. Upon substituting for \( \dot{P}_{n}/P_{n} \) and \( V_{EP}/V_{EE}D_{n} \), equation (A1) simplifies to

\[
(1 - \gamma k)\dot{E} = \tau E(r - \rho). \tag{A3}
\]

During the TM phase the dynamics are governed by (9), (10), (13) and (A3), a 4x4 system involving \( m, b, F \) and \( E \). To relate \( \chi, \pi \) and \( r \) to these variables, assume \( \phi(m, vF) \) is a linearly homogeneous CES function, with \( \sigma \) being the elasticity of substitution between \( m \) and \( vF \). In this case, equations (6') and (7') in the text yield

\[
\frac{\tau \theta_{f} + \sigma \theta_{m}}{\sigma} \dot{m} - \frac{(\tau - \sigma) \theta_{f}}{\sigma} \dot{F} = \left[ 1 - \frac{\gamma k}{\sigma}(\sigma \theta_{m} + \tau \theta_{f}) \right] \dot{E} - \frac{\tau}{i}(dr + d\pi), \tag{A4}
\]

\[
\frac{(\sigma - \tau) \theta_{m}}{\sigma} \ddot{m} + \frac{\tau \theta_{m} + \sigma \theta_{f}}{\sigma} \ddot{F} = \left[ 1 - \frac{\gamma k}{\sigma}(\sigma - \tau) \theta_{m} \right] \dot{E} + \frac{\tau}{i - \chi}(dr + d\pi - d\chi), \tag{A5}
\]

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where $i \equiv r + \pi$ is the nominal interest rate and
\begin{align*}
\theta_m & \equiv \frac{\phi_m m}{\phi} = \frac{\phi_m m}{\phi_m m + \phi_f v F} \\
\theta_f & \equiv \frac{\phi_f v F}{\phi} = \frac{\phi_f v F}{\phi_m m + \phi_f v F}
\end{align*}
are the shares of liquidity services provided by domestic and foreign currency. (The second equality in the expressions for $\theta_m$ and $\theta_f$ follows from linear homogeneity of $\phi$, which implies $\phi = \phi_m m + \phi_f v F$.)

One more equation is needed. This is supplied by the definition of the inflation rate, which links the path of $\pi$ to the paths of $r$ and $\chi$. From (1), (15) and (16),
\begin{align*}
\pi & = \chi + \gamma \hat{P}_n/P_n, \\
\Rightarrow \pi & = \frac{\gamma k}{1 - \gamma k}(r - \rho) + \chi.
\end{align*}
Equations (A4)-(A6) can be solved for $\pi$, $\chi$ and $r$ as a function of $m$, $E$ and $F$. Anticipating future needs, evaluate the solutions at a steady state where $r = \rho$ and $\pi = \chi$. The solutions for $\pi$ and $r$ are then
\begin{align*}
\frac{dr}{dt} & = n_2 \dot{m} - n_3 \dot{F} + n_4 \dot{E}, \\
\frac{d\pi}{dt} & = n_5 \dot{m} - n_6 \dot{F} + n_7 \dot{E},
\end{align*}
where
\begin{align*}
n_1 & = \frac{\gamma k \tau}{1 - \gamma k}, \\
n_2 & = \frac{\rho (\tau - \sigma) \theta_m}{[\sigma \tau (1 + n_1)]}, \\
n_3 & = \frac{\rho (\tau \theta_m + \sigma \theta_f)}{[\sigma \tau (1 + n_1)]}, \\
n_4 & = \frac{[\sigma - \gamma k (\sigma - \tau) \theta_m]}{[\sigma \tau (1 + n_1)]}, \\
n_5 & = n_3 + i(\tau - \sigma) \theta_f / \sigma \tau, \\
n_6 & = i[\sigma - \gamma k (\sigma \theta_m + \tau \theta_f)]/\sigma \tau - n_4.
\end{align*}
Now linearize (9), (10), (13) and (A3) around the stationary equilibrium $(m^*, b^*, F^*, E^*)$. After choosing units so that $v = 1$ initially and exploiting the information in (A2) and
(A6)-(A8), there emerges

\[
\begin{bmatrix}
\dot{m} \\
\dot{b} \\
\dot{E} \\
\dot{F}
\end{bmatrix} = 
\begin{bmatrix}
n_6 & 0 & -n_7 m/E & -n_5 m/F \\
n_2 b/m - \pi & \rho & c_1 & -n_3 b/F \\
0 & 0 & \gamma k - 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{m} - m^* \\
\dot{b} - b^* \\
\dot{E} - E^* \\
\dot{F} - F^*
\end{bmatrix},
\]  

