Borrowing for Growth: Big Pushes and Debt Sustainability in Low-Income Countries

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Abstract

We analyze big push borrowing-and-investment programs in a new model-based framework that answers the most trenchant criticisms IMF and World Bank debt sustainability analysis. The new framework is grounded in a fully-articulated, dynamic macroeconomic model. It allows for financing schemes that mix concessional, external commercial, and domestic debt, while taking into account the impact of public investment on growth and constraints on the speed and magnitude of fiscal adjustment. Supplementing concessional loans with nonconcessional borrowing in world capital markets is generally a high-risk, high-return strategy. It may greatly enhance the prospects for debt sustainability or lead to spectacular failure; much depends on the fine details governing debt contracts, the dynamics of growth, and the speed of fiscal adjustment.

Keywords: debt sustainability, public investment, fiscal policy, infrastructure, growth.

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1. Introduction

There is a large investment gap in LICs, particularly in infrastructure. Addressing it is critical to increasing potential growth. Aid is limited and LICS will likely rely increasingly on domestic resources and noncessional external borrowing. A wide range of issues . . . merit attention in this context. (IMF, 2012)

The past decade has seen a steady increase in nonconcessional borrowing by low-income countries (LICs). In retrospect, this development was predictable. The Highly Indebted Poor Country (HIPC) and Multilateral Debt Relief initiatives reduced the external debt of the poorest countries by eighty percent. Concurrently the borrowing environment improved as new, non-traditional creditors emerged, domestic debt markets continued to develop, and world capital markets became more accessible and in some cases positively welcoming.1 LICs in Sub-Saharan Africa (SSA) and elsewhere have responded to these developments by borrowing aggressively to scale up public investment programs and anti-poverty programs. Coming so soon after the HIPC initiative, the rapid accumulation of new debt and the increasing reliance on more expensive commercial debt are cause for concern (Arnone and Presbitero, 2010; IMF, 2013; Prizzon and Mustapha, 2014; Reuters, 2014). Debt indicators are still far below the levels seen in the mid-90s, but there is a growing fear that the resumption of large-scale borrowing may mark the start of a new lend-and-forgive cycle. Debt sustainability analysis (DSA) is back in vogue for a reason.

1.1 Debt Sustainability Analysis: A Quick Tour of the Literature

In principle, DSA is a straightforward matter of fiscal accounting. The overall impact of a borrowing and investment program on the government’s intertemporal budget constraint coincides with its impact on public sector net worth. If net worth increases, future fiscal surpluses cover the cost of debt service. There is no need to cut other expenditures or raise tax rates; the investment program pays for itself. Conversely, if net worth declines, some supporting fiscal adjustment is required to maintain solvency and prevent explosive growth in government debt.

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1 A recent survey by Citigroup describes the new borrowing environment: “Knowing they want to borrow money to spend on projects to close the infrastructure deficit, governments in SSA have faced a wave of lenders looking to get money out of the door and into their pockets: whether investment bankers extolling the virtues of issuing Eurobonds; apparently cheap BRIC country loans, but with long-term catches on payment implications . . . .” (Cowan, 2010, p.9)
While solvency is the correct criterion in theory, it has proven difficult to apply fruitfully in practice. Several approaches have been tried in the literature. Applications for developed countries have concentrated on estimation of impulse responses in structural VAR models that capture feedback effects between tax revenue, output, public investment, and private investment. Perotti (2004) finds that infrastructure investment is not self-amortizing in models estimated for Germany, the United States, Canada, the United Kingdom, and Australia. Pereira and Pinho (2006) reach the same conclusion for seven of twelve countries in the Euro zone, but in the other five countries the long-run fiscal impact is neutral or positive. By contrast, the estimates for Portugal in Pereira and Andraz (2010) and for Brazil in Ferreira and Araujo (2008) are highly encouraging: in the long run, tax revenues increase 3-7 times more than investment outlays.

The take-away for LICs in these studies is unclear. For several reasons, the methodology and results are suspect. First, all of the models — including that for Brazil — presume a closed economy; effects stemming from variations in the real exchange rate are conspicuous by their absence. Second, there is no good way to handle recurrent expenditure. When recurrent expenditure is left out of the system, as in Pereira and Pinho (2006), Ferreira and Araujo (2008), and Pereira and Andraz (2010), the estimates are biased. (Recurrent expenditure is usually correlated with other variables in the system, including public investment.) When it is included, it is impossible to conduct the right policy experiment because the impulse responses commingle the effects of different expenditures. Thus Perotti’s estimates of the impact on public sector net worth mix the increase in infrastructure investment with induced increases in government consumption and transfers payments. Third, the estimates for the long-run output multiplier are not credible. In Perroti (2004), the long-run multiplier is positive only for Germany; the other four multipliers are negative, with three being large in absolute terms. Highly negative rates of return also show up in four of the twelve estimates in Pereira and Pinho (2006), alongside small positive returns in the other cases; with the exception of Spain, however, none of the confidence bands exclude zero. The estimates for Portugal and Brazil in Pereira and Andraz (2010) and Ferreira and Araujo (2008) suffer from the opposite credibility problem: the positive multipliers are absurdly large. Any number of factors could explain the eccentric variance in these results. Most notably, standard errors
in the impulse responses are invariably large at long time horizons. Judging from the results in Kamps (2004), the problem is endemic to VAR models.\textsuperscript{2} Better models that capture open-economy interactions will help some. But the difficulties in generating precise, sensible estimates for the long-run multiplier and in separating the effects of recurrent expenditure from the effects of capital expenditure cast doubt on the entire enterprise. The future may bring a different verdict, but, for now, structural VARs and DSA should part company.

Another strand of the literature analyzes debt sustainability in simple quasi-reduced form models. Garcia and Rigabon (2005) and Celasun et al. (2007) estimate VARs and carry out Monte Carlo simulations to compute probabilities of debt distress (i.e., probabilities debt will exceed certain thresholds) in emerging economies. Bohn (1998, 2008), Mendoza and Oviedo (2004), Abiad and Ostry (2005), Mendoza and Ostry (2008), and Ghosh et al. (2011) investigate the conditions for debt stability in emerging-economy/developed-country models that consist essentially of two equations, one that tracks debt accumulation and a second that describes the response of the primary fiscal balance to growth in the debt. Real output, public revenues, and the real interest rate are strictly exogenous.\textsuperscript{3}

The models in this literature convey various insights but they are not suitable for the analysis of debt sustainability in big push investment programs in LICs. Too much is missing or hidden in opaque reduced-form solutions. When governments borrow to promote growth, expansion of tax bases, variations in the real exchange rate, the efficiency of public investment, fluctuations in the real interest rate on internal debt (important even when domestic debt does not help finance the investment program), cost overruns stemming from absorptive capacity constraints, and the mix of concessional and nonconcessional loans all affect the path of the fiscal deficit and debt sustainability. We also believe that socio-political constraints on fiscal adjustment should be modeled directly as constraints on fiscal instruments, not as an upper bound on the primary fiscal surplus.\textsuperscript{4} It is not so difficult,

\textsuperscript{2} The estimate for the long-run multiplier is statistically insignificant in 20 of 22 OECD countries. In contrast to Pereira and Andraz (2010), the estimated multiplier is negative for Portugal.

\textsuperscript{3} Mendoza and Oviedo (2004) and Ghosh et al. (2010) allow the external risk premium to vary with the level of debt.

\textsuperscript{4} Ghosh et al. (2011) present evidence that "fiscal fatigue" may limit the capacity for further increases in the primary fiscal surplus at high levels of debt. [See also Abiad and Ostry (2005) and Mendoza and Ostry (2008).] This is similar to our criterion that the borrowing + investment program is sustainable only if debt converges to a stationary level without violating socio-political constraints on how much and how fast fiscal policy can change.
after all, to accept endogenous, growth-driven improvements in the primary fiscal surplus; the hard, stressful part is increasing the surplus through higher tax rates and expenditure cuts. Debt sustainability analysis of big push scenarios is inherently complicated; it requires a medium-sized macroeconomic model with substantial structural detail, numerous policy instruments, and realistic constraints on the speed and magnitude of fiscal adjustment.

Summing up, the literature on debt sustainability is bereft of useful analytical tools and reliable, informative results. Neither empirical studies nor existing theory provide much guidance to policy makers. LDCs borrowing for growth are flying blind (Carranza et al., 2014).

1.2 A New Model-Based Framework for DSA

Our objective in this paper is to help policy makers see better. The visual aid on offer is a transparent, fully-articulated theoretical model of debt sustainability recently developed by four IMF staff and one alien. The model has been tested in pilot projects and applied in nineteen countries (mainly in Sub-Saharan Africa). We are now placing the new model-based framework for debt sustainability analysis (MBDSA) in the public domain for inspection by outside parties. Criticisms and suggested improvements are truly welcome. The MBDSA is meant to address the most serious shortcomings in the existing IMF-World Bank debt sustainability framework (see Box 1).

The main body of the paper is organized into six sections. Sections 2 and 3 explicate the new MBDSA. We construct and calibrate an optimizing intertemporal model of debt sustainability that features sector-specific private capital, productivity-enhancing infrastructure, concessional loans with a variable grant element, domestic debt, external commercial debt, an absorptive capacity constraint, variable efficiency of public investment, and poor hand-to-mouth consumers. Since the model tracks the path of debt over an infinite horizon, we can test for stability and use the results to derive rigorous, theory-based indicative thresholds. These correspond to the peak level of debt on trajectories that skirt the inside of the boundary line for stable debt dynamics.\(^5\)

\(^5\) Most policy makers will want to build in a cushion against potential adverse shocks. This determines how close the peak level of debt is allowed to get to the stability boundary in the baseline projection.
Sections 4 - 6 address various aspects of big-push investment programs, employing a mix of analytical and numerical methods. In Section 4 we simplify the model and use pen and paper to derive a set of propositions for short- vs. long-run crowding in of private investment and for the long-run impact on the fiscal budget. Following this, Sections 5 and 6 analyze debt sustainability in the fully-loaded model under different assumptions about the flexibility of fiscal adjustment and for different mixes of concessional and commercial debt. Our results, previewed below, speak directly to many of the issues that have preoccupied the literature:

- Despite the low tax-take in LICs, increases in infrastructure investment are often self-financing in the long run. The favorable long-run effect on the fiscal budget reflects extra increases in output and revenue associated with strong crowding in of private capital.

- Although infrastructure investment is often self-financing across steady states, the revenue gains from growth do not materialize soon enough to obviate the need for difficult fiscal adjustments in the short and medium run. When concessional loans finance one-half of total investment, the transition path always includes an extended phase in which the tax rate on consumption increases by 3-6 percentage points.

- Supplementing concessional loans with nonconcessional borrowing in world capital markets can smooth away difficult fiscal adjustments, reconciling scaling up of public investment with constraints on feasible increases in tax rates and feasible cuts in non-investment expenditure, when user fees recoup all recurrent costs, the return on infrastructure is high (20-30%), and cuts in noninvestment expenditure offset growth in public sector wages. But the strategy is risky. If public investment is inefficient, if fiscal adjustment is too slow, or if absorptive capacity constraints cause temporary cost overruns, it can easily lead to explosive growth of the external debt.

- The prospects for success are excellent in one important special case. In SSA average user fees equal average recurrent costs for operation and maintenance of infrastructure assets. Due to low collection rates, however, revenue from user fees barely covers half of recurrent costs. If this can change — even slowly — the efficacy of fiscal smoothing improves dramatically. In the base case, where noninvestment expenditure is downwardly inflexible and user fees pay for 50% of recurrent costs, borrowing on commercial terms to smooth fiscal adjustment does more harm than good: the consumption tax rises from 15% to 19% and stays there forever (i.e., until year 91). By contrast, when the collection rate increases gradually from 50 to 100% over ten years, policy makers can smooth away all increases in the consumption tax. Moreover, a small increase in the tax to 15.5% insures debt sustainability against a variety of large adverse financial shocks (substantial cost overruns, highly inefficient public investment, big unexpected increases in borrowing rates, etc.)

- The sign of the crowding-in coefficient for private investment varies over time. The coefficient is positive and large in the long run, but negative in the short and medium run. Moreover, both the depth and the duration of temporary crowding out are greater the
larger the increase in the private capital stock across steady states. Temporary crowding out is averted in some cases where nonconcessional borrowing smooths the path of fiscal adjustment; but crowding in is weak, at best, and cumulative growth of the private capital stock badly lags growth in the stock of infrastructure at the 20-year horizon. The big push in public investment does not trigger a "growth takeoff."\textsuperscript{6}

- The sustainable level of debt is surprisingly robust to the fraction of recurrent costs financed by user fees and to variations in the values of deep parameters that describe preferences and technology, including the return on infrastructure. Nevertheless, it is hard to justify uniform thresholds for sustainable debt. Indicative thresholds vary by 10-50% of GDP across scenarios that differ in the size of the investment program, borrowing rates, initial conditions, absorptive capacity constraints, and the efficiency of public investment. Debt sustainability analysis is inevitably case specific.\textsuperscript{7}

The paper concludes in Section 7 with a discussion of ongoing research to develop variants of the benchmark DSF that can accommodate the demand for more diverse country- and program-specific DSA. We are optimistic that a portfolio of models can strike the right balance: one size does not fit all, but a few sizes may fit many.

2. The Model

Our framework for DSA is the standard two-sector model of a small open economy embellished with multiple types of public sector debt and multiple tax and spending variables. The country produces a composite traded good and a nontraded good from capital $k$, labor $L$, and government-supplied infrastructure $z$. The traded good is the numeraire and $x$ and $n$ subscripts refer to the tradables and nontradables sectors. All quantity variables except labor are detrended by $(1+g)^t$, where $g$ is the exogenous long-run growth rate of real GDP. Since the time horizon is 20+ years, the model abstracts from money and all nominal rigidities.

We lay out the model in stages, starting with the specification of technology.

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\textsuperscript{6} This is consistent with the empirical evidence on big push investment programs, which are rarely associated with growth takeoffs. See Easterly (2006) and Warner (2014).

\textsuperscript{7} The data show that LDCs have experienced debt servicing problems at widely different levels of debt. See Table 9.1 in Montiel (2010).
Technology

Cobb-Douglas production functions convert inputs into output, with infrastructure entering as a public good that enhances productivity in both sectors:

\begin{align*}
q_{x,t} &= a_x z^{\alpha_x} k_{x,t-1}^{\alpha_x} L_{x,t}^{1-\alpha_x}, \\
q_{n,t} &= a_n z^{\alpha_n} k_{n,t-1}^{\alpha_n} L_{n,t}^{1-\alpha_n}.
\end{align*}

(1)

(2)

Factories and infrastructure are built by combining one imported machine with \( a_j \) (\( j = k, z \)) units of a nontraded input (e.g., construction). The supply prices of private capital and infrastructure are thus

\begin{align*}
P_{k,t} &= 1 + a_k P_{n,t}, \\
P_{z,t} &= 1 + a_z P_{n,t},
\end{align*}

(3)

(4)

where \( P_n \) is the relative price of the nontraded good.

Factor Demands

Competitive profit-maximizing firms equate the marginal value product of each input to its factor price. This yields

\begin{align*}
P_{n,t}(1 - \alpha_n) q_{n,t}/L_{n,t} &= w_t, \\
(1 - \alpha_x) q_{x,t}/L_{x,t} &= w_t, \\
P_{n,t}\alpha_n q_{n,t}/k_{n,t-1} &= r_{n,t}, \\
\alpha_x q_{x,t}/k_{x,t-1} &= r_{x,t},
\end{align*}

(5)

(6)

(7)

(8)

where \( w \) is the wage and \( r_j \) is the rental earned by capital in sector \( j \). Labor is intersectorally mobile, so the same wage appears in (5) and (6). Capital is sector-specific, but \( r_x \) differs from \( r_n \) only on the transition path. After adjustment is complete and \( k_x \) and \( k_n \) have settled at their equilibrium levels, the rentals are equal.

Private Sector Optimization Problems

There are two types of private agents, savers and non-savers (with the latter distinguished by a 1 subscript). Labor supply of savers is fixed at \( L \) while that of non-savers is \( L_1 = aL \). Preferences of the two agents qua consumers are described by a linearly homogeneous CES
utility function. The optimal choices for consumption of the traded and nontraded goods are subsumed in the composite consumer goods $c$ and $c_1$. The exact consumer price index is $P = [\kappa+(1-\kappa)P_n^{1-\epsilon}]^{1/(1-\epsilon)}$, where $\kappa$ and $\epsilon$ are CES distribution and substitution parameters.

Non-savers live hand to mouth, consuming all of their income from wages and transfers ($T$) each period. Let $h$ denote the consumption value added tax (VAT) and assume transfers are proportional to the agent’s share in aggregate employment. The non-saver’s budget constraint then reads

$$(1 + h_t)P_t c_{1,t} = w_t L_1 + \frac{a}{1 + a} T_t. \quad (9)$$

Savers’ behavior is more complicated. They solve the problem

$$\max \sum_{t=0}^{\infty} \beta^t (c_t)^{1-1/\tau} \frac{1}{1 - 1/\tau}, \quad (10)$$

subject to

$$P_t b_t - b_t^* = r_{x,t} k_{x,t-1} + r_{n,t-1} k_{n,t-1} + w_t L_t + \frac{T_t}{1 + a} - \frac{1 + r^*}{1 + g} b_{t-1}^* + \frac{1 + r_{t-1}}{1 + g} P_k b_{t-1} - P_k \left[ i_{x,t} + i_{n,t} + \frac{v}{2} \left( \frac{i_{x,t}}{k_{x,t-1}} - \delta - g \right)^2 k_{x,t-1} \right] + \frac{v}{2} \left( \frac{i_{n,t}}{k_{n,t-1}} - \delta - g \right)^2 k_{n,t-1} - \frac{\eta}{2} (b_t^* - b^*)^2 - P_t c_t (1 + h_t) - \mu_t z_{t-1}; \quad (11)$$

$$(1 + g) k_{x,t} = i_{x,t} + (1 - \delta) k_{x,t-1}; \quad (12)$$

$$(1 + g) k_{n,t} = i_{n,t} + (1 - \delta) k_{n,t-1}; \quad (13)$$

where $\beta = 1/[(1 + \rho)(1 + g)^{(1-\tau)/\tau}]$ is the discount factor; $\rho$ is the pure time preference rate; $\tau$ is the intertemporal elasticity of substitution; $b$ is the stock of domestic bonds; $i_j$ is gross investment in sector $j$; $\delta$ is the depreciation rate; $r^*$ is the exogenous real interest rate on foreign loans $b^*$; $r$ is the real interest rate on domestic bonds; $\mu$ is the user fee charged for infrastructure services; and $a_o$ and $v$ are positive constants. In the budget constraint (11), $v(\cdot)^2 k_{j,t-1}/2$ captures adjustment costs incurred in changing the capital stock; $^8 \eta(\cdot)^2/2$ measures portfolio adjustment costs associated with deviations of foreign loans from their steady-state level ($\tilde{b}^*$), and $P_t$ multiplies $b_t$ and $b_{t-1}$ because domestic bonds are indexed

$^8$ For simplicity, we assume that adjustment costs are zero when the capital stock grows at the trend growth rate $g$. This ensures that adjustment costs are zero across steady states as in models that ignore trend growth.
to the price level. Observe also that the trend growth rate appears in several places in (11)-(13), reflecting the fact that some variables are dated at \( t \) and others at \( t-1 \).

The choice variables in the optimization problem are \( e_t, b_t, b_t^*, i_{j,t}, \) and \( k_{j,t} \). Routine manipulations of the first-order conditions deliver

\[
c_t = c_{t+1} \left( \beta_1 \frac{1 + r_t}{1 + g_t} \right)^{-\tau},
\]

\[
\eta(b_t^* - \bar{b}^*) = 1 - \frac{1 + r_t}{1 + \bar{r}} \frac{P_t}{P_{t+1}},
\]

and

\[
(1 + r_t) \frac{P_{t+1}}{P_t} \frac{P_{k,t}}{P_{k,t+1}} \left[ 1 + v \left( \frac{i_{x,t}}{k_{x,t-1}} - \delta - g \right) \right] = \frac{r_{x,t+1}}{P_{k,t+1}} + 1 - \delta
\]

\[
+ v \left( \frac{i_{x,t+1}}{k_{x,t}} - \delta - g \right) \left( \frac{i_{x,t+1}}{k_{x,t}} + 1 - \delta \right) - \frac{v}{2} \left( \frac{i_{x,t+1}}{k_{x,t}} - \delta - g \right)^2,
\]

\[
(1 + r_t) \frac{P_{t+1}}{P_t} \frac{P_{k,t}}{P_{k,t+1}} \left[ 1 + v \left( \frac{i_{n,t}}{k_{n,t-1}} - \delta - g \right) \right] = \frac{r_{n,t+1}}{P_{k,t+1}} + 1 - \delta
\]

\[
+ v \left( \frac{i_{n,t+1}}{k_{n,t}} - \delta - g \right) \left( \frac{i_{n,t+1}}{k_{n,t}} + 1 - \delta \right) - \frac{v}{2} \left( \frac{i_{n,t+1}}{k_{n,t}} - \delta - g \right)^2.
\]

Each of these equations admits of a straightforward intuitive interpretation. Equation (14) is a slightly irregular Euler equation in which the slope of the consumption path depends on the real interest rate adjusted for trend growth and on changes in the VAT. The other three equations are arbitrage conditions. Equations (16) and (17) require the return on capital, net of marginal adjustment costs, to equal the real interest rate. Similarly, equation (15) says that marginal transactions costs offset the interest differential between domestic bonds and foreign loans.

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9 The nominal value of government bonds carried over from the previous period is \( B_{t-1} \). This is marked-up through indexation to \( (P_{t-1}/P_{t-1})B_{t-1} \), where \( P_t = P_0 + (1 - \kappa)P_{t-1}^{1/(1-\kappa)} \). After dividing by \( P_{t-1} \) (the traded good is the numeraire), we get \( (P_t/P_{t-1}) \times (B_{t-1}/P_{t-1}) = P_b_{t-1} \) in the private agent’s budget constraint.

10 The convention for detrending the capital stocks differs from that for other variables. Because \( K_{j,t-1} \) (the capital stock before detrending) is the capital stock in use at time \( t \), we define \( k_{j,t-1} \equiv K_{j,t-1}/(1+g)^t \). Under this convention, \( i_j = (\delta + g)k_j \) in the long run — as required for the capital stock to grow at the trend growth rate \( g \).
Public Investment Efficiency and Accumulation of Infrastructure

In the case of infrastructure, success will require *breaking with the past* by applying greater scrutiny of projects at the selection stage, integrity in procurement, and effective post-completion management to ensure maintenance and efficient operation and continuing accountability to users. (Ndulu, 2006; our emphasis)

Casual observation and indirect empirical evidence support the conjecture in Hulten (1996) and Pritchett (2000) that often the productivity of infrastructure is high but the return on public investment very low for the simple reason that a good deal of public investment *spending* does not increase the stock of productive capital.\(^{11}\) The model allows for this slip betwixt cup and lip. Public investment \(i_z\) increases the stock of physical infrastructure \(\tilde{z}\):

\[(1 + g)\tilde{z}_t = i_{z,t} + (1 - \delta)\tilde{z}_{t-1}.\]  
(18)

Some of the newly built infrastructure, however, may not be economically valuable, productive infrastructure:\(^{12}\)

\[z_t = z_o + s(\tilde{z}_t - \tilde{z}_o), \quad s \leq 1.\]  
(19)

**Fiscal Adjustment and the Public Sector Budget Constraint**

The government spends on transfers, debt service, and infrastructure investment. It collects revenue from user fees for infrastructure services and from the consumption VAT. When revenues fall short of expenditures, the resulting deficit is financed through external borrowing, viz.:

\[dc_t - dc_{t-1} + d_t - d_{t-1} = P_{z,t} \left[ 1 + \frac{i_{z,t}}{\tilde{z}_{t-1}} - \delta - g \right] (i_{z,t} - i_{z,o}) + i_{z,o} \]

\[+ \frac{r_d - g}{1 + g} d_{t-1} + \frac{r_{de} - g}{1 + g} dc_{t-1} + \frac{r_{l-1} - g}{1 + g} P_t b_{t-1} \]

\[+ T_t - h_t P_t (c_t + c_{1,t}) - \mu_t \tilde{z}_{t-1}.\]  
(20)

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\(^{11}\) In Hulten’s (1996) growth accounting decompositions, the index of infrastructure effectiveness is more important than any other factor, accounting for 40% of the difference in the growth rates of high- vs. low-growth LDCs. Measurement error introduced by equating growth of productive capital with net investment can also explain why empirical studies generally find a much stronger positive relationship between growth and physical indicators of infrastructure than between growth and capital stock series calculated via the perpetual inventory method (Straub, 2008).

\(^{12}\) Note that \(z_o\), the initial stock of productive infrastructure, equals \(s_1 \tilde{z}_o\), where \(s_1 \neq s\) when the efficiency of past investment differs from the efficiency of new investment.
where $r_d$ and $r_{dc}$ are the interest rates (in dollars) on concessional debt $d$ and commercial debt $dc$.

The term $P_{z,t}[,\ldots]$ needs some explanation. Because skilled administrators are in scarce supply in LICs, ambitious public investment programs are often plagued by poor planning, weak oversight, and myriad coordination problems, all of which contribute to large cost overruns during the implementation phase.\textsuperscript{13} To capture this, we multiply new investment $(i_{z,t} - i_{z,o})$ by $(1 + \frac{i_{z,t}}{\bar{z}_{t-1}} - \delta - g)\phi \approx [1 + (\bar{z}_t - \bar{z}_{t-1})/\bar{z}_{t-1}]\phi$, where $\phi \geq 0$ determines the severity of the the absorptive capacity constraint in the public sector. Cost overruns depend on both $\phi$ and the speed of scaling up, as measured by the growth rate of the infrastructure stock. The absorptive capacity constraint affects only implementation costs for new projects as $i_{z}/\bar{z} - \delta - g = 0$ in a steady state.

Policy makers happily accept all concessional loans proffered by official creditors. The borrowing + amortization schedule for these loans is fixed exogenously. Thus, in any given year, the \textit{ex ante} financing gap (GAP) is

\begin{equation}
\text{GAP}_t = P_{z,t} \left[1 + \frac{i_{z,t}}{\bar{z}_{t-1}} - \delta - g\right]^{\phi} (i_{z,t} - i_{z,o}) + i_{z,o} + T_o + \frac{1 + r_d}{1 + g} d_{t-1} - d_t
\end{equation}

\begin{equation}
+ \frac{r_{dc} - g}{1 + g} d_{c,t-1} + \frac{r_{t-1} - g}{1 + g} P_t b_{t-1} - h_o P_t (c_t + c_{1,t}) - x_t - \mu t z_{t-1},
\end{equation}

while the stock of commercial debt grows at the rate

\begin{equation}
dc_t - dc_{t-1} = P_{z,t} \left[1 + \frac{i_{z,t}}{\bar{z}_{t-1}} - \delta - g\right]^{\phi} (i_{z,t} - i_{z,o}) + i_{z,o} + \frac{1 + r_d}{1 + g} d_{t-1} - d_t
\end{equation}

\begin{equation}
+ \frac{r_{dc} - g}{1 + g} d_{c,t-1} + \frac{r_{t-1} - g}{1 + g} P_t b_{t-1} - h_i P_t (c_t + c_{1,t}) - x_t - \mu t z_{t-1},
\end{equation}

GAP is revenue collected at the initial tax rate ($h_o$) and current user fee less the sum of infrastructure investment, net concessional borrowing, interest payments on the debt, and initial transfers ($T_o$). In the short/medium run, part of this gap can be financed by commercial borrowing. Debt sustainability requires, however, that the VAT and transfers eventually adjust to cover the entire gap. We let policy makers divide the burden of adjustment (net windfall when GAP < 0) between spending cuts and tax increases. The debt-stabilizing

\textsuperscript{13} Development agencies report that cost overruns of 35% and more are common for new projects in Africa. The most important factor by far is inadequate competitive bidding for tendered contracts (Foster and Bricendo-Garmendia, 2010).
values for transfers and the VAT — their long-run target values — are

\[ T_{\text{target},t} = T_o - \lambda \text{GAP}_t, \quad 0 \leq \lambda \leq 1, \quad (22) \]

\[ h_{\text{target},t} = h_o + (1 - \lambda) \frac{\text{GAP}_t}{P_t(c_t + c_{1,t})}, \quad 0 \leq \lambda \leq 1, \quad (23) \]

where policy makers’ preferences fix \( \lambda \).

Equations (22) and (23) are paired with targets for the long-run levels of domestic and external commercial debt. The reactions functions that govern the paths of \( h \) and \( T \) incorporate these targets as well as socio-political constraints on how much and how fast fiscal policy can change:

\[ h_t = \text{Min} \left\{ h_{t-1} + \lambda_1 (h_{\text{target},t} - h_{t-1}) + \lambda_2 \frac{dc_{t-1} - dc_{\text{target}}}{P_{n,t}q_{n,t} + q_{x,t}}, \bar{h} \right\}, \quad (24) \]

\[ T_t = \text{Max} \left\{ T_{t-1} + \lambda_3 (T_{\text{target},t} - T_{t-1}) - \lambda_4 (dc_{t-1} - dc_{\text{target}}), \bar{T} \right\}. \quad (25) \]

\( \bar{h} \) is the upper bound on the VAT and \( \bar{T} \) is the lower bound on transfers.\(^{14} \) Inside the bounds, the parameters \( \lambda_1-\lambda_4 \) determine whether policy adjustment is fast or slow. Under "slow" adjustment, \( dc \) may rise above its target level in the time it takes \( h \) and \( T \) to reach \( h_{\text{target}} \) and \( T_{\text{target}} \). When this happens, the transition path includes a phase in which \( T < T_{\text{target}} \) and \( h > h_{\text{target}} \) to generate the fiscal surpluses needed to pay down the debt.

The reaction functions embody the core policy dilemma. Fiscal adjustment is painful, especially when administered suddenly in large doses. The government would prefer therefore to phase-in tax increases and expenditure cuts slowly. But if it moves too slowly, or if the bounds on \( h \) and \( T \) constrain adjustment too much, interest payments will rise faster than revenue net of transfers, causing the debt to grow explosively. Large debt-financed increases in public investment are undeniably risky — the economy converges to a stationary equilibrium only if policy makers win the race against time.

In the interests of saving space, we decided not to analyze domestic borrowing and investment programs. Results for this scenario are available, however, in Buffie et al. (2012).

\(^{14} \) \( \bar{T} \) may be rising over time, as in the case where other cuts in non-investment expenditure do not offset growth in public sector wages.
Market-Clearing Conditions

Flexible wages and prices ensure that demand continuously equals supply in the labor market

\[ L_x + L_n = L + L_1 \]  

(26)

and in the nontradedables market

\[ q_{nt} = (1 - \kappa) \left( \frac{P_{nt}}{P_t} \right)^\epsilon (c_t + c_{1,t}) + a_k \left[ i_{x,t} + i_{n,t} + \frac{v}{2} \left( \frac{i_{x,t}}{k_{x,t-1}} - \delta - g \right)^2 k_{x,t-1} \right. \]

\[ + \left. \frac{v}{2} \left( \frac{i_{n,t}}{k_{n,t-1}} - \delta - g \right)^2 k_{n,t-1} \right] + a_z \left[ \left( 1 + \frac{i_{z,t}}{z_{t-1}} - \delta - g \right)^\phi \left( i_{z,t} - i_{z,o} \right) + i_{z,o} \right]. \]  

(27)

The first term to the right of the equal sign in (26) is demand for nontraded consumer goods (retrieved from private agents’ indirect utility functions via Roy’s identity), while the second and third terms link public and private investment to orders for new capital goods. On the right side in (26), both components of labor supply are fixed.

External Debt Accumulation and the Current Account

Finally, adding the public and private sector budget constraints produces the accounting identity that growth in the country’s net foreign debt equals the difference between national spending and national income:

\[ d_t - d_{t-1} + dc_t - dc_{t-1} + b_t^* - b_{t-1} = P_t(c_t + c_{1,t}) + P_{zt} \left[ \left( 1 + \frac{i_{zt}}{z_{t-1}} - \delta - g \right)^\phi \left( i_{z,t} - i_{z,o} \right) + i_{z,o} \right] \]

\[ + P_{kt} i_{x,t} + i_{n,t} + \frac{v}{2} (i_{x,t}/k_{x,t-1} - \delta - g)^2 k_{x,t-1} \]

\[ + \frac{v}{2} (i_{n,t}/k_{n,t-1} - \delta - g)^2 k_{n,t-1} + \frac{r_d - g}{1 + g} d_{t-1} + \frac{r_{dc} - g}{1 + g} dc_{t-1} \]

\[ + \frac{r^* - g}{1 + g} b_{t-1}^* + \frac{\eta}{2} (b_t^* - \bar{b}^*)^2 - P_{nt} q_{nt} - q_{x,t}. \]  

(28)

Equation (28) includes extra terms that reflect the impact of trend growth on real interest costs. The textbook identity emerges when \( g = 0 \).