(A9)

where

\[
c_1 = n_4 b/E - \gamma k X - z(1 - \gamma k),
\]
\[
c_2 = \frac{\rho(\tau - \sigma)\theta_m}{\sigma[1 + (\tau - 1)\gamma k]m/E},
\]
\[
c_3 = \frac{\rho}{1 + (\tau - 1)\gamma k} \left[ 1 - \frac{\gamma k(\sigma - \tau)\theta_m}{\sigma} \right],
\]
\[
c_4 = \frac{\rho(\tau\theta_m + 1\sigma\theta_f)}{\sigma[1 + (\tau - 1)\gamma k]F/E}.
\]

The general solution to (A9) is

\[
m(t) - m_o = m^* - m_o + s_{11} h_1 e^{\lambda_1 t} + s_{12} h_2 e^{\lambda_2 t} + s_{13} h_3 e^{\lambda_3 t} + s_{14} h_4 e^{\lambda_4 t}, \quad t < t_1 \]  

(A10)

\[
b(t) - b_o = b^* - b_o + s_{21} h_1 e^{\lambda_1 t} + s_{22} h_2 e^{\lambda_2 t} + s_{23} h_3 e^{\lambda_3 t} + s_{24} h_4 e^{\lambda_4 t}, \quad t < t_1 \]

(A11)

\[
E(t) - E_o = E^* - E_o + s_{31} h_1 e^{\lambda_1 t} + s_{32} h_2 e^{\lambda_2 t} + s_{33} h_3 e^{\lambda_3 t} + s_{34} h_4 e^{\lambda_4 t}, \quad t < t_1 \]

(A12)

\[
F(t) - F_o = F^* - F_o + s_{41} h_1 e^{\lambda_1 t} + s_{42} h_2 e^{\lambda_2 t} + s_{43} h_3 e^{\lambda_3 t} + s_{44} h_4 e^{\lambda_4 t}, \quad t < t_1 \]

(A13)

where an o subscript indicates the initial value of a variable, \( \lambda_1 - \lambda_4 \) are the eigenvalues of the system, \( s_{ij} \) is the \( i \)th component of the eigenvector associated with the eigenvalue \( \lambda_j \), and \( h_1 - h_4 \) are constants determined by boundary conditions.

**The Dynamic System After Bond Sales/Purchases Cease**

When bond-financing ends at time \( t_1 \), the system defined by (11), (13) and (A3) takes over. The linearized versions of (13) and (A3) are the same as in the third and fourth rows of (A9), but the Taylor series expansions are around the new steady state \((\bar{m}, \bar{E}, \bar{F})\). Thus

\[
\dot{E} = c_2(m - \bar{m}) + c_3(E - \bar{E}) - c_4(F - \bar{F}),
\]

(A14)
\[ \dot{F} = (\gamma k - 1)(E - \bar{E}), \]  
\[ \dot{m} = \left( \frac{b}{m} n_2 + n_6 - \pi \right)(m - \bar{m}) + \left[ \frac{b}{E} n_4 - \frac{m}{E} n_7 - \gamma k X - z(1 - \gamma k) \right] (E - \bar{E}) \]
+ \left( \frac{b}{F} n_3 + \frac{m}{F} n_5 \right) (F - \bar{F}). \]

(A15)

(A16)

To rule out perverse comparative statics results and ensure well-behaved dynamics, I assume the economy operates on the upward-sloping portion of the seigniorage Laffer curve where the elasticity of money demand with respect to inflation is less than unity. Under this restriction, the system in (A14)-(A16) has two positive eigenvalues and one negative eigenvalue. Since $F$ is the only predetermined variable, the system is saddlepoint stable. On the convergent saddle path that leads to the new steady state

\[ m(t) - m_o = \bar{m} - m_o + \left\{ \frac{b}{E} n_4 - \frac{m}{E} n_7 - \gamma k X - z(1 - \gamma k) \right\} (\gamma k - 1) \]
\[ -\lambda_5 \left( \frac{b}{F} n_3 + \frac{m}{F} n_5 \right) \frac{\lambda_5}{n_2 b/m + n_6 - \pi - \lambda_5 h_5 e^{\lambda_5 t}}, \quad t \geq t_1 \]  