2.1 Comments on the Model

Three aspects of the model merit brief comment. First, the assumption that portfolio adjustment costs differentiate foreign loans from domestic bonds provides a simple, flexible way to represent the degree to which the private sector is integrated into world capital.
markets. The value of $\eta$ in (15) controls the degree of capital mobility and thus variability of the interest rate spread during the transition from the old to the new steady state. For some emerging market economies, a small $\eta$ may be appropriate. Elastic capital flows then keep the domestic rate close to the foreign rate. In LICs, where $\eta$ is comparatively large, the spread fluctuates much more.

The main alternative to portfolio adjustment costs is an upward-sloping supply curve for foreign loans. This specification has plusses and minuses. On the debt side of the ledger, the slope of the loan supply curve fixes both the cost of external borrowing for the government and the elasticity of private capital flows. We think the two should be kept separate. The fact that the private sector is not well integrated into world capital markets in many LICs does not imply that the risk premium rises sharply when the government borrows to finance a credible, fiscally sustainable investment program. Doubtless, however, the risk premium is debt-sensitive in some countries.\footnote{The empirical evidence is distinctly mixed. See Section 6.} Accordingly, the computer program that runs the model allows the user to choose their preferred specification. We analyze the sensitivity of the results to the alternative specifications in Section 6.

More choice in the \textit{composition} of public investment is also desirable. The benchmark model assumes that all spending goes for new investment instead of a mix of new investment and maintenance. In analytical terms, the two types of investment are virtually identical: net investment may be increased either by increasing gross investment ($i_z$) or by spending more on maintenance to slow depreciation of the existing stock. The split is important if — as appears to be the case in many LICs — the return to maintenance is much higher than the return on new investment. Just how important will become clear in Section 6.\footnote{Practitioners can choose a program that runs the benchmark model or another program that allows them to specify the split between new investment and maintenance, the time paths for each type of investment, and the value of a parameter that determines whether (at the margin) maintenance pays a higher or lower return than new investment.}

Finally, the base case assumes that infrastructure enters as a shift factor in production functions that exhibit constant returns scale in labor and the private capital stock. In our view, the empirical evidence favors this specification over one that postulates constant returns to infrastructure and private inputs. But there is no need to consult a long list of citations. We show in Section 6 that the two specifications generate similar results. The
impact on growth depends mainly on the efficiency of public investment, the return on infrastructure, and the cost of underfunding maintenance, not on whether the factor shares for private capital and labor sum to unity or to $1 - \psi$.

3. Calibration of the Model and Solution Technique

To prepare the model for calibration, we need to (i) link the adjustment cost parameters for changes in the capital stock and foreign bonds/loans to observable elasticities and to (ii) pin down the relationship between the return on infrastructure, the parameter $\psi$, and private sector output. This is readily done. Starting with the first item on the list, note that in each sector the first-order condition for investment reads $[1 + v(i_t/k_{t-1} - \delta - g)]\lambda_{1,t}P_{k,t} = \lambda_{2,t}$, where $\lambda_1$ and $\lambda_2$ are the multipliers associated with the budget constraint and the law of motion for the capital stock. Since $\lambda_2/\lambda_1$ is the shadow price of $K$ measured in dollars, $\lambda_2/\lambda_1 P_k$ is effectively Tobin’s $q$, the ratio of the demand price to the supply price of capital. Let $\Omega \equiv \hat{I}/\hat{q}$ denote the $q$-elasticity of investment spending. Evaluated at a stationary equilibrium, we then have $v(\delta + g)\Omega = 1$. Hence $v = 1/(\delta + g)\Omega$.

We employ a similar strategy to link the portfolio adjustment cost parameter to an elasticity that reflects the degree of substitutability between domestic bonds and foreign loans. Write equation (15) as

$$\eta \frac{(b_t^* - \tilde{b}^*)/Y}{IRD} = \frac{1}{Y},$$

where $Y$ is annual GDP and $IRD \equiv [(1 + r_t)P_t/P_{t+1} - (1 + r^*)]/(1 + r_t)P_t/P_{t+1}$ is the interest rate differential. The term $\xi \equiv (b_t^* - \tilde{b}^*)/Y \div IRD$ is the ratio of capital flows, measured as a percentage of annual GDP, to the IRD. Since $\eta$ is not observable or unit free, we calibrate to $\xi$.

The only inputs needed to calculate the return on infrastructure are the purchase price $P_z$ and the shadow rental $r_z$. The latter is simply the marginal value product of infrastructure at constant prices. From (1) and (2),

$$r_z = \psi_x q_x + \frac{P_n \psi_n q_n}{z}.$$
The return, net of depreciation, is

\[ R = \frac{r_z}{P_z} - \delta = \psi_x q_x + \psi_n P_n q_n - \delta, \]  

(29)

\( \delta \) is set directly, while \( q_n, q_x, z, \) and \( P_z \) are derived from the values of other variables. \( (P_n = 1 \) by choice of units.) This leaves \( R, \psi_x, \) and \( \psi_n \) as unknowns in equation (29). We assign values to \( R \) and \( \psi_n/\psi_x \) and instruct the computer to solve (29) for \( \psi_x \).

### 3.1 Base Case Calibration

The values in Table 1 are based on a mixture of data and guesstimates. We discuss below the rationale for the value assigned to each parameter and the problems that arose in calibrating certain parts of the model:

- **Consumption share of the nontraded good \( (\gamma_n) \).** The share of services + construction in GDP is a good guide to the share of nontradables production in GDP. The former is typically around 50%, so we set the consumption share of nontradables at .5. This and the values assigned to other parameters yield a share of nontradables in GDP of 50.3% in the base case.

- **Intertemporal elasticity of substitution \( (\tau) \).** Most estimates of \( \tau \) for LDCs lie between .1 and .5 (Agenor and Montiel, 1999). The value in the base case, .34, equals the average estimate for LICs in Ogaki et al. (1996).

- **Elasticity of substitution in consumption \( (\epsilon) \).** We fix \( \epsilon \) at .50 as estimates of compensated elasticities of demand tend to be small at high levels of aggregation, especially when food claims a large share of total consumption.\(^{17}\)

- **Capital’s share in value added \( (\alpha_n, \alpha_x) \).** Data on factor shares may be found in social accounting matrices assembled by GTAP (Global Trade Analysis Project) and IFPRI (International Food Policy Research Institute). The GTAP database for SSA suggests a capital share of 55-60% in the nontradables sector and 35-40% in the tradables sector.\(^{18}\) The data in Thurlow et al. (2004) and Perrault et al. (2010) suggest similar numbers.\(^{19}\) Accordingly, we set \( \alpha_n = .55 \) and \( \alpha_x = .40 \).

- **Depreciation rate \( (\delta) \).** Our choice of 5% is in line with estimated depreciation rates for private capital in developed countries (Musgrave, 1992; Nadiri and Prucha, 1996) and for various types of infrastructure in LDCs (Pereira and Ferreira, 2007; Ianchovichina et al., 2012).

\( ^{17} \) See Lluch et al. (1977, chapter 3), Deaton and Muellbauer (1980, p.71), Blundell (1988, p.35), and Blundell et al. (1993, Table 3b).

\( ^{18} \) The nontradables sector comprises trade + transport, private services, dwellings, and construction. The tradables sector consists of agriculture and manufacturing.

\( ^{19} \) The average factor shares cited here conceal tremendous variation. For example, the value added share of capital in the services sector is 59% in Zambia (Thurlow et al., 2004) but only 27% in Malawi (Thurlow et al., 2008).
• *q*-elasticity of investment spending ($\Omega$). There are few reliable estimates of this elasticity for LDCs. The assigned value, 2, is at the high end of estimates for developed countries and in line with the estimates for Egypt in Shafik (1992) and for Korea in Dailami (1986), Kong (1998), and Kim et al. (2015). The results do not change substantively when $\Omega$ equals .5 or 10.

• **Real interest rates on concessional and nonconcessional loans** ($r_{dc}, r_{rd}$). In recent years, Ghana, Senegal, Cote d’Ivoire, Ethiopia, and Kenya have floated Eurobonds paying 5.6-9%. Today’s interest rates are unusually low, however. Gueye and Sy (2010) estimate that SSA paid an average interest rate of 8.55% on debt raised in external capital markets between 2000 and 2009. The IMF’s DSAs show an average interest rate of 2.3% on non-concessional loans taken out by LICs in 2009-2010. Assuming 2.5% inflation in world prices of traded goods, the corresponding real rates in dollars are 6% and 0.

• **Trend growth rate** ($g$). The trend growth rate of 1.5% equals the 1990-2008 per capita growth rate for SSA reported in African Development Indicators.

• **The pure time preference rate** ($\rho$), the **real interest rate** ($r$) on domestic bonds, and the **real return on private capital**. Across steady states, the real interest rate on domestic debt and the real return on private capital equal $(1 + \rho)(1 + g)^\tau - 1$. We choose $\rho$ jointly with $\tau$ and $g$ so that the domestic real interest rate is 10% at the initial equilibrium. This is consistent with the data for SSA in Fedelino and Kudina (2003), with the estimated return on private capital in Isham and Kaufmann (1999), Dalgaard and Hansen (2005), and Marshall (2012), with the stylized fact that domestic debt in low- and middle-income countries is usually much more expensive than external commercial debt, and with the range of real loan rates in LDCs (generally 7-15%) reported in World Development Indicators. There is tremendous variation in real interest rates across countries and time periods, however. To test the robustness of the results, we also carry out runs for $r = .07$.

• **Ratio of user fees to recurrent costs per unit of infrastructure** ($f$). The user fee for infrastructure services is a fixed multiple/fraction $f$ of recurrent costs ($\mu = f \delta P_{zo}$). Fuel taxes (earmarked for road maintenance and construction), electricity tariffs, and user charges for water and sanitation are low but not trivial in LICs. On average, user fees recoup 50% of recurrent costs in SSA (Briceno-Garmendia et al., 2008). Again, however, there is considerable variation — Zambia’s average electricity tariff was three cents per kWh in 2008. We decided therefore to let $f$ vary from .2 to unity, with $f = .5$ in the base case.

• **Consumption VAT** ($h$). The consumption VAT in the model proxies for the average indirect tax rate. Our rate of 15% is about the same as the average indirect tax rate on consumption in Ghana. Combined revenue from the VAT and user fees ranges from 12.6% to 16.1% of GDP depending on the value assigned to the user fee. This is com-

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20 8.55% is the average excluding the Seychelles (where the distress rate was estimated at 32.2%) and South Africa.

21 For 2003-2006, consumption, indirect taxes, and trade taxes averaged 81%, 9.6%, and 4.6% of GDP, respectively. If duties on consumer imports accounted for half of trade taxes, then the average consumption tax was 14.7%. (Data from IMF, 2007b)
parable to the range of total domestic revenue to GDP in SSA [12.2-15.5% for different LIC groups in Briceno-Garmendia et al. (2008)].

- **Real interest rate on foreign debt held by the private sector** ($r^*$). Equation (15) implies that $r^* = r$ across steady states. Hence $r^* = .10$, a value that incorporates a substantial risk premium.\(^{22}\)

- **Domestic debt** ($b$). Different datasets give different numbers for the ratio of domestic debt to GDP in LICs. We settled on 20% by averaging the figures reported in IMF(2009a), Panizza (2008), and Arnone and Presbitero (2010).

- **Private and public external debt** ($d, b^*, dc$). Concessional loans and the public external debt are 50% of GDP at the initial equilibrium.\(^{23}\) The ratio of total public debt to GDP and the share of concessional loans in total debt are 70% and 71.4%, respectively, vs. 74% and 69% for LICs in 2007-2008 (IMF, 2009a and IMF staff calculations). Since little is known about the likely value of private foreign debt (or assets) in LICs, we chose $b^* = 0$ for the base case.

- **Initial ratio of infrastructure investment to GDP** ($i_{z,o}/GDP_o$). Outlays on operations and maintenance (O+M) of infrastructure average 3.1% of GDP for LICs in SSA. But true O+M costs are 40-50% higher (Briceno-Garmendia et al., 2008).\(^{24}\) After taking this and net investment associated with trend growth into account, we get 6% for the initial ratio of infrastructure investment to GDP. The figure for LICs in SSA was 6.09% in 2008 (Briceno-Garmendia et al., 2008).

- **Ratio of labor supply of non-savers to labor supply of savers** ($a$). Sixty percent of consumers are nonsavers ($a = 1.5$).\(^{25}\) This agrees with the estimates for LDCS in Lopez et al. (2000) holds in LDCs and with the survey findings for LICs in Steadman Group (2009). Estimates for the consumption share of non-savers in the Phillipines, Thailand, and Korea by Chyi and Huang (1997) range from .33 to .44, which maps to $a = x - y$ for the calibration in our model.\(^{26}\) To the best of our knowledge, these are the only estimates available for LDCs. Numerous estimates exist, however, for developed countries; most place the share of non-saving households between 25 and 60 percent (see Table 2 in Buffie, 2013).

- **Efficiency of public investment** ($s$) and the absorptive capacity constraint ($\phi$). The base case assumes that public investment is efficient and that scaling up does not strain absorptive capacity ($\phi = 0$). Motivated by the findings in Hulten (1996) and Foster and Briceno-Garmendia (2010), we also investigate scenarios with extreme inefficiency ($s = .7$)

\(^{22}\) We have also analyzed the case in which $b^*$ is a foreign financial asset that enters the utility function. The marginal nonpecuniary yield on $b^*$ then compensates for the higher interest rate paid by domestic bonds ($r > r^*$). This variant of the model produces results nearly identical to those in the current model.

\(^{23}\) The initial values of $d$ and $dc$ affect the initial value of transfers but little else. There is virtually no impact on the solution path.

\(^{24}\) Because of underspending on O+M, 30% of Africa’s infrastructure assets are in need of rehabilitation (Foster and Briceno-Garmendia, Overview and chapter 2).

\(^{25}\) The results are insensitive to the share of nonsavers in the economy. See Tables 5 and 7 in Section 6.

\(^{26}\) The consumption share of non-saving households underestimates $a$ when average consumption by saving households exceeds average consumption by non-saving households.
and a tight absorptive capacity constraint ($\phi = 5.19$).\footnote{In Africa, large cost overruns stemming from planning/coordination/management problems and low capital budget execution ratios (average = 66\%) suggest that absorptive capacity may be a binding constraint in many countries (Foster and Bricendo-Garmendia, 2010, Overview).}

- **Cost share of nontraded inputs in the production of capital goods ($\alpha_k, \alpha_z$).** Data on the ratio of imported machinery and equipment to aggregate investment indicate that $\alpha_k$ is around .50 in SSA. One-half is also the guesstimate used by the IMF (2007a) in its analysis of scaling up public investment in Nigeria.

- **Interest elasticity of private capital flows ($\xi$).** Empirical evidence on the likely value of $\xi$ is limited and indirect. The value in the base case, unity, implies that a one percentage point increase in the domestic interest rate induces capital inflows equal to one percent of annual GDP. This intermediate degree of capital mobility is consistent with the estimate of $\xi$ for emerging market economies in Chang et al. (2013), with the estimated response of capital flows to the interest rate differential in De Gregorio et al. (2000), with the finding that the offset coefficient for capital flows is far below unity (.15-.60) in LDCs and transition economies (Ljubaj et al., 2010), and with abundant casual evidence that capital flows in LICs are not elastic enough to prevent large fluctuations in domestic bond rates.\footnote{It should also be noted here that in 20-25\% of LICs the elasticity of non-FDI private capital flows is comparable to that for emerging markets (Araujo et al., 2015).}

- **Return on infrastructure ($R$).** Estimates of the return on infrastructure are all over the map, but the weight of the evidence in both micro and macro studies points to a high average return. The median rate of return on World Bank projects circa 2001 was 20\% in SSA and 15-29\% for various sub-categories of infrastructure investment. In the Bank’s recently-completed, comprehensive study of infrastructure in Africa, estimated returns for electricity, water and sanitation, irrigation, and roads range from 17\% to 24\% (Foster and Briceno-Garmendia, 2010, chapter 2). Similarly, the macro-based estimates in Dalgaard and Hansen (2005) cluster between 15\% and 30\% for a wide array of different estimators. Hulten et al. (2006), Escribano et al. (2008), Calderon et al. (2009), and Calderon and Serven (2010) supply additional evidence of high returns. Some growth regressions suggest low or insignificant returns, but these are dominated by studies that use cumulative public investment instead of physical indicators to measure the stock of infrastructure. \footnote{Most studies that use the perpetual inventory method to compute the aggregate capital stock do not account for inefficient public investment. The resulting measurement error biases the estimated return on infrastructure downward: the studies estimate the return on investment spending, $Rs$, not the return on infrastructure, $R$. A similar bias plagues empirical studies that regress growth on the ratio of infrastructure investment to GDP. The coefficient on the investment rate is actually the return on infrastructure scaled down by $s$. The returns estimated in Dalgaard and Hansen (2005) are high but lower than the true return (on infrastructure). For further discussion of these issues, see Berg et al. (2015).}

All of this adds up to a presumption that high returns are the norm. Our low-, average-, and high-return scenarios assume therefore initial returns of 10\%, 20\%, and 30\%, respectively. Thirty percent may raise some eyebrows, but it is not as big as some of the numbers thrown around in the literature. Curiously, in two of its more visible publications, the IMF allows that the return might be as high as 50\%.\footnote{See the scaling-up exercise in Box 4.1 in Barkbu et al. (2008) and Gupta, Powell, and Yang (2006).}
Elasticities of sectoral output with respect to the stock of infrastructure \((\psi_x, \psi_n)\). The ratio \(\psi_x/\psi_n\) is set independently. This ratio and other values assigned in calibrating the model — most notably, the return on infrastructure — pin down \(\psi_n\) and \(\psi_x\). We assume \(\psi_x/\psi_n = 1\) in all runs.\(^{31}\)

Division of fiscal adjustment between expenditure cuts and tax increases \((\lambda)\). Across steady states, noninvestment expenditure and taxes share the burden of fiscal adjustment equally \((\lambda = .5)\).

The numerical simulations are free of approximation error — in all scenarios, they simulations track the global nonlinear saddle path. The solutions were generated by set of programs written in Matlab 7.10 and Dynare 4.1.1.

4. Insights From a Simplified Model

Although the model in Section 2 has many moving parts, it is not a black box. In this section, we radically simplify the model in order to demonstrate analytically that increases in infrastructure investment are often self-financing in the long run and that temporary crowding out of private investment is inevitable regardless (almost) of the elasticity of private capital flows and the amount of concessional loans available to the government. Both results figure prominently in the numerical simulations presented later in Sections 5 and 6.

4.1 The Short-Run Crowding-Out Problem

Consider a stripped-down model cast in continuous time that ignores trend growth, non-traded goods, private capital flows, absorptive capacity constraints, adjustment costs to changes in the capital stock, and non-saving households. In this setup, the representative private agent chooses \(k\) and \(c\) to maximize

\[
U = \int_0^\infty \frac{c^{1-1/\tau}}{1-1/\tau} e^{-\rho t}dt
\]

subject to

\[
\dot{k} + \dot{b} = q(k, z) + T + rb - (1 + h)c - \mu z - \delta k;
\]

where

\[
q(k, z) = az^{\psi} k^\alpha L^{1-\alpha}
\]

\(^{31}\) The value of \(\psi\) in the base case (where \(R = .20\)) is .2308.
is the production function for GDP and \( \rho \) is the pure time preference rate. On an optimal path,

\[
\begin{align*}
  r + \delta &= q_k = a\alpha z^\psi k^{a-1} L^{1-\alpha}, \quad (33) \\
  \dot{c} &= \tau c(q_k - \rho - \delta). \quad (34)
\end{align*}
\]

The government adjusts lump-sum transfers to maintain a balanced budget while investing to increase the stock of infrastructure:

\[
\begin{align*}
  \dot{z} &= i_z - \delta z, \quad (35) \\
  T &= hc + \mu z - i_z - rb. \quad (36)
\end{align*}
\]

Substituting for \( T \) in (31) and imposing \( \dot{b} = 0 \) gives

\[
\dot{k} = q(k, z) - i_z - c - \delta k. \quad (37)
\]

Since the capital account is closed all around, the national budget constraint requires the country to run a zero trade balance each period.

To close the model, we assume that infrastructure investment jumps immediately to its new steady-state level at \( t = 0 \). Equations (34), (35), and (37) then comprise a self-contained system of three differential equations in \( k \), \( z \), and \( c \). The linearized system is

\[
\begin{bmatrix}
  \dot{c} \\
  \dot{k} \\
  \dot{z}
\end{bmatrix} =
\begin{bmatrix}
  0 & \tau c q_k k & \tau c q_k z \\
  -1 & \rho & q_z \\
  0 & 0 & -\delta
\end{bmatrix}
\begin{bmatrix}
  c - c^* \\
  k - k^* \\
  z - z^*
\end{bmatrix}.
\]

(38)

The stationary equilibrium \( (c^*, k^*, z^*) \) is a saddle point, with two state variables \( (k \) and \( c) \) and two negative eigenvalues:

\[
\begin{align*}
  \lambda_1 &= -\delta, \\
  \lambda_2 &= \frac{\rho - \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k}}{2}.
\end{align*}
\]

We solve for the complete transition path in Appendix A. The analysis is confined to small changes (i.e., \( x - x^* \) is a differential).
Prolonged Crowding Out Precedes Strong Crowding In

After slight manipulation, the solutions for \( c \) and \( k \) may be written as

\[
c - c^* = \frac{R + \delta}{J} \left[ \tau c/k + \frac{\lambda_2 - \rho}{\rho + \delta} (\delta + \tau \alpha c/k) \right] (z - z^*) + (\rho - \lambda_2)(k - k^*), \tag{39}
\]

\[
k - k^* = \frac{R + \delta}{(\rho + \delta)J} \left[ \frac{\delta}{1 - \alpha} e^{\lambda_2 t} - (\delta + \tau \alpha c/k) e^{-\delta t} \right] (z^* - z_o), \tag{40}
\]

where

\[
J \equiv \tau (1 - \alpha)c/k - \delta
\]

and \( R = q_z - \delta \) is the net return on infrastructure. The address for \( k^* \) follows directly from (34). In a steady state,

\[
a \alpha z^\psi k^{\alpha - 1} L^{1-\alpha} = \rho + \delta.
\]

Hence

\[
k^* - k_o = \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} (z^* - z_o). \tag{41}
\]

Since \( z \) enters the production function as a shift factor, growth in the stock of infrastructure stimulates private investment by increasing the marginal product of capital. Equation (41) tells us, in addition, that the long-run crowding-in effect may be quantitatively large. The ratio of the gross return on infrastructure to the gross return on private capital, \((R + \delta)/(\rho + \delta)\), multiplies a term that lies somewhere between .42 and 1 \((\alpha = .3 - .5)\). Thus, if empirical estimates are right and the mean return on infrastructure is much higher than the mean return on private capital, the long-run crowding-in coefficient will often approach or exceed two. Suppose, for example, that \( \rho = .10, \delta = .05, \) and \( \alpha = .475 \) as in the base case in Table 1. \([\alpha = (\alpha_x + \alpha_n)/2 \) and \( \rho = .10 \) because the net return on private capital and the real interest rate both equal 10%\]. The crowding-in coefficient then ranges from 1.21 to 2.11 when \( R = .15 - .30 \). Productive infrastructure and private capital are very strong complements in the long run.

But this is hard to see without the aid of a chrystal ball. Figure 1 depicts the path to the new steady state. The \( \dot{k} = 0 \) schedule

\[
\left. \frac{k - k^*}{z - z^*} \right|_{\dot{k} = 0} = \frac{(R + \delta)(\lambda_2 + \delta)}{(\rho + \delta)\lambda_2 J}(\delta + \tau \alpha c/k), \tag{42}
\]

22
is positively sloped as $\lambda_2 + \delta$ and $J$ are opposite in sign.\(^{32}\) When $i_z$ jumps to $i_z^*$, the schedule shifts down, generating a path like ABC, iff

$$\frac{\hat{k}(0)}{\delta(z^* - z_o)} = \frac{i(0) - i_o}{i_z^* - i_{z,o}} = \frac{R + \delta}{(\rho + \delta)(1 - \alpha)} \left( \frac{\lambda_2 + \delta}{J} + \alpha \right) < 0. \quad (43)$$

Two conflicting effects operate [$\alpha > 0$ vs. $(\lambda_2 + \delta)/J < 0$]. The scaling up of public investment increases future income. This together with the temporary sharp cut in transfer payments creates a strong incentive for the private agent to smooth the path of consumption by reducing investment. On the other hand, the positive impact of increases in the stock of infrastructure on future productivity could spur an immediate increase in private investment. The positive pull of the long-run fundamentals dominates the consumption-smoothing motive, however, only when the intertemporal elasticity of substitution is unbelievably large. In Appendix A we prove

**Proposition 1** An increase in infrastructure investment crowds out private investment in the short run iff

$$\tau < \tau^* \equiv \frac{\rho + \delta(1 - \alpha)}{\alpha(\rho + \delta)c/q},$$

where $c/q = 1 - \delta\alpha/(\rho + \delta) - i_z/q$.

**Corollary 1** If $\tau < 1/\alpha$ private investment decreases in the short run.

The import of Proposition 1 and Corollary 1 is that the intertemporal elasticity of substitution has to be far above unity to prevent temporary crowding out of private investment. It is a small step from this to the conclusion that crowding out is quantitatively large for realistic values of $\tau$. In fact, the crowding-out coefficient may exceed one when infrastructure pays a higher return than private capital. The solution in (43) provides

$$\lim_{\tau \to 0} \frac{i(0) - i_o}{i_z^* - i_{z,o}} = -\frac{R + \delta}{\rho + \delta} < -1 \text{ for } R > \rho.$$

To grasp the logic of such extreme crowding out, suppose the private agent holds consumption constant at $t = 0$ and cuts investment by the same amount as the increase in infrastructure investment. If $R > \rho$, consumption rises monotonically toward its higher steady-state level. For some $\tau$ sufficiently small, this path is inferior to a smoother path in

\(^{32}\) To construct the $\hat{k} = 0$ schedule, note first from (38) that $\hat{k} = -(c - c^*) + \rho(k - k^*) + (R + \delta)(z - z^*)$. Substituting for $(c - c^*)$ from (39) then produces the solution in (42).
which consumption rises at $t = 0$. But if consumption rises on impact, the sum of public and private investment must decline. Solving (43) for the $\tau$ that is "sufficiently small" yields (see Appendix A)

**Proposition 2**  
An increase in infrastructure investment reduces total investment in the short run iff

$$\tau < \bar{\tau} \equiv \frac{\alpha(1 - H)[\rho + \delta(1 - \alpha)(1 - H)]}{\{\alpha^2 + H(1 - \alpha)[H(1 - \alpha) + 2\alpha](\rho + \delta)c/q\}}$$

where $H \equiv (\rho + \delta)/(R + \delta)$.

The logic informing Proposition 2 points to another important result. Observe from (40) and (41) that

$$k - k_0 = \frac{R + \delta}{\rho + \delta} \left\{ \frac{\alpha}{1 - \alpha} + J^{-1} \left[ \frac{\delta}{1 - \alpha}e^{\lambda zt} - (\delta + \tau \alpha c/k)e^{-\delta t} \right] \right\} (z^* - z_0). \quad (40')$$

Since $R$ does not appear anywhere in $\{\cdot\}$, long-run crowding in, short-run crowding out, and the duration of phase AB in Figure 1 all increase with the return on infrastructure. Ironically, prolonged, deep crowding out of private investment in the short run is a natural corollary of strong crowding in of private investment in the long run:

**Proposition 3**  
An increase in infrastructure investment stimulates private capital accumulation in the long run but crowds out private investment in the short run when $\tau < \tau^*$. Ceteris paribus, long-run crowding in and the depth and duration of temporary crowding out are greater the higher the return on infrastructure.

Table 2 supplements Propositions 1 and 2 with quantitative information on $\tau^*$, $\bar{\tau}$, the short- and long-run crowding-in coefficients ($CIC_{SR}$, $CIC_{LR}$), and $t_2$, the time it takes the capital stock to recover to its pre-shock level. The numbers are dismaying. All is well in the long run: the crowding-in coefficients are large, ranging from .97 to 3.18 for normal/high rates of return on infrastructure ($R = .20 - .30$). But the path to the new steady state is not direct; invariably, it includes an initial phase in which infrastructure and private capital appear to be strong substitutes. Every value of $\tau^*$ exceeds two, and short-run crowding-out coefficients above unity predominate when $R$ reaches 20%. Moreover, recovery from temporary crowding-out is painfully slow: the AB phase in Figure 1 lasts 8-32 years when $\tau = .25 - .75$.

Two factors underlie the crowding-out problem: the high return on infrastructure, which imparts a steep upward slope to the consumption path, and the tight national budget con-
straint, which forces private sector spending to contract dollar for dollar in the short run as public investment increases. Obviously, nothing can or should be done about the first factor. (Investing in low-return projects to minimize temporary crowding out seems like a bad idea.) Tapping into world capital markets, however, relaxes the national budget constraint. Access to external credit allows the private sector to smooth the path of consumption without decreasing private investment, improving the prospects for continuous crowding in. Public sector external borrowing works in much the same way by delaying tax increases until growth in the stock of infrastructure substantially increases productivity and private sector income. Both schemes sound promising, but neither is likely to solve the crowding-out problem. The numbers in Table 2 are daunting. Normally a one point reduction in the threshold value of \( \tau \) would be a big deal — more than enough to flip a sign and greatly magnify quantitative effects. But when the starting point is \( \tau^* = 3 \), getting to \( \tau^* = 2 \) is inconsequential. The battle against temporary crowding out is long and all uphill. In the next two sections we show it is a battle that external finance, whether in the form of elastic private capital flows or an ample supply of concessional loans, cannot win.

4.1.1 Opening the Private Capital Account

When the private capital account is open, 

\[
\eta (b^* - \bar{b}^*) = r - r^* \tag{44}
\]

joins the set of first-order conditions and (37) changes to

\[
\dot{k} = q(k, z) + \dot{b}^* - i_z - c - r^* b^* - \delta k - \eta (b^* - \bar{b}^*)^2 / 2. \tag{45}
\]

Equations (33) and (44) tie the path of \( b^* \) to the paths of \( k \) and \( z \):

\[
\dot{b}^* = \xi q_k \left[ q_k \frac{\alpha - 1}{\alpha} \dot{k} + (R + \delta) \dot{z} \right], \tag{46}
\]

where, to repeat, \( \xi \) is the elasticity of capital flows (measured as a percentage of GDP) with respect to the interest rate differential. [As in (15'), \( \xi = 1/\eta q \).]

The Euler equation for consumption and the law of motion for the stock of infrastructure are unchanged. To get (45) in the right form, substitute for \( \dot{b}^* \) and solve for \( \dot{k} \). Straightfor-
ward algebra delivers
\[
\dot{k} = \frac{\alpha}{\alpha + (1 - \alpha)\xi q_k^2} [q(k, z) + q_k(R + \delta)(i_z - \delta z) - c - i_z - r^*b^* - \delta k - (b^* - \bar{b}^*)^2 / 2\xi q].
\] (47)
The linearized version is
\[
\dot{k} = \frac{\alpha}{\alpha + (1 - \alpha)\xi (\rho + \delta)^2} \{\rho[1 + \xi(1 - \alpha)(\rho + \delta)^2/\alpha](k - k^*)
+ (R + \delta)[1 - \xi(\rho + \delta)^2](z - z^*) - (c - c^*)\}. (48)
\]
\(\xi = 0\) gives back the solution for the closed economy. At the opposite extreme, perfect capital mobility (\(\xi \to \infty\)) implies
\[
\dot{k} = \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} (i_z^* - \delta z) > 0, \forall t,\] (49)
\[
c = c^*, \forall t. (50)
\]
Consumption jumps immediately to its new steady-state level and the capital stock rises in synch with the stock of infrastructure. There is no transitory decrease in private investment because access to a perfect, frictionless world capital market allows the private agent to completely smooth the path of consumption.