(A17)

\[ F(t) - F_o = \bar{F} - F_o + \frac{\gamma k - 1}{\lambda_5} h_5 e^{\lambda_5 t}, \quad t \geq t_1 \]  

(A18)

\[ E(t) - E_o = \bar{E} - E_o + h_5 e^{\lambda_5 t}, \quad t \geq t_1 \]  

(A19)

where $\lambda_5$ is the system’s negative eigenvalue and $h_5$ is a constant fixed by initial conditions.

**Locating the Stationary Equilibria of the Two Policy Regimes**

Two tasks remain. The first is to locate the stationary equilibria associated with the different policy regimes. Fortunately, most of this work is already done. Observe from (9), (12), (13), (A3) and (A6) that across steady states money growth, currency depreciation, and inflation are equal, and that real expenditure, the relative price of the nontraded good, the real exchange rate, and the real interest rate are all constant:

\[ E^* = \bar{E} = E_o, \quad \pi = \mu = \chi. \]
Imposing these conditions in (A4) and (A5) gives

\[ \hat{m}_{\text{steady state}} = -\epsilon \hat{\pi}_{\text{steady state}} \]  
(A20)

\[ \hat{F}_{\text{steady state}} = (\sigma - \tau) \frac{\pi}{\rho + \pi} \theta_m \hat{\pi}_{\text{steady state}} \]  
(A21)

where

\[ \epsilon = \frac{\pi}{\rho + \pi} (\tau \theta_m + \sigma \theta_f) < 1 \]

is the elasticity of money demand with respect to inflation.

Equations (A20) and (A21) are common to the two policy regimes. What differs is how steady-state inflation is determined. In the TM regime, we can simply replace \( \hat{\pi} \) in (A20) and (A21) with \( \hat{\mu} \) because money growth is an exogenous policy variable. It follows directly from this and the government budget constraint (10) that

\[ m^* - m_o = -\epsilon m \hat{\mu}, \]  
(A22)

\[ b^* - b_o = \frac{\pi m}{\rho} (1 - \epsilon) \hat{\mu}, \]  
(A23)

\[ F^* - F_o = (\sigma - \tau) \frac{\pi}{\rho + \pi} \theta_m F \hat{\mu}. \]  
(A24)

In the full-monetization policy regime, the steady-state values of \( \pi \) and \( \mu \) are determined endogenously by the amount of debt inherited at time \( t_1 \). Equations (11), (A20) and (A21) yield

\[ \bar{m} - m_o = -\frac{\rho \epsilon}{\pi (1 - \epsilon)} [b(t_1) - b_o], \]  
(A25)

\[ \bar{\pi} - \pi_o = \frac{\rho}{m (1 - \epsilon)} [b(t_1) - b_o], \]  
(A26)

\[ \bar{F} - F_o = \frac{(\sigma - \tau) \theta_m \rho F}{(\rho + \pi) m (1 - \epsilon)} [b(t_1) - b_o]. \]  
(A27)

The Boundary Conditions that Link the Transition Paths in the Two Policy Regimes

The second task is to find six boundary conditions that tie down \( h_1-h_4 \) in (A10)-(13), \( h_5 \)
in \((A17)-(A19)\), and \(b(t_1) - b_o\) in \((A25)-(A27)\). The boundary conditions identify the unique path in the TM regime that is consistent with optimizing behavior and hooks up with the convergent saddle path the economy traverses after the return to full monetization of the fiscal deficit at time \(t_1\).