In the general case, crowding out gives way to crowding in at some threshold level of capital mobility \(\xi^*\). It is evident from inspection of inspection of (48), however, that \(\xi^*\) is out of reach for LDCs. All of the terms involving \(\xi\) are multiplied by \((\rho + \delta)^2\), a number on the order of \(.01 - .02\). Consequently, even elastic capital flows do not help very much: *near-perfect* capital mobility is required to eliminate temporary crowding out, and for believable values of \(\xi\) the results are quantitatively similar to those in the closed economy. Table 3 elaborates. Since \(\xi^* = 15 - 61\) for \(\tau = .25 - 1\), the conclusion that private decreases in the short run seems safe. Nor is there much of a reduction in the magnitude or the duration of crowding out. Even when \(\xi\) equals five— an absurdly large value for a LIC — the numbers for \(CIC_{SR}\) and \(t_2\) are only 5-15% lower than in Table 2.
4.1.2 Concessional Borrowing

Suppose concessional loans are available from IFIs at a zero real interest rate to finance a fraction $\chi$ of the investment program. During the borrowing period,

$$\dot{k} = q(k, z) + \chi(i^*_z - i_{z,o}) - i^*_z - c - \delta k, \quad 0 \leq t \leq t_1. \quad (51a)$$

The loans are repaid in equal installments in the next $t_1$ years

$$\dot{k} = q(k, z) - \chi(i^*_z - i_{z,o}) - i^*_z - c - \delta k, \quad t_1 \leq t \leq 2t_1, \quad (51b)$$

after which

$$\dot{k} = q(k, z) - i^*_z - c - \delta k, \quad t \geq 2t_1. \quad (51c)$$

Equations (33), (34), and (50a)-(50c) define the core dynamic system. The economy follows distinct noncovergent paths in phases $(0, t_1)$ and $(t_1, 2t_1)$. From time $2t_1$ onward, it rides the saddle path to the steady state $(c^*, k^*, z^*)$.

Once again, it proves hard to escape temporary crowding out. In Appendix B we demonstrate that

$$\frac{i(0) - i_o}{i^*_z - i_{z,o}} \geq 0 \quad \text{as} \quad \frac{\lambda_2 + \delta}{J} \alpha + \frac{\rho + \delta}{R + \delta} (2 - e^{-\lambda_3 t_1}) e^{-\lambda_3 t_1} (1 - \alpha) \chi \geq 0, \quad (52)$$

where

$$\lambda_3 = \frac{\rho + \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k}}{2}.$$

The problem, visible to the naked eye, is that the coefficient multiplying $\chi$ is the product of three fractions all smaller than one when $R > \rho$. This suggests that for normal/high rates of return on infrastructure $\chi = 1$ will not push $\tau^*$ below .75 (viz., the empirically relevant range). Table 4 confirms the conjecture: 100% concessional financing is insufficient; excess borrowing — borrowing to pay for the investment program plus temporary tax cuts — is required to prevent temporary crowding out. Intuitively, the consumption-smoothing motive is weaker but still potent. Consider a country that secures 100% financing for its big-push program. Given the long amortization period characteristic of concessional loans, steady increases in disposable income will continue past the borrowing period into the repayment phase as long as infrastructure investment pays a decent return. Although disposable income does not temporarily decrease as in the no-borrowing case, there is still a strong incentive
to consume some future income gains today by reducing investment; ergo $\tau^*$ is smaller than in Table 2, but still large in absolute terms.

The news is much better in the second and third panels of Table 4. In contrast to private capital inflows, concessional loans significantly decrease the severity of crowding out. Increasing the share of concessional finance from zero to 50-100% in the case (in the case where $\rho = .10$, $\tau = .50$, $R = .20$) reduces the crowding-out coefficient at $t = 0$ from 1.0 to .41-.56 and shortens the crowding-out phase by 5.8-8.4 years. All of the gains disappear, however, during the repayment period. After year nineteen, the path for the capital stock is virtually identical to the path without borrowing.

4.1.3 Sectoral Factor Intensities

Adjustment costs to changing the capital stock, positive real interest rates on external loans (reflecting a mix of concessional and non-concessional borrowing), and poor, non-saving households reduce temporary crowding out of private investment. For believable empirical values, these variables affect only the quantitative results. Introducing all of them into the simplified model does not overturn the conclusion that private investment decreases in the short run when infrastructure pays a high return.

The simplified model misses one important effect, however. In the fully-loaded model, an increase in public investment raises the relative price of the nontraded good for the first 5-10 years of the transition path. The jump in $P_n$ creates a powerful pro-investment effect when, as in our calibration for the base case, the nontradables sector is substantially more capital intensive than the tradables sector. Moreover, since $P_n$ increases with aggregate spending, external borrowing strongly magnifies the relative price effect. Crowding out still wins if concessional borrowing and taxes pay equally for the investment program. But when

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33 Positive real interest rates on external loans imply higher taxes and lower disposable income in the future. The existence of poor, non-saving households who pay some of the taxes that finance infrastructure investment reduces capitalists' real income loss in the short run. Both effects flatten the path of capitalists' after-tax income and thereby weaken the incentive to smooth consumption by temporarily reducing investment. Adjustment costs to changing the capital stock decrease the magnitude of short-run crowding out but do not alter the threshold value of $\tau$ at which crowding out occurs. (Adjustment costs are zero at $\tau = \tau^*$.)

34 The Stolper-Samuelson theorem predicts this result. In the two-sector Heckscher-Ohlin-Samuelson trade model (which assumes mobile capital but a fixed aggregate capital stock), the theorem states that an increase in the relative price of the capital-intensive good raises the own capital rental in both sectors. In a dynamic model, the increase in the return on capital spurs an increase in aggregate investment.
nonconcessional borrowing substitutes for tax increases (in the short run), the balance in the base case finally shifts in favor of continuous crowding in.

This victory does not count for much. The key insight conveyed by the simplified model remains valid. Extensive sensitivity tests presented in Section 6 show that, regardless of whether private investment rises or falls in the short run, the consumption-smoothing motive substantially neutralizes the positive pull of the fundamentals for a long time. Even with immediate crowding in of private investment, growth of the private capital stock badly lags growth in the stock of infrastructure for 20+ years.

4.2 The Long-Run Fiscal Impact

Across steady states,

$$\rho + \delta = \alpha a \psi z k^{\xi+\alpha-1} L^{1-\alpha},$$  \hspace{1cm} (53)$$

$$z = i_z / \delta,$$  \hspace{1cm} (54)$$

$$\delta z (1-f) = hc - T - \rho b,$$  \hspace{1cm} (55)$$

$$c = q - \delta (k + z).$$  \hspace{1cm} (56)$$

These four equations and the production function in (32) can be solved for $q, k, z, c$ and $T$ as a function of $h, i_z$, and $f$. In converting (36) into (55), we replaced $\mu$ with $f \delta$. (Recall that $f$ is the ratio of user fees to recurrent costs.)

More investment in infrastructure increases NNP and revenue from the consumption VAT and user fees by

$$d \text{NNP} = \left( \frac{R + \delta}{\rho + \delta \frac{\alpha}{1-\alpha} + R} \right) \frac{di_z}{\delta},$$  \hspace{1cm} (57)$$

$$hdc + f \delta dz = \left[ \frac{h}{\delta} \left( \frac{R + \delta}{\rho + \delta \frac{\alpha}{1-\alpha} + R} \right) + f \right] di_z$$  \hspace{1cm} (58)$$

in the long run. The revenue gain pays for the additional investment and leaves something left over to finance higher transfer payments when

$$\frac{h}{\delta} \left( \frac{R + \delta}{\rho + \delta \frac{\alpha}{1-\alpha} + R} \right) + f > 1,$$

or

$$R > \frac{\delta (1 + h - f) (1 - \alpha)}{h [1 - \alpha \delta / (\rho + \delta)]} - \delta.$$  \hspace{1cm} (59)$$

29
The crucial implication of (59) is that \( R \) does not have to be unusually high for the increase in infrastructure investment to be self-financing. For the base case values in Table 1, the borderline value of \( R \) is a modest 8.5%. The base case assumes that user fees cover half of recurrent costs. Even with \( f = 0 \), however, (59) holds for \( R > 18.8\% \). This is high but well within the range of empirical estimates.

Lest proponents of scaling up public investment make too much of these results, we issue two caveats. First, the simplified model assumes that public investment is efficient. As shown later, if this assumption fails to hold, the fiscal windfall becomes a fiscal shortfall.\(^{35}\) Second, difficulties may arise on the transition path. Efficient public investment in high-return infrastructure projects is self-financing only in the long run. Over a medium run that extends into the repayment period for concessional loans, expenditure cuts and tax increases are unavoidable. In some cases, nonconcessional borrowing can smooth away difficult fiscal adjustments that threaten to undermine support for the investment program. But if the capacity for fiscal adjustment is too limited and the government borrows too much in the short/medium run, there is a risk that future revenue gains will not arrive fast enough to prevent explosive growth of the debt. The viability of the borrowing-for-development strategy depends not only on the efficiency of public investment but also on whether policy makers can solve the problem of how to get from here to there.

5. Nonconcessional External Borrowing

Increasingly LICs have access to external debt markets. In recent years, Uganda, Tanzania, Senegal, Ghana, Angola, Congo DRC, Mali, Mauritania, and Rwanda have all entered into nonconcessional loan agreements or issued sovereign bonds in international capital markets. Recognizing that the trend is likely to continue, the World Bank and the IMF have adopted a more flexible approach to nonconcessional borrowing. The World Bank has introduced

\(^{35}\) This statement comes with an important qualification. Recall that we calibrate to \( R \) and back out the value of \( \psi \) from equation (29). The assumption that \( R \) is independent of \( s \) is valid when past investment was efficient. (\( z \) and hence \( \psi \) are same as in the base case because \( s < 1 \) only for new investment). By contrast, when \( s < 1 \) for past and future investment, \( R \) is higher in the economy with low \( s \) [\( z \) is lower in (29)]. It can be shown that the higher value for \( R \) exactly offsets the lower value of \( s \) and that the impact on growth and the fiscal budget are the same as in the base case. See the invariance result in Berg et al. (2015).
the IDA Nonconcessional Borrowing Policy and the IMF routinely approves supplemental commercial loans for critical, large-scale infrastructure projects.

5.1 The Counterfactual

The time lines for infrastructure investment and concessional borrowing are:

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ... | 27 | 28 | ∞ |
|------|---|---|---|---|---|---|---|---|---|----|     |    |    |   |
| \( \frac{P_{zo}(i_z - i_{zo})}{GDP_o} \) | 6 | 7 | 8 | 8 | 8 | 7.5 | 6.5 | 5.5 | 4 | 4 | ... | ... | ... | 4 |
| \( \frac{d_t - d_{t-1}}{GDP_o} \) | 4 | 4 | 4 | 4 | 4 | 3 | 1.75 | .5 | -1.329 | -1.329 | ... | -1.329 | 0 | 0 |

The government aims for a transformative big push. Infrastructure investment jumps 6% of GDP in year 1, rises to 8% in years 3-5, and then decreases to its steady-state level. Concessional borrowing equals 4% of initial GDP for five years, after which net inflows decrease in step with investment. Repayment occurs in equal payments over eighteen years after an eight-year grace period. Cumulative borrowing in years 1-8 equals 45% of cumulative investment.

We need a comparison run to evaluate the pros and cons of schemes that supplement nonconcessional borrowing with commercial borrowing. This is provided by a run in which the government refrains from nonconcessional borrowing. The counterfactual assumes therefore that taxes and transfers adjust continuously to close the ex ante financing gap.

36 These correspond to the average maturity and grace-period for new concessional loans to LICs in 2009-2010, based on available IMF and World Bank DSAs. We apply an equal principal payment formula to generate the repayment profile.

37 The big push is on the scale of ambitious programs recently initiated by several LICs in SSA (Mozambique, Rwanda, Tanzania, and Uganda). The financing share of concessional loans and the time path of public investment is based on data from applications of the MBDSA to diverse countries by IMF staff.

38 Setting \( \lambda_1 = \lambda_3 = 1 \) and \( \lambda_2 = \lambda_4 = 0 \) in (24) and (25) yields:

\[
T_{t, \text{counterfactual}} = T_{\text{target}, t} = T_0 - \lambda \text{GAP}_t, \\
h_{t, \text{counterfactual}} = h_{\text{target}, t} = h_0 + (1 - \lambda) \frac{\text{GAP}_t}{H(\alpha_t + \alpha_{t-1})}.
\]


5.2 The Base Case

Figure 2 is discouraging. In the counterfactual, the VAT rises to 17.8% at year three, while transfers decrease by 2.1% of initial GDP. Conditions improve thereafter, but very slowly; after two decades, the VAT is still 17%.

The government’s protracted fiscal problems stem from the disappointing medium-run (i.e. 10-20 year) impact on private capital accumulation and growth. Unfortunately, disappointment is not specific to the base case but rather a highly robust feature of the transition path — see the results in the simplified model. Higher taxes and lower transfer payments reduce private sector disposable income by 2.3-4.4% in the short run. Because future returns rise, it might be optimal to maintain or increase investment at the same time the budget constraint shifts inward. But this is improbable: it requires private agents highly averse to non-smooth consumption paths — $\tau$ is low in LICs — to cut current consumption at least 4% even though future income will be much higher. In the case at hand, the private sector prefers to temporarily dissave. Consumption decreases 1% and aggregate investment $(i_x + i_n)$ falls 3.5% in the first year. Private investment stays depressed for another six years and then rises a feeble .5% per annum until debt service ends at year 27. Due to the lengthy period of lower/stagnant investment, growth of the private capital stock and real income lag far behind growth in the stock of infrastructure. At the 20-year horizon, $z$ has increased 58% (87% of the 67% increase across steady states), but $k$ and $y$ are only 5.2% and 13.6% higher (vs. 25% higher across steady states).39 And because income and the tax base grow slowly the revenue demands of debt service keep the VAT high and transfers low until the last concessional loan is paid off. Eventually policy makers are rewarded for their perseverance with a tiny revenue windfall and a large increase in the private capital stock, but this occurs in a distant future too far away to matter.

The alternative to the counterfactual is to smooth the path of fiscal adjustment by borrowing against future revenue gains in the commercial debt market. This strategy fares poorly in the base case because the revenue gains are too small for too long. There are benefits on the real side: crowding-in of private investment is immediate, and real wages and

39 It is common practice to discount econometric estimates of the q-elasticity of investment and assign $\Omega$ a value of 5-10 to speed up adjustment of the capital stock. That does not work in our case. When $\Omega = 10$, the increases in $k$ and $y$ at the 20-year horizon rise from 5.2% and 13.6% to 6.3% and 14.1%.
real GDP are slightly higher at the 10-20 year benchmarks. The fiscal bind persists, however. External commercial borrowing merely delays the day of fiscal reckoning; two decades out, the VAT and expenditure cuts exceed their peak/trough values in the counterfactual.

Problematic as they are, the results in the base case may be overly optimistic. We worry most about whether expenditure will really help share the burden of fiscal adjustment. This is perhaps the shakiest assumption in the base case. After investment, interest payments, and untouchable anti-poverty programs are put to one side, there is not much left to cut except the wage bill. But if public sector unions demand raises in line with those granted in the private sector, it will be difficult to prevent expenditure from increasing, let alone enforce cuts on the order of 2% of initial GDP.

In future sections we aim for greater realism. Most of the analysis will concentrate on two scenarios with downwardly inflexible expenditure. In the first, transfers are constant until growth generates a fiscal windfall:

\[ T_t = \text{Max}\{T^1_t, T_o\}, \]  

where \( T^1 \) is the value of \( T \) in (24). The second, more pessimistic scenario assumes the government is unable to cut other expenditures to offset increases in public sector wages. This implies

\[ T_t = \text{Max}\{T^1_t, T_o + .05 GDP_o (w_t - w_o)/w_o\} \]

when the initial wage bill is 5% of GDP and raises in the public sector match raises in the private sector.

5.3 Relatively Optimistic Scenarios

Our criterion for success is that debt sustainability prove compatible with modest increases in the VAT when either (60) or (61) determines the path of transfers. Obviously the base case fails this criterion (Figures 3a-3b). If success is possible, it requires higher user fees and/or higher returns on infrastructure. Faster revenue growth may then reconcile rapid scaling up with politically acceptable increases in the VAT.

Figures 4a-4b show the outcome when user fees pay for all recurrent costs, the initial return on infrastructure is 20-30%, and cuts in transfer payments are infeasible (\( \bar{T} = T_o \)).
These paths are considerably better than the paths in the base case. In the runs with $R = .30$, another six percentage points are added to the gains in real output and the real wage at the 20-year horizon. More important, the increase in the VAT is limited to 1.4 - 2.2 points. By contrast, in the counterfactual runs the VAT peaks at 20% and stays above 18% for 10 - 15 years.

Regrettably, the case for BOAF (borrowing on all fronts) is much weaker in Figures 5a-5b. When growth in the public wage bill fuels continuous growth in noninvestment expenditure [i.e., (61) applies], the ceiling on the VAT has to increase another full percentage point (to 17.5 - 18.2%) to preserve debt sustainability. This pushes the boundary of feasible adjustment; we suspect that policy makers in most LICs would tell their economic team to go away and work on a new plan.

5.4 Debt Blowups

It is easy to unintentionally abuse the flexibility afforded by borrowing in the commercial debt market. Policy makers might overestimate their capacity for fiscal adjustment, finance low-return projects, misplace the blueprints for reforms to improve governance and the efficiency of public investment, or repeatedly succumb to the temptation to put off necessary but unpopular tax increases and expenditure cuts. In which event we get the dreadful results on view in Figure 6. These runs and the comparison runs lifted from Figures 4a-4b assume optimistically that user fees cover all recurrent costs ($f = 1$) and that the return to infrastructure is high ($R = .20 - .30$). In Panel A politicians improve the original plan by ordering increases in the VAT to be delayed for five years. Because of the delay, policy makers lose the race against time and the debt blows up. In Panel B the government borrows heavily in the commercial market as it pays off its concessional debt, counting on growth in future tax revenues to ensure debt sustainability. But since public investment is inefficient ($s = .7$), future revenue gains are too small to stabilize the debt even though policy makers subsequently summon the courage to increase the VAT from 16.5% to 18.5%. A similar problem arises in Panel C. The actual return, although quite respectable (20%), is ten points below the expected return. When the lower-than-expected return maps into lower-than-expected revenue growth, the external debt keeps rising after the path in the
comparison run turns south; just three years after concessional loans are repaid, the ratio of the commercial debt to GDP hits 61%. Finally, borrowing to cover temporary cost overruns proves a bad gamble in Panel D; again, a belated attempt to regain fiscal control by further increasing the VAT comes too late to save the day.\footnote{In these runs we have assumed that unanticipated repudiation of the debt far out on the transition path enables the economy to converge to the steady state. This is unrealistic but not misleading or otherwise consequential. At some point, foreign creditors will realize that default is inevitable. The risk premium then rises sharply and the debt grows faster than shown in Figure 6. The bias in our results is thus quantitative, not qualitative. Figure 6 gives the correct answer to the question of whether the scaling-up program is feasible.}

\section*{5.5 An Interim Assessment}

There is a wide variety of scenarios and results in Figures 2-6. Taking stock, what do they tell us about the risks and rewards of nonconcessional borrowing? Is BOAF a sensible strategy for LICs?

Given the magnitude of the ex ante financing gap (4-5.3\% of initial GDP), it is difficult to rapidly scale up public investment when concessional loans are the only source of external funds. This is transparent from the counterfactual runs. Few, if any, policy makers will rush to embrace a program that increases the VAT 4-6 points for 10-20 years. Access to the commercial debt market can therefore be the difference between success and failure. Early on, domestic opposition to the program will be less if nonconcessional borrowing allows tax increases and expenditure cuts to be phased in more slowly. Later, when the limits of fiscal adjustment have been reached, new borrowing can avert default or a collapse in public investment.

The catch is that the path to success is narrow and slippery. If the return on infrastructure is high, if public investment is efficient, if user fees cover all recurrent costs, and if new projects can be implemented quickly on a large scale without cost overruns (i.e., absorptive capacity is sufficient), then an increase in the VAT of 1.5-2 points for five years suffices to make scaling up work. But this is too many ifs. In all other scenarios, either the debt blows up or the VAT rises to levels that threaten social and political stability. Borrowing-on-all-fronts (BOAF) is a high-risk, high-return strategy. It may greatly enhance the prospects for debt sustainability or lead to spectacular failure.
While much depends on the fine details governing debt contracts, the dynamics of growth, and the speed of fiscal adjustment, overall the results have a decidedly negative slant. But we are not done yet. There is one more case to consider — an important case that goes a long way to even the balance between positive and negative results.

5.6 An Exceptionally Important Scenario: Can SSA Increase the Collection Rate?

Average user fees equal or slightly exceed average costs for operation and maintenance of physical infrastructure in SSA. Because of low collection rates, however, revenues cover only half of recurrent costs (Briceno-Garmendia et al., 2008; Eberhard et al., 2008).

The low collection rate represents both a problem and an opportunity. Raising the collection rate from 50 to 100% would capture more revenue not only from the expansion of infrastructure services but also from services supplied by the existing network. Of course the prevalence of low collection rates suggests that the problem is not easy to solve (Foster and Briceno-Garmendia, 2010, chapter 3). But the transition from partial to full collection does not have to occur overnight to greatly reduce the demands made on other fiscal instruments. In Figures 7a-7b it takes a decade to complete the task. The VAT does not increase at all in three of the runs and rises only half a percentage point in the fourth. Moreover, while the commercial debt stays at a high level for a long time when public wages increase apace with private wages, it peaks earlier at a much lower level (23-36%) in the runs where the government holds non-investment expenditure constant.

Achieving a big push in public investment without increases in the VAT or cuts in non-investment expenditure is not the only strong selling point for gradually increasing the collection rate. Equally important, modest but steady increases in collections provide substantial insurance against debt blowups in the troublesome scenarios examined in the previous section. Figures 8a-8b compare outcomes with and without increases in the collection rate when public investment is inefficient and projects incur large cost overruns. In one case the debt blows up even as the VAT rises to 20%; in the other, a small increase in the rate to 15.5% preserves debt sustainability.
These results look too good to be true, and maybe they are. Undoubtedly some governments will succeed in raising the collection rate in upcoming decades. That said, it is not clear in our experiments whether the collection rate would be expected to rise or fall over time. The complicating factor is that scaling up public investment adds to the collection task by increasing the share of the population with access to infrastructure services. If collection capacity does not increase as fast as the supply of services, the collection rate might fall. The runs in Figures 7 and 8 are relevant and important, but also highly speculative.

6. Sensitivity Analysis

There is a good deal of information in Figures 2-8 about how the return on infrastructure, the efficiency of public investment, revenue from user fees, and constraints on expenditure cuts affect the path of debt accumulation. Below we summarize the findings from other runs undertaken to test the robustness of the results:

- **Small differences in fiscal effort often translate into large differences in indebtedness over the medium and long run.** In many of the runs the magnitude and the duration of the increase in the public commercial debt is discomfiting. Consider, for example, the comparison run (solid line) in Figure 9. Certainly strong nerves will be needed to stick with the program as the commercial debt rises toward 50% of GDP. If the government feels uncomfortable carrying this much debt, then it should raise the ceiling on the VAT from 15% to 15.5%. The debt then crests much earlier at a much lower level (32%); moreover, descent from the peak is rapid, with the debt falling to just 18% of GDP at \( t = 40 \).

- **The results are insensitive to variations in deep parameters that describe preferences, technology, and the general structure of the economy.** We have carried out runs for a wide range of alternative values of the intertemporal elasticity of substitution, the interest-elasticity of private capital flows, the consumption share of traded goods, the \( q \)-elasticity of investment spending, the elasticity of substitution between traded and nontraded goods, the percentage of hand-to-mouth consumers, the time preference rate, and factor cost shares. None of these parameters significantly affect the peak level of debt accumulation or the minimum increase in the VAT consistent with debt sustainability (Table 5).

- **Small differences in borrowing rates for concessional and commercial loans have big effects on debt accumulation and the amount of fiscal adjustment required for debt sustainability.** The real interest rate on public commercial loans is 6\% in the base case. Lowering the rate to 4\% reduces the minimum increase in the VAT required for debt sustainability by 1.4 percentage points (Table 5). For the same VAT, the peak ratio of the debt to GDP decreases 18–20 percentage points (Table 6). Changes in the real rate charged on
concessional loans have similarly large effects.

- **Growth in the private capital stock invariably lags far behind growth in the stock of infrastructure.** Table 7 shows how key deep parameters affect private investment, the private capital stock, and real output. There is some meaningful variation in the response of private investment in year one. This never translates, however, into significant variation in the private capital stock or real output at the 10-20 year horizon. The big-push program builds 76% of new infrastructure in the first ten years, increasing the total stock 50%. During the same period, the private capital stock increases only 2.2-3.9%. The pace of capital accumulation picks up in the second decade, but even at year twenty the private capital stock has covered only 19-42% of the ground to its new steady state level. Public investment is worth doing. It pays a high return and strongly increases the profitability of private investment. It should not be expected, however, to produce a growth miracle. The development gains accrue slowly, over a very long period of time.

- **The fiscal effort required for debt sustainability increases substantially when the risk premium is sensitive to the ratio of debt to GDP.** In the runs labeled ERP (short for Endogenous Risk Premium),

\[
\begin{align*}
\delta_{dc,t} = r_f + 0.03e^{\chi(D_t/GDP_t-D_0/GDP_0)},
\end{align*}
\]

where \(r_f = 0.03\) is the risk-free rate and \(D \equiv d + dc + b^*\) is the sum of public and private external debt.\(^{41}\) The parameter \(\chi\) sets the slope of the inverse loan supply curve. Some estimates in the literature support our choice for the base \((\chi \approx 0)\); others find \(\chi = 0.25-2.\)\(^{42}\) In the run where \(\chi = 1\), a ten percentage point increase in the debt-GDP ratio raises the borrowing rate from 6% to 6.31%. For \(\chi = 2\), the rate increases to 6.66%.

Moving up the loan supply curve is costly in big-push programs. The reason is simply that the debt rises to and stays at a high level for a long time. This increases the borrowing rate 1-1.4 percentage points for 56 years in the run \(\chi = 1\) and 2-3.4 percentage points for 49 years in the run \(\chi = 2\). As a result, the minimum VAT compatible with debt sustainability jumps from 19% to 19.9-21.5% (Table 5). The sharp increase implies, inter alia, the potential for sunspot equilibria. Suppose, for example, that a VAT above 20% is politically intolerable. When lenders believe the program is viable, they lend the requisite amount at 6% and the country repays every dollar it borrows without raising the VAT above 19%. But if fears of default drive the interest rate above 7.5%, the VAT hits the 20% ceiling and the country either defaults, validating the risk premium, or re-sizes its ambitions, substituting a small push for a big push.\(^{43}\) The fate of the program rests with lenders’ beliefs: optimism leads to success, pessimism to failure.

\(^{41}\) The risk-free rate equals the average real interest rate on 10-year U.S. treasury bonds for 1993-2007. Endogenous variations in the risk premium affect borrowing rates for both the public and private sector.

\(^{42}\) Cantor and Packer do not find any significant relationship between yield spreads and the ratio of debt to GDP, while the estimate in Dailami et al. (2005) is .009, and the point estimate in Hilscher and Nosbusch (2010) implies \(\chi = 0.01\). The coefficient on the public debt-GDP ratio is statistically insignificant in five of eight estimates in Borio and Packer (2004) and is either insignificant or of the wrong sign for Emerging Europe, Asia and Africa in Westphalen (2001). The estimates in Min (1998), Eichengreen and Mody (2000), Ferrucci (2003), Akitoby and Stratmann (2008), van der Ploeg and Venables (2011), Jaramillo and Tejada (2011), and Araujo et al. (2013) range from .25 to 2.

\(^{43}\) The probability attached to these two responses determine the size of the risk premium.
6.1 Alternative Investment Programs

Investment programs come in a variety of shapes, sizes, and colors. At the outset, the government has to decide whether to go big or small, whether to scale up quickly or slowly, and whether to invest solely in new projects or in a mix of new projects and capital maintenance. While there is no case for scaling up slowly and only a weak case for going small, the return to getting the right mix of new investment and maintenance is potentially very high:

- **The sustainable level of debt is lower for smaller investment programs.** Small investment programs increase future revenue less than large investment programs. Consequently, the sustainable level of debt is lower. When both investment and concessional borrowing are reduced 50%, the minimum VAT decreases from 19% to 17% and the peak sustainable increase in the ratio of commercial debt to GDP drops from 48.3% to 24.1%. The price for playing it safe (i.e., for reducing $h$ to levels that do not strain the capacity for fiscal adjustment) is very high — 7% of GDP at year twenty.

- **Fast scaling up is superior to gradual scaling up.** If the investment program is sound, the government should scale up as fast as absorptive capacity constraints permit. Gradual scaling up sacrifices 3.7% of GDP at year twenty without decreasing the amount of fiscal adjustment required for debt sustainability. ($h = .19$ in both the base case and the gradual scaling up scenario.)

- **If empirical estimates are correct, programs that rectify underfunding of maintenance pay a very high return.** More spending on maintenance increases the supply of infrastructure by extending the service life of the existing stock and newly created infrastructure. In the language of the model, the depreciation rate is a decreasing function of the ratio of real maintenance expenditure $m$ to the stock of infrastructure:

$$
(1 + g)z_t = i_{z,t} + [1 - \delta_o e^{-\Lambda m_t/z_t}]z_{t-1}, \quad \Lambda > 0.
$$

(63)

Maintenance is underfunded relative to new investment when $\Lambda > e^{Am/z} / \delta_o$. Development economists universally agree that this condition holds with margin to spare. According to empirical estimates and countless case studies, the return to $m$ is 2-5 times higher than the return to $i_z$.\(^{45}\)

We set $\Lambda$ equal to two or three and increased maintenance spending from zero to two percent of GDP by reducing new investment.\(^{46}\) Unsurprisingly, the more efficient investment program yields impressive gains: real output rises another 2-7 percentage points at year twenty and $h$ decreases from 17.8-19% to 16.1-18.1%.

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\(^{44}\) The supply price of maintenance is assumed to be the same as the supply price of infrastructure. The specification in (63) follows Rioja (2003) and Kalaitzidakis and Kalyvitis (2004). See Adam and Bevan (2014) for a more in-depth analysis of maintenance investment in a variant of the current model.

\(^{45}\) Heggie and Vickers (1998) estimate the return on road maintenance in World Bank projects at 38.6%. Foster and Briceno-Garmendia (2010, Chapter 2) estimate the return in Sub-Saharan Africa to be 139%. Gramlich (1994) cites CBO estimates that the return to highway maintenance in the United States was 35% in the 1980s.

\(^{46}\) The increase in maintenance expenditure occurs in a single step in year one.
7. Concluding Remarks

In this paper we have developed a new model-based framework for DSA (MBDSA) in an effort to answer the most trenchant criticisms of IMFWB debt sustainability analysis. The MBDSA is grounded in a fully-articulated, dynamic macroeconomic model. It allows for financing schemes that mix concessional and external commercial debt, while taking into account the impact of public investment on growth and constraints on the speed and magnitude of fiscal adjustment. The borrowing + investment program is judged sustainable if it satisfies the natural and theoretically correct criterion that the debt eventually converge to a stationary level.

The new MBDSA is far from perfect. The most common complaint we receive is that the model omits too much. Policy makers generally welcome the opportunity to supplement the existing DSF with analysis from the new MBDSA, but often want *more* bells and whistles. As usual, one size does not fit all.