The initial values of the predetermined variables \(F\) and \(b\) supply restrictions at the start of each policy regime. At \(t = 0\), equations \((A11)\) and \((A13)\) give

\[
\begin{align*}
b^* - b_o + s_{21} h_1 + s_{22} h_2 + s_{23} h_3 + s_{24} h_4 &= 0, \\
F^* - F_o + s_{41} h_1 + s_{42} h_2 + s_{43} h_3 + s_{44} h_4 &= 0.
\end{align*}
\]

At \(t = t_1\), the stock of internal debt settles at its new steady-state level (\(\dot{b} = 0\) for \(t > t_1\)); moreover, the solution for \(F(t_1)\) in \((A13)\) must be the same as the solution in \((A18)\). Thus

\[
\begin{align*}
b(t_1) - b_o &= b^* - b_o + s_{21} h_1 e^{\lambda_1 t_1} + s_{22} h_2 e^{\lambda_2 t_1} + s_{23} h_3 e^{\lambda_3 t_1} + s_{24} h_4 e^{\lambda_4 t_1}, \\
F^* - F_o &= s_{41} h_1 e^{\lambda_1 t_1} + s_{42} h_2 e^{\lambda_2 t_1} + s_{43} h_3 e^{\lambda_3 t_1} + s_{44} h_4 e^{\lambda_4 t_1} + \frac{1 - \gamma k}{\lambda_5} h_5 e^{\lambda_5 t_1}.
\end{align*}
\]

Two more boundary conditions can be derived from the fact that foreseen jumps in real expenditure, the nominal price of the nontraded good, and the nominal exchange rate are inconsistent with optimizing behavior.\(^{42}\) Note also that the path of real money balances is continuous for \(t > 0\). (Nominal money balances \(M\) are predetermined and the path of the price level \(P\) is continuous for \(t > 0\).) Hence, both \(E\) and \(m\) are constant at \(t_1\). This implies that equations \((A10)\) and \((A12)\) must deliver the same solution at \(t_1\) as their counterparts \((A17)\) and \((A19)\), viz.:

\[
\begin{align*}
m^* - m^* &= s_{11} h_1 e^{\lambda_1 t_1} + s_{12} h_2 e^{\lambda_2 t_1} + s_{13} h_3 e^{\lambda_3 t_1} + s_{14} h_4 e^{\lambda_4 t_1} \\
&- \left\{ \frac{b}{E} n_4 - \frac{m}{E} n_7 - \gamma k X - z (1 - \gamma k) \right\} (\gamma k - 1) \\
&- \lambda_5 \left( \frac{b}{F} n_3 + \frac{m}{F} n_5 \right) \frac{\lambda_5}{n_2 b/m + n_6 - \pi - \lambda_5} h_5 e^{\lambda_5 t_1}, \\
\end{align*}
\]

\[
\begin{align*}
h_5 e^{\lambda_5 t_1} &= s_{31} h_1 e^{\lambda_1 t_1} + s_{32} h_2 e^{\lambda_2 t_1} + s_{33} h_3 e^{\lambda_3 t_1} + s_{34} h_4 e^{\lambda_4 t_1}.
\end{align*}
\]

Equations \((A28)-(A33)\) can be solved jointly for \(h_1-h_5\) and \(b(t_1) - b_o\). These solutions, the
solutions for the eigenvalues $\lambda_1$-$\lambda_5$, the solutions for the eigenvectors $(s_{1j}, s_{2j}, s_{3j}, s_{4j})$ ($j = 1$-$4$), equations (A10)-(A13) and (A17)-(A19) define the complete transition path. Once the solutions for $F(t)$, $m(t)$ and $E(t)$ are in hand, the paths for $r$ and $\pi$ can be retrieved via (A7) and (A8).

**The Deficit-Share Formulation of Tight Monetary Policy**

The model is solved in the same way but with (22)-(23) replacing (9)-(10) and

$$m^* - m_o = -\frac{eb}{1 - \epsilon} \frac{1 + m/b}{1 - \alpha[1 - \pi/\rho - b/m(1 - \epsilon)]} d\alpha,$$

(A34)

$$b^* - b_o = \frac{\pi b(1 + m/b)}{\rho[1 - \alpha[1 - \pi/\rho - b/m(1 - \epsilon)]]} d\alpha,$$

(A35)

$$F^* - F_o = \frac{(\sigma - \tau)\theta mF}{(\rho + \pi)m(1 - \epsilon)} \frac{\pi b(1 + m/b)}{\rho[1 - \alpha[1 - \pi/\rho - b/m(1 - \epsilon)]]} d\alpha,$$

(A36)

replacing (A22)-(A24).
NOTES

1. Amoako-Tuffour (1999) emphasizes the importance of this link in Ghana.

2. The theoretically correct measure of the real interest rate is the nominal interest rate less the relevant forward-looking inflation rate. When calculated as the nominal rate on 91-day T-Bills minus the 3-month ahead inflation rate, the real interest rate was 4.1% in May, 29.3% in June, 34.3% in July and 41.2% in August. If the contemporaneous (annualized) monthly inflation rate is substituted for the 3-month ahead inflation rate, the numbers change to 17.9%, -55.3%, 60.6% and 27.5%. [Source: Author's calculations from data in Monthly Economic Review (Central Bank of Kenya) and Economic Survey (Central Bureau of Statistics).]