To meet user demands for richer, more diverse country- and program-specific DSA, Fund research teams have developed, or are in the process of developing, variants of the benchmark model that incorporate (i) uncertainty; (ii) terms of trade shocks; (iii) investments in energy and human capital; (iv) additional fiscal instruments, including government consumption of traded/nontraded goods and taxes on wage income and profits; (v) underemployment and dualistic labor markets; and (vi) money and seigniorage. Choice in the suite of models is not, however, choice among competing frameworks for DSA. All of the models adhere to the general analytical approach in the benchmark model presented here. Numerous details differ across the models, but the fundamentals of DSA are the same.
Appendix A

The saddle point solution for the system in (37) is

\[ c - c^* = X_{11} h_1 e^{-\delta t} + (\rho - \lambda_2) h_2 e^{\lambda_2 t}, \]  
(A1)

\[ k - k^* = X_{21} h_1 e^{-\delta t} + h_2 e^{\lambda_2 t}, \]  
(A2)

\[ z - z^* = h_1 e^{-\delta t}, \]  
(A3)

where

\[ X_{11} = \frac{\tau(R + \delta) c/k}{J}, \]
\[ X_{21} = \frac{(R + \delta)(\delta + \tau c/k)}{(\rho + \delta) J}, \]
\[ J = \tau(1 - \alpha)c/k - \delta, \]
\[ \lambda_2 = \rho - \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k}, \]

and \( h_1 \) and \( h_2 \) are constants determined by initial conditions. To pin down \( h_1 \) and \( h_2 \), evaluate (A2) and (A3) at \( t = 0 \):

\[ h_1 = z_o - z^*, \]
\[ h_2 = X_{21}(z^* - z_o) - (k^* - k_o), \]

\[ \implies h_2 = \frac{(R + \delta) \delta}{(\rho + \delta) J (1 - \alpha)} (z^* - z_o), \]

where the last step makes use of

\[ k^* - k_o = \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} (z^* - z_o). \]  
(41)

Substituting for \( h_1 \) and \( h_2 \) in (A1) and (A2) gives

\[ c - c^* = \frac{R + \delta}{J} \left[ \frac{\tau c/k + \lambda_2 - \rho}{\rho + \delta} (\delta + \tau c/k) \right] (z - z^*) + (\rho - \lambda_2) (k - k^*), \]  
(39)

\[ k - k^* = \frac{R + \delta}{(\rho + \delta) J} \left[ \frac{\delta}{1 - \alpha} e^{\lambda_2 t} - (\delta + \tau c/k) e^{-\delta t} \right] (z^* - z_o). \]  
(40)
From (40),

\[
\frac{\dot{k}(0)}{\delta(z^* - z_o)} = \frac{(R + \delta)}{(\rho + \delta)J} \left( \frac{\lambda_2}{1 - \alpha} + \delta + \tau \alpha c/k \right),
\]

(A4)

\[
\Rightarrow \frac{\dot{k}(0)}{\delta(z^* - z_o)} = \frac{i(0) - i_o}{i^*_z - i_{z,o}} = \frac{R + \delta}{(\rho + \delta)(1 - \alpha)} \left( \frac{\lambda_2 + \delta}{J} + \alpha \right).
\]

(42)

Private investment decreases at \( t = 0 \) iff

\[
\frac{\lambda_2 + \delta}{J} + \alpha < 0
\]

or, equivalently,

\[
\frac{\lambda_2 + (1 - \alpha)(\delta + \tau \alpha c/k)}{J} < 0.
\]

(A5)

The above condition holds when

\[
\alpha^2 J < \rho(1 - \alpha) + \delta(1 - 2\alpha).
\]

After canceling terms, this simplifies to

\[
\tau < \frac{\rho + \delta(1 - \alpha)}{\alpha(\rho + \delta)c/q},
\]

(A6)

the condition stated in Proposition 1 in the text.

Turning back to (A4), the crowding-out coefficient exceeds unity in absolute value when

\[
\frac{(R + \delta)V}{2(\rho + \delta)J(1 - \alpha)} < -1,
\]

(A7)

where

\[
V \equiv \rho - \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k + 2(1 - \alpha)(\delta + \tau \alpha c/k)}.
\]

For \( J > 0 \), (A7) requires

\[
\rho + 2(X + F) < \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k},
\]

where

\[
X \equiv (1 - \alpha)(\delta + \tau \alpha c/k),
\]

\[
F \equiv (\rho + \delta)J(1 - \alpha)/(R + \delta).
\]
Squaring both sides produces
\[ \alpha^2 J + \frac{\rho + \delta}{R + \delta}(1 - \alpha) \left[ \rho + 2X + \frac{\rho + \delta}{R + \delta}(1 - \alpha)J \right] < \rho(1 - \alpha) + \delta(1 - 2\alpha), \]
\[ \implies J \left\{ \alpha^2 + \frac{\rho + \delta}{R + \delta}(1 - \alpha) \left[ \frac{\rho + \delta}{R + \delta}(1 - \alpha) + 2\alpha \right] \right\} < \rho(1 - \alpha) \left( 1 - \frac{\rho + \delta}{R + \delta} \right) \]
\[ + \delta \left\{ 1 - 2 \left[ \alpha + \frac{\rho + \delta}{R + \delta}(1 - \alpha) \right] \right\}. \quad (A9) \]

Further simplification leads to
\[ \tau(1 - \alpha)c/k < \delta + \frac{(\rho + \delta)(1 - \alpha)[1 - (\rho + \delta)/(R + \delta)] - \delta[\alpha + (\rho + \delta)(1 - \alpha)/(R + \delta)]}{1 - [1 - (\rho + \delta)/(R + \delta)](1 - \alpha)[1 + \alpha + (\rho + \delta)(1 - \alpha)/(R + \delta)]}, \]
and then
\[ \tau < \frac{\alpha(1 - H)[\rho + \delta(1 - \alpha)(1 - H)]}{\{\alpha^2 + H(1 - \alpha)[H(1 - \alpha) + 2\alpha](\rho + \delta)c/q\}}, \quad (A10) \]
where \( H \equiv (\rho + \delta)/(R + \delta). \)

**Appendix B**

When the government finances part of the investment program through concessional borrowing, the economy follows distinct nonconvergent paths during the borrowing and repayment phases before connecting to the saddle path at time \( 2t_1 \). The solutions for \( c \) and \( k \) in the three phases are\(^{47}\)
\[ c - \bar{c} = X_{11}(z_o - z^*)e^{-\delta t} + (\rho - \lambda_2)h_2e^{\lambda_2 t} + (\rho - \lambda_3)h_3e^{\lambda_3 t}, \quad 0 \leq t \leq t_1, \quad (B1) \]
\[ k - k^* = X_{21}(z_o - z^*)e^{-\delta t} + h_2e^{\lambda_2 t} + h_3e^{\lambda_3 t}, \quad 0 \leq t \leq t_1, \quad (B2) \]
\[ c - \bar{c} = X_{11}(z_o - z^*)e^{-\delta t} + (\rho - \lambda_2)g_2e^{\lambda_2 t} + (\rho - \lambda_3)g_3e^{\lambda_3 t}, \quad t_1 \leq t \leq 2t_1, \quad (B3) \]
\[ k - k^* = X_{21}(z_o - z^*)e^{-\delta t} + g_2e^{\lambda_2 t} + g_3e^{\lambda_3 t}, \quad t_1 \leq t \leq 2t_1, \quad (B4) \]
\[ c - c^* = X_{11}(z_o - z^*)e^{-\delta t} + (\rho - \lambda_2)v_2e^{\lambda_2 t}, \quad t \geq 2t_1, \quad (B5) \]
\[ k - k^* = X_{21}(z_o - z^*)e^{-\delta t} + v_2e^{\lambda_2 t}, \quad t \geq 2t_1, \quad (B6) \]

\(^{47}\)Since the analysis pertains to small changes, \( X_{11}, X_{21}, \) and \( \lambda_1 - \lambda_3 \) are the same in the three systems in (B1) - (B6).
where
\[
\lambda_3 = \frac{\rho + \sqrt{\rho^2 + 4\tau(1 - \alpha)(\rho + \delta)c/k}}{2}.
\]
The stationary value for \( k \) is the same in each system. The solutions for \( \bar{c}, \tilde{c}, \) and \( c^* \) follow directly from (51a)-(51c). Setting \( \dot{k} = 0 \) and making use of (41) gives
\[
\bar{c} - c_o = \left( \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} + R + \delta \chi \right) (z^* - z_o), \quad (B7)
\]
\[
\tilde{c} - c_o = \left( \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} + R - \delta \chi \right) (z^* - z_o), \quad (B8)
\]
\[
c^* - c_o = \left( \frac{R + \delta}{\rho + \delta} \frac{\alpha}{1 - \alpha} + R \right) (z^* - z_o). \quad (B9)
\]
We need five boundary conditions to tie down \( h_2, h_3, g_2, g_3, \) and \( v_2 \). Evaluating (B2) at \( t = 0 \) provides
\[
h_2 + h_3 = \frac{(R + \delta)\delta}{(\rho + \delta)J(1 - \alpha)} (z^* - z_o). \quad (B10)
\]
Since the capital stock is predetermined, equations (B2) and (B4) must yield the same solution for \( k \) at time \( t_1 \). Similarly, equations (B4) and (B6) have to return the same solution at time \( 2t_1 \). Thus
\[
g_2e^{\lambda_2t_1} + g_3e^{\lambda_3t_1} = h_2e^{\lambda_2t_1} + h_3e^{\lambda_3t_1}, \quad (B11)
\]
\[
g_2e^{\lambda_22t_1} + g_3e^{\lambda_32t_1} = v_2e^{\lambda_22t_1}. \quad (B12)
\]
To derive the last two boundary conditions, consult the first-order condition for consumption. This is
\[
c^{-1/\tau} = \theta(1 + h), \quad (B13)
\]
where \( \theta \) is the multiplier attached to the budget constraint. The VAT tax is constant, and foreseen jumps in \( \theta \) are inconsistent with optimizing behavior. Hence \( c(t^-_1) = c(t^+_1) \) and \( c(2t^-_1) = c(2t^+_1) \). Imposing these no-jump conditions in (B1), (B3), and (B5) yields
\[
\bar{c} + (\rho - \lambda_2)h_2e^{\lambda_2t_1} + (\rho - \lambda_3)h_3e^{\lambda_3t_1} = \bar{c} + (\rho - \lambda_2)g_2e^{\lambda_2t_1} + (\rho - \lambda_3)g_3e^{\lambda_3t_1}, \quad (B14)
\]
\[
c^* + (\rho - \lambda_2)v_2e^{\lambda_22t_1} = \bar{c} + (\rho - \lambda_2)g_2e^{\lambda_22t_1} + (\rho - \lambda_3)g_3e^{\lambda_32t_1}. \quad (B15)
\]
Equations (B10)-(B12), (B14), and (B15) can be solved for $h_2$, $h_3$, $g_2$, $g_3$, and $v_2$. The solutions are

$$h_3 = (2 - e^{-\lambda_3 t_1}) \frac{\delta \chi}{\lambda_3 - \lambda_2} e^{-\lambda_3 t_1} (z^* - z_0),$$

$$h_2 = \frac{(R + \delta) \delta}{(\rho + \delta) J (1 - \alpha)} (z^* - z_o) - h_3,$$

$$g_3 = \frac{\delta \chi}{\lambda_2 - \lambda_3} e^{-\lambda_3 t_1} (z^* - z_o),$$

$$g_2 = h_2 + (h_3 - g_3) e^{(\lambda_3 - \lambda_2) t_1},$$

$$v_2 = g_2 + g_3 e^{(\lambda_3 - \lambda_2) t_1}.$$

At $t = 0$, equation (B2) gives

$$\dot{k}(0) = \delta X_{21} (z_o - z^*) + h_2 \lambda_2 + h_3 \lambda_3. \quad \text{(B16)}$$

Substituting for $h_2$ and $h_3$ delivers

$$\frac{\dot{k}(0)}{\delta (z^* - z_o)} = \frac{R + \delta}{(\rho + \delta) (1 - \alpha)} \left( \frac{\lambda_2 + \delta}{J} + \alpha \right) + (2 - e^{-\lambda_3 t_1}) e^{-\lambda_3 t_1} \chi \quad \text{(B17)}$$

and the condition stated in equation (52) in the text.
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The large gap separating the positions of the IMFWB and the critics reflects in part a mismatch between what the DSF can supply and what is needed for detailed, informed policy discussion. The DSF is heavily empirical. It feeds cross-country data into a parsimonious probit model to estimate how the probability of debt distress depends on the debt burden, GDP growth, and a variable (the CPIA score) that proxies for the quality of institutions and the capacity for effective policy adjustment. The indicative debt threshold is then derived by solving for the ratio of debt to GDP consistent with a specified probability of debt distress. Interactive dummy variables control for differences between MICs and LICs, while stress tests evaluate the robustness of the baseline projection to various macroeconomic shocks.

The indicative debt threshold and the results from the stress tests do not mechanically determine a country's borrowing limit or its risk rating (IMF, 2012). They simply delineate danger zones where historically LDCs have often struggled to maintain debt service. Fund staff supplement this information with a good deal of judgment when assessing the risk of debt distress. But how often this is done and exactly how it is done is far from clear. Adding to the confusion, the baseline scenario becomes ill-defined at country-level analysis. Since the estimated probit model incorporates the effects of "average" shocks and the "average" reaction function in the cross-country sample, it is impossible to tell whether the indicative debt threshold derives from good or bad policies or whether the shocks in the aggregate sample are representative of historical shocks for the country under examination. It is not surprising therefore that the policy debate is both contentious and impoverished. The DSF is almost completely opaque; nobody really knows what lies inside the black box.
Table 1: Calibration of the Model.

<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Value in Base Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption share of the nontraded good ($\gamma_n$)</td>
<td>.5</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution ($\tau$)</td>
<td>.34</td>
</tr>
<tr>
<td>Elasticity of substitution between traded and nontraded consumer goods ($\varepsilon$)</td>
<td>.5</td>
</tr>
<tr>
<td>Capital’s share in value added ($\alpha_n, \alpha_x$)</td>
<td>$\alpha_n = .55, \alpha_x = .40$</td>
</tr>
<tr>
<td>q-elasticity of investment spending ($\Omega$)</td>
<td>2</td>
</tr>
<tr>
<td>Long-run target for external commercial debt ($dc_{\text{target}}$)</td>
<td>0</td>
</tr>
<tr>
<td>Real interest rate on concessional loans ($r_d$)</td>
<td>0</td>
</tr>
<tr>
<td>Real interest rate on non-concessional loans ($r_{dc}$)</td>
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</tr>
<tr>
<td>Trend growth rate ($g$)</td>
<td>.015</td>
</tr>
<tr>
<td>Real interest rate on domestic bonds ($r$)</td>
<td>.10</td>
</tr>
<tr>
<td>Ratio of user fees to recurrent costs ($f$)</td>
<td>.5</td>
</tr>
<tr>
<td>Consumption VAT ($h$)</td>
<td>.15</td>
</tr>
<tr>
<td>Real interest rate on foreign loans held by the private sector ($r^*$)</td>
<td>.10</td>
</tr>
<tr>
<td>Ratio of private foreign loans and public external debt ($d, b^*, dc$) to initial GDP</td>
<td>$d = .5, \ dc = 0, b^* = 0$</td>
</tr>
<tr>
<td>Ratio of infrastructure investment to GDP ($i_{z0}/GDP_0$)</td>
<td>.06</td>
</tr>
<tr>
<td>Ratio of labor supply of non-savers to labor supply of savers ($a$)</td>
<td>.60</td>
</tr>
<tr>
<td>Efficiency of public investment ($s$)</td>
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</tr>
<tr>
<td>Absorptive capacity constraint ($\phi$)</td>
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</tr>
<tr>
<td>Cost share of nontraded inputs in the production of capital goods ($\alpha_k = \alpha_z$)</td>
<td>.5</td>
</tr>
<tr>
<td>Interest elasticity of private capital flows ($\xi$)</td>
<td>1</td>
</tr>
<tr>
<td>Return on infrastructure ($R$)</td>
<td>.20</td>
</tr>
<tr>
<td>Ratio of elasticities of sectoral output with respect to the stock of infrastructure ($\psi_x/\psi_n$)</td>
<td>1</td>
</tr>
<tr>
<td>Division of fiscal adjustment between expenditure cuts and tax increases ($\lambda$)</td>
<td>.5</td>
</tr>
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</table>
Table 2: Crowding Out in the Simplified Model.1

<table>
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<tr>
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<th>( \tau^* )</th>
<th>( \bar{\tau} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R = .10 )</td>
<td>( R = .20 )</td>
</tr>
<tr>
<td>( \rho = .06 )</td>
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</tr>
<tr>
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<td>3.54</td>
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<td>2.66</td>
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<td>2.14</td>
</tr>
<tr>
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<td></td>
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<tr>
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<td>3.53</td>
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<table>
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<tr>
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<th>( \text{CIC}_{SR} ) when ( \tau = .50 )</th>
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<td>-.80</td>
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<tr>
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<td>-.85</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = .30 )</td>
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<td>-.93</td>
</tr>
<tr>
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<td>-1.06</td>
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<th>( \tau = .50 )</th>
<th>( \tau = .75 )</th>
<th>( \tau = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = .30 )</td>
<td>28.6</td>
<td>15.6</td>
<td>10.3</td>
<td>7.3</td>
</tr>
<tr>
<td>( \alpha = .40 )</td>
<td>30.1</td>
<td>15.8</td>
<td>9.9</td>
<td>6.7</td>
</tr>
<tr>
<td>( \alpha = .50 )</td>
<td>31.8</td>
<td>16.0</td>
<td>9.6</td>
<td>6.1</td>
</tr>
<tr>
<td>( \rho = .10 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = .30 )</td>
<td>23.5</td>
<td>12.4</td>
<td>8.0</td>
<td>5.6</td>
</tr>
<tr>
<td>( \alpha = .40 )</td>
<td>24.8</td>
<td>12.6</td>
<td>7.8</td>
<td>5.2</td>
</tr>
<tr>
<td>( \alpha = .50 )</td>
<td>26.1</td>
<td>12.7</td>
<td>7.5</td>
<td>4.7</td>
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</table>