3. Since the inflation rate fluctuated wildly from month to month, calculations of the real interest rate are highly sensitive to whether expected inflation is approximated by contemporaneous or future inflation. If contemporaneous monthly inflation is the right proxy, then the real interest rate was 58% in March and 60% in July but highly negative in April and June. When the 3-month ahead inflation rate replaces the contemporaneous inflation rate, the real interest rate in March changes to -76% and the real rate in June jumps above 100%. (Because of deflation in October, the forward-looking real rate for July is absurdly high; however, the public probably did not anticipate that a cash budget and drastic expenditure cuts would be imposed 2-3 months later.)

4. I am indebted to Chris Adam for supplying me with the monthly CPI series used to calculate these figures.

5. Another point needs to be kept in mind: the raw data do not give an accurate picture of how TM affected capital flows unless adjustments are made for initial conditions and the impact of partial liberalization of the capital account. If net inflows were positive in the first half of 1993, they may have reflected a one-time adjustment to the relaxation of exchange controls and the severe liquidity squeeze in preceding years that had lowered the ratio of M2 to GDP by 55% since 1987.

6. These numbers are the period-end inflation rates reported in Economic and Financial Review (Central Bank of Nigeria). There appears to be a lot of uncertainty, however, about what the true inflation rate was in 1988 and 1989. In Moser, Rogers and van Til (1997), the period-end inflation rate is 39% for 1988 and 44.7% for 1989. According to International Financial Statistics the period-average inflation rate rose from 38.2% in 1988 to 50.5% in 1989; but in Economic and Financial Review the increase is only from 38.3% to 40.9%, and in Moser, Rogers and van Til (1997) the period-average rate decreases slightly from 54.5% to 50.5%.

7. Data from other sources corroborate the official figures for the capital account. Cross-border deposits held by the non-bank public rose from $1.96bn. in 1988 to $2.66 bn. in 1989 and to $3.53bn. in 1990 (International Financial Statistics).

8. The assumption of indexed bonds is standard in the literature on monetarist arithmetic. It eliminates the risk that the government will repudiate part of its debt by engineering
an unexpected increase in the price level.

9. I do not assume Cobb-Douglas preferences. Since the analysis is based on small changes, however, $\gamma$ can be treated as constant and equation (1) yields the same solution for changes in the price level as the exact consumer price index.

10. The Kenyan authorities described the exchange rate system in this period as a “partial float.” (See the central bank’s *Economic Report for the Financial Year Ended June 30, 1993.*). Nigeria may be another case of a near float. Between 1986 and 1993, the government appeared to follow a policy of adjusting the official exchange rate in line with the parallel rate (Azam, 1999). But adjustment occurred with a lag and there were large variations in foreign exchange reserves in some periods.

11. On the downward sloping portion of the seigniorage Laffer curve, higher fiscal deficits are associated with lower inflation and the dynamics are indeterminate because an infinity of paths converge to the stationary equilibrium.

12. The values for all other variables ($\eta, \theta_m, \theta_f$, etc.) are pinned down by the values stated in the text and restrictions implied by the first-order conditions and the steady-state conditions.

13. The combined weight of rent, recreation, entertainment and education, transport and communications, and health and personal care in the the CPI is 40.8%. If half of miscellaneous goods and services, fuel and power, and household equipment and operations are nontradable, then the weight rights to 47.4%.

14. In the late eighties, the World Bank’s import restrictiveness index was much higher for Nigeria than for any other country in SSA (World Bank, 1994, p.230).

15. The lowish value of $\gamma$ is consistent with the extremely high weight for food in the CPI. (The weight is 69.1%. This seems too high; the weight for food in Kenya’s CPI is 38.7%.)

16. A value of 10% for $r$ is consistent with lower values for $\rho$ if Calvo-type repudiation risk is present or if there is exogenous growth $\alpha$ from technical progress. In the latter case, $r = \rho + \alpha/\tau$ across steady states, and the government borrows effectively at the interest rate $\rho + (1 - \tau)\alpha/\tau$.

17. Revenue from indirect taxes was 22.2% of consumption spending in 1992-93 (Statistical Abstract 1998).

18. This measure is superior to total reserve money (line 14 in IFS) as it does not include reserve money held by “other public entities.” (The inflation tax is levied only on the private sector.) The stock of high-powered money was unusually high (10.1% of GDP) in Kenya in 1992 because of extra spending before the elections. Much of the extra money was withdrawn through open market sales in the first two months of 1993 (before the start of the TM episode).

19. I wish to thank Catherine Pattillo for sending me the estimates for Nigeria and Kenya.

20. TM results in high inflation and high real interest rates when the elasticity of substitution
between domestic and foreign currency ($\sigma$) is sufficiently large. The ratio of foreign to domestic currency affects mainly the location of the zone where inflation and real interest rates increase sharply. In Table 3, for example, the zone of high real interest rates is $\sigma = 3.75 - 4.2$ when $\tau = .25$ and $t_1 = .5$. The zone drops down to $\sigma = 2.75 - 3.22$, with the values for the elasticity of inflation and the semi-elasticity of the real interest rate being similar in magnitude, when the stock of foreign currency is initially 12% of GDP instead of 8%.

21. This figure may be too low because new debt issues were considerably large than the outstanding stock of debt during the TM period. Garba (1996, p.93) calls this the “domestic debt puzzle.”

22. The solutions presented in the next section are elasticities with respect to $\mu$ (nominal money growth) for small changes based on a linearized version of the model. This creates problems in tracking the term involving real interest payments in the government budget constraint. The true variation in the term $rb$ is $d(rb) = r_t db + b_t dr$. The linearized approximation, however, is $\rho db + b^* dr$, where $b^* = b(t_1)$ is the new steady-state value of government debt. For small changes, $b^* \approx b_0$; but then both terms in the linear approximation are considerably smaller than in the true variation ($r_t \gg \rho$ and $b_t \gg b_0$). Four months into the TM program, on June 30, 1993, the value of the internal debt was 31% of GNP. This figure is larger than $b_0$ but much less than $b(t_1)$ — the TM regime lasted another two months and real interest rates peaked in July. Thus the solutions from the linearized model where the debt is 31% of GDP certainly underestimate the adverse macroeconomic effects in the later stages of the TM regime and probably in the earlier stages as well. ($b^* dr > b_t dr$ until $t > .25$, but $\rho db < r_t db$, and with forward-looking agents underestimation of the future impact on inflation and the real interest rate results in underestimation of the impact in earlier periods.)

23. See Agenor and Montiel (1996, Table 10.1) for a brief review of the literature.

24. When Sargent and Wallace assert that “... if the interest rate on bonds is greater than the economy’s growth rate, the real stock of bonds will grow faster than the size of the economy” (1981, p.2), they presume too much. In the present model, the impact on steady-state debt is not known beforehand: it depends on how changes in the real interest rate, the real exchange rate, and tax revenue alter the fiscal deficit relative to seigniorage on the transition path. Even in closed-economy models where the primary fiscal deficit is constant, Sargent and Wallace’s claim is overly strong. See Buffie (2001a) for a proof that transitory increases in seigniorage alone can give rise to pleasant monetarist arithmetic.

25. This statement is a bit loose. Obviously money growth is permanently lower on equilibrium paths characterized by Pleasant Monetarist Arithmetic. But slower money growth after $t_1$ is guaranteed only if the government can precommit to a sufficiently long initial period of TM.

26. In some countries the deficit-share rule was explicit. The central bank of Turkey, for example, did not commit to a particular growth rate of the money supply in 1989; instead it announced that the share of the deficit financed by printing money would be reduced.
27. The attempt in 1986 to finance more of the fiscal deficit through daily T-Bill auctions resulted in immediate, large increases in the nominal interest rate and the fiscal deficit, accelerating inflation and depreciation of the currency, and loss of monetary control (Fardi, 1991).

28. Due to lack of data and the extreme instability of the economy in 1992 — inflation, the fiscal deficit and seigniorage were at unsustainable levels — I was not able to calibrate the model to Zambia’s 1993 TM episode.

29. The solutions for $\pi$ and $b$ are elasticities with respect to $\alpha$, while those for $r$ are semi-elasticities. When comparing results across tables, note that at the initial equilibrium a 10% reduction in $\alpha$ generates an equal 10% reduction in money growth. Ex post, however, the reduction in money growth will differ under the deficit-share and fixed money growth rules.

30. The stock of government debt does not affect $P_n, r_i, P_k, \pi$ or $\chi$. In addition, $m$ and $F$ do not enter into the solutions for $r_i, P_n$ and $P_k$.

31. In the saddle point solution that prevails after $t_1$, the four constants associated with the four positive eigenvalues equal zero.

32. Cross-section estimates cluster around unity while those based on time series data are typically close to .5. See Gaude (1975), White (1978), and Mansur and Whalley (1984).

33. The figure for Kenya is one minus the share of imported machinery in private investment [(calculated from data on pp.51, 60, 98 in Statistical Abstract 1998 (Central Bureau of Statistics)]. For Nigeria, I had access only to aggregate (public + private) data.

34. The 28% figure for $\theta^n_L$ is labor’s combined share in manufacturing, services and construction. (Manufacturing was effectively nontradable in 1988.)

35. In Nigeria, private tradables production in 1988 was essentially agricultural production. The state-owned oil sector enters the model only through the variable $X$.

36. Technically speaking, the labor market clears only asymptotically. After 1.5 years, however, the change in the real wage is 80-82% of the change in the market-clearing wage. (The lower number is for Nigeria.)

37. The estimates of Summers (1981), Hayashi (1982), and Alonso-Borrego and Bertolila (1994) imply, respectively, q-elastitcicies of .30-1.02, .85, and .42-.54 when (as in the model) $I = \delta K$ and $\delta = .05$. Abel and Blanchard (1986) estimate the q-elasticity to lie between .1 and .3. However, the elasticity with respect to the marginal profit component of q exceeds unity. Engel and Foley’s (1975) estimates range from .78 to .87 for producer’s durable equipment and from 2 to 2.3 for nonresidential construction. Malkiel et al. (1986) estimate the q-elasticity for 12 two-digit SIC industries and obtain values ranging from 0 to 1.85, with half of the estimates equaling or exceeding unity. The estimates for $\phi''$ in Galeoti and Schiantarelli (1991) — the “a” parameter in their cost function — yield a q-elasticity of 1.07-1.6 in the present model.
38. A few estimates also exist for LDCs, but it is doubtful that the small, thin stock markets in these studies — the source for data on the demand price of capital — accurately reflect information about the fundamentals that drive private investment decisions.

39. Real wage concessions may not be forthcoming even when the union maximizes utility of its representative member. Much depends on how the slope of the labor demand schedule changes when the schedule shifts to the left.

40. Real wage rigidity is confined to the formal sector in most LDCs. The conclusion that real output and the aggregate capital stock decrease permanently also holds, however, when a flex-wage informal sector co-exists with a formal sector where the real wage is rigid. What may differ is the impact on employment. If workers laid off in the formal sector refuse to take jobs in the informal sector, then the results are unchanged; if they accept the jobs, then underemployment increases instead of open unemployment.

41. This remark is specific to the Kenya 1993 variant of the model. In other plausible calibrations of the model, PMA can occur when $\epsilon$ is $3$.4.

42. Since foreseen jumps in the multiplier $\beta$ are inconsistent with optimizing behavior, equations (5) and (12) imply that $P_n$ and $E$ do not jump at time $t_1$. To see that the nominal exchange rate $e$ does not jump at $t_1$, examine the first-order condition associated with the optimal choice of $E$ when the private agent’s budget constraint is specified in nominal terms. This is $V_E(P_n, E) = \tilde{\beta}e(1 + z)$. Optimizing behavior and perfect foresight again imply that $\tilde{\beta}$ (the multiplier associated with the nominal budget constraint) does not jump at $t_1$. Since $P_n$ and $E$ are constant at $t_1$, the nominal exchange $e$ and the nominal price of the nontraded good must be constant as well.
References