<table>
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<th>( R = .20 )</th>
<th>( R = .30 )</th>
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<td></td>
<td></td>
</tr>
<tr>
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<td>.58</td>
<td>.97</td>
<td>1.36</td>
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<tr>
<td>( \alpha = .40 )</td>
<td>.91</td>
<td>1.52</td>
<td>2.12</td>
</tr>
<tr>
<td>( \alpha = .50 )</td>
<td>1.36</td>
<td>2.27</td>
<td>c3.18</td>
</tr>
<tr>
<td>( \rho = .10 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = .30 )</td>
<td>.43</td>
<td>.71</td>
<td>1.00</td>
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<td>( \alpha = .40 )</td>
<td>.67</td>
<td>1.11</td>
<td>1.56</td>
</tr>
<tr>
<td>( \alpha = .50 )</td>
<td>1.00</td>
<td>1.67</td>
<td>2.33</td>
</tr>
</tbody>
</table>

1 The depreciation rate and the ratio of infrastructure investment to GDP equal 5% in all runs. The value of \( t_2 \) is independent of the return on infrastructure (R).
Table 3: Crowding Out in the Simplified Model: Open Private Capital Account.\(^1\)

<table>
<thead>
<tr>
<th>(\rho)</th>
<th>(\alpha)</th>
<th>(\xi^*) = 1</th>
<th>(\xi^*) = 3</th>
<th>(\xi^*) = 5</th>
<th>(\xi^*) = 1</th>
<th>(\xi^*) = 3</th>
<th>(\xi^*) = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\tau = .25)</td>
<td>(\tau = .50)</td>
<td>(\tau = .75)</td>
<td>(\tau = 1)</td>
<td>(\tau = .25)</td>
<td>(\tau = .50)</td>
<td>(\tau = .75)</td>
</tr>
<tr>
<td>(.06)</td>
<td>(.30)</td>
<td>60.9</td>
<td>47.8</td>
<td>38.5</td>
<td>31.4</td>
<td>29.8</td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>(.40)</td>
<td>61.0</td>
<td>47.1</td>
<td>36.9</td>
<td>29.0</td>
<td>29.2</td>
<td>15.0</td>
</tr>
<tr>
<td></td>
<td>(.50)</td>
<td>61.2</td>
<td>46.5</td>
<td>35.3</td>
<td>26.4</td>
<td>28.5</td>
<td>14.5</td>
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<tr>
<td>(.10)</td>
<td>(.30)</td>
<td>33.6</td>
<td>26.6</td>
<td>21.5</td>
<td>17.5</td>
<td>23.4</td>
<td>12.2</td>
</tr>
<tr>
<td></td>
<td>(.40)</td>
<td>33.6</td>
<td>26.2</td>
<td>20.6</td>
<td>16.2</td>
<td>22.4</td>
<td>11.4</td>
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<tr>
<td></td>
<td>(.50)</td>
<td>33.6</td>
<td>25.8</td>
<td>19.7</td>
<td>14.7</td>
<td>25.8</td>
<td>10.6</td>
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</tbody>
</table>

\(^1\) The depreciation rate and the ratio of infrastructure investment to GDP equal 5% in all runs. The values for \(\xi^*\) and \(t_2\) are independent of the return on infrastructure.
Table 4: Crowding Out in the Simplified Model: Concessional Borrowing.\(^1\)

<table>
<thead>
<tr>
<th>(\rho) = .06</th>
<th>(\rho) = .10</th>
<th>(\chi) = .5</th>
<th>(\chi) = 1</th>
<th>(\chi) = 1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R = .10)</td>
<td>(R = .20)</td>
<td>(R = .30)</td>
<td>(R = .10)</td>
<td>(R = .20)</td>
</tr>
<tr>
<td>(\tau = .25)</td>
<td>(\tau = .50)</td>
<td>(\tau = .75)</td>
<td>(\tau = 1)</td>
<td>(\tau = .50)</td>
</tr>
<tr>
<td>(\chi) = .50</td>
<td>(\chi) = .75</td>
<td>(\chi) = 1</td>
<td>(\chi) = 1.25</td>
<td>(\chi) = .1.50</td>
</tr>
<tr>
<td>1.73</td>
<td>1.20</td>
<td>.69</td>
<td>.30</td>
<td>.09</td>
</tr>
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<td>2.12</td>
<td>1.83</td>
<td>1.52</td>
<td>1.20</td>
<td>.89</td>
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<td>2.28</td>
<td>2.08</td>
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<td>1.66</td>
<td>1.43</td>
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<td>2.17</td>
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<td>1.47</td>
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<td>1.64</td>
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<td>2.36</td>
<td>2.24</td>
<td>2.13</td>
<td>2.00</td>
</tr>
</tbody>
</table>

\(^1\) The depreciation rate and the ratio of infrastructure investment to GDP equal 5%, the income share of capital (\(\alpha\)) is 40%, and the borrowing + repayment period is 20 years (\(t_1 = 10\)) in all runs. The value of \(t_2\) is independent of the return on infrastructure (\(R\)).
Table 5: Minimum VAT and peak levels of external public sector debt compatible with debt sustainability when cuts in transfer payments are not feasible.\(^1\)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Deep Parameters</th>
<th>Commercial Debt</th>
<th>Total External Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>η</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base Case</td>
<td>.19</td>
<td>48.3</td>
<td>88.2</td>
</tr>
<tr>
<td>τ = .20</td>
<td>.192</td>
<td>48.5</td>
<td>88.5</td>
</tr>
<tr>
<td>τ = .75</td>
<td>.188</td>
<td>46.3</td>
<td>87.6</td>
</tr>
<tr>
<td>Ω = .50</td>
<td>.192</td>
<td>45.5</td>
<td>88.9</td>
</tr>
<tr>
<td>Ω = 10</td>
<td>.189</td>
<td>46.6</td>
<td>86.6</td>
</tr>
<tr>
<td>ξ = .10</td>
<td>.19</td>
<td>48.6</td>
<td>88.3</td>
</tr>
<tr>
<td>ξ = 5</td>
<td>.19</td>
<td>48.0</td>
<td>87.6</td>
</tr>
<tr>
<td>No non-savers (a = 0)</td>
<td>.191</td>
<td>52.1</td>
<td>91.5</td>
</tr>
<tr>
<td>ε = .20</td>
<td>.19</td>
<td>51.8</td>
<td>90.5</td>
</tr>
<tr>
<td>ε = .75</td>
<td>.19</td>
<td>47.1</td>
<td>87.6</td>
</tr>
<tr>
<td>Factor shares = 1(^2)</td>
<td>.19</td>
<td>45.9</td>
<td>90.6</td>
</tr>
<tr>
<td>α(_x) = .55, α(_n) = .40</td>
<td>.194</td>
<td>46.5</td>
<td>88.6</td>
</tr>
<tr>
<td>α(_x) = α(_n) = .30</td>
<td>.188</td>
<td>42.6</td>
<td>85.4</td>
</tr>
<tr>
<td>R = .30</td>
<td>.178</td>
<td>50.1</td>
<td>86.9</td>
</tr>
<tr>
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<td>.205</td>
<td>45.9</td>
<td>92.4</td>
</tr>
<tr>
<td>r = .07</td>
<td>.196</td>
<td>45.9</td>
<td>88.7</td>
</tr>
<tr>
<td>γ(_x) = .25</td>
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<td>47.6</td>
<td>90.7</td>
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<table>
<thead>
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<th>Borrowing Costs</th>
<th>Commercial Debt</th>
<th>Total External Debt</th>
</tr>
</thead>
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<tr>
<td></td>
<td>η</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r(_{dc}) = .04</td>
<td>.176</td>
<td>53.5</td>
<td>92.9</td>
</tr>
<tr>
<td>r(_d) = .02</td>
<td>.196</td>
<td>55.4</td>
<td>89.7</td>
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<td>ERP(^3), Λ = 1</td>
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<td>46.9</td>
<td>89.0</td>
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<tr>
<td>ERP, Λ = 2</td>
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<td>46.6</td>
<td>90.3</td>
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Table 5 (continued)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\tilde{h}$</th>
<th>Commercial Debt</th>
<th>Total External Debt</th>
</tr>
</thead>
<tbody>
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<td>$s = .70$</td>
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<td>90.7</td>
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<tr>
<td>$\phi = 5.19$</td>
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<td>62.0</td>
<td>101.5</td>
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</table>

<table>
<thead>
<tr>
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<th>$\tilde{h}$</th>
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<th>Total External Debt</th>
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<td>53.1</td>
<td>92.5</td>
</tr>
<tr>
<td>$f = 1$</td>
<td>.172</td>
<td>44.1</td>
<td>85.7</td>
</tr>
<tr>
<td>Gradual Scaling Up$^4$</td>
<td>.19</td>
<td>44.3</td>
<td>84.3</td>
</tr>
<tr>
<td>Program 50% Smaller$^5$</td>
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<td>24.1</td>
<td>69.4</td>
</tr>
<tr>
<td>Maintenance$^6$</td>
<td>$\Lambda = 2$</td>
<td>.181</td>
<td>51.5</td>
</tr>
<tr>
<td></td>
<td>$\Lambda = 3$</td>
<td>.173</td>
<td>56.5</td>
</tr>
<tr>
<td></td>
<td>$\Lambda = 2, R = .30$</td>
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<td>54.5</td>
</tr>
<tr>
<td></td>
<td>$\Lambda = 3, R = .30$</td>
<td>.161</td>
<td>60.8</td>
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</table>

1 At the initial equilibrium, the VAT is 15%, the return on infrastructure is 20%, and the ratio of revenue from user fees to recurrent costs ($f$) is .5. The values for all other parameters in the base case are the same as in Table 1.

2 Production function exhibits constant returns to scale, with factor shares for labor and capital marked down by one minus the factor share of infrastructure.

3 Runs where the risk premium is an increasing function of the ratio of debt to GDP. A ten percentage point increase in the debt ratio increases the external borrowing rate by 50 basis points when $\eta = 1.57$ and by 75 basis points when $\eta = 2.23$.

4 Investment in infrastructure increases by 2.4% of initial GDP in year one and then rises in approximately equal steps to its steady-state level in year eight. Concessional borrowing rises steadily with investment in the first eight years, with total borrowing the same as for rapid scaling up.

5 Values for infrastructure investment and concessional borrowing in each year are half as large as in the base case.

6 Maintenance investment is 2% of initial GDP each year. Total investment is unchanged.
Table 6: Peak increases in public sector debt when cuts in transfer payments are not feasible and (i) the external loan rate is 4% vs. 6% or (ii) the concessional loan rate is 2% vs. 0.1

<table>
<thead>
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<th>Commercial Debt</th>
<th>Total External Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{dc} = .06$</td>
<td>$r_{dc} = .04$</td>
</tr>
<tr>
<td>$R = .20, \bar{h} = .190$</td>
<td>48.3</td>
<td>29.9</td>
</tr>
<tr>
<td>$R = .30, \bar{h} = .178$</td>
<td>50.1</td>
<td>30.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Commercial Debt</th>
<th>Total External Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_d = 0$</td>
<td>$r_d = .02$</td>
</tr>
<tr>
<td>$R = .20, \bar{h} = .196$</td>
<td>37.1</td>
<td>55.4</td>
</tr>
<tr>
<td>$R = .30, \bar{h} = .183$</td>
<td>37.5</td>
<td>60.4</td>
</tr>
</tbody>
</table>

1 Figures are the ratio of public sector debt to GDP. At the initial equilibrium, the external commercial debt is zero and total external public debt is 50% of GDP. Notation: $\bar{h}$ is the ceiling on the VAT; and $r_d$ and $r_{dc}$ are the interest rates on concessional and non-concessional loans.
Table 7: Private investment, the private capital stock, and real output at different time horizons.*

<table>
<thead>
<tr>
<th>Scenario</th>
<th>I(1)</th>
<th>I(10)</th>
<th>K(10)</th>
<th>K(20)</th>
<th>Y(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>1.2</td>
<td>10.3</td>
<td>3.6</td>
<td>7.7</td>
<td>14.8</td>
</tr>
<tr>
<td>(\tau = .20)</td>
<td>-0.7</td>
<td>7.5</td>
<td>2.4</td>
<td>5.6</td>
<td>13.8</td>
</tr>
<tr>
<td>(\tau = .75)</td>
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<td>14.0</td>
<td>5.2</td>
<td>10.4</td>
<td>16.2</td>
</tr>
<tr>
<td>(\Omega = .50)</td>
<td>1.3</td>
<td>6.2</td>
<td>2.2</td>
<td>4.8</td>
<td>13.4</td>
</tr>
<tr>
<td>(\Omega = 10)</td>
<td>-1.6</td>
<td>12.1</td>
<td>4.1</td>
<td>9.1</td>
<td>15.5</td>
</tr>
<tr>
<td>(\xi = .10)</td>
<td>-0.5</td>
<td>9.3</td>
<td>3.5</td>
<td>7.6</td>
<td>14.8</td>
</tr>
<tr>
<td>(\xi = 5)</td>
<td>2.6</td>
<td>11.5</td>
<td>3.8</td>
<td>8.0</td>
<td>15.0</td>
</tr>
<tr>
<td>No non-savers ((a = 0))</td>
<td>-2.5</td>
<td>9.1</td>
<td>2.4</td>
<td>6.6</td>
<td>14.3</td>
</tr>
<tr>
<td>Factor shares = 1</td>
<td>1.4</td>
<td>10.8</td>
<td>3.9</td>
<td>7.7</td>
<td>14.0</td>
</tr>
<tr>
<td>(a_x = .55, a_n = .40)</td>
<td>-2.3</td>
<td>9.8</td>
<td>2.6</td>
<td>7.1</td>
<td>14.5</td>
</tr>
<tr>
<td>(a_x = a_n = .30)</td>
<td>-0.5</td>
<td>10.7</td>
<td>3.3</td>
<td>7.6</td>
<td>13.4</td>
</tr>
<tr>
<td>R = .30</td>
<td>-2.4</td>
<td>13.1</td>
<td>3.6</td>
<td>9.8</td>
<td>20.8</td>
</tr>
<tr>
<td>R = .10</td>
<td>4.8</td>
<td>8.2</td>
<td>3.7</td>
<td>5.7</td>
<td>9.3</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Alternative Investment Programs</th>
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<tr>
<td>Program 50% Smaller</td>
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<td>(\Lambda = 2, R = .30)</td>
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* Notation: I, K, and Y refer to total private investment, the total private capital stock, and real output. The solutions show the percentage deviation of the variable from its initial steady-state value at different time horizons. See the notes to Table 5 for explanations of the runs Factor Shares = 1, Program 50% Smaller, Gradual Scaling Up, and Maintenance.
Figure 1: Transition path for the private capital stock.
Figure 2: The transition path in the base case when the government takes out only concessional loans. The fiscal deficit and debt are expressed as percentages of current GDP while the plot for transfers (T) shows the change in transfers as a percentage of initial GDP. All other plots, except those for taxes (h) and the interest rate (r), show the percentage deviation of the variable from its initial steady-state value.
Figure 3a: Paths of the VAT rate, transfers, and debt in the base case when the ceiling on the VAT is 19% and cuts in transfers are not feasible.

Figure 3b: Paths of the VAT rate, transfers, and debt in the base case when the ceiling on the VAT is 20.3% and public sector wage increases match private wage increases.
Figure 4a: Paths of the VAT rate, transfers, and debt when user fees finance all recurrent costs and the return on infrastructure is 30%. The ceiling on the VAT is 16.4% and cuts in transfers are not feasible.

Figure 4b: Paths of the VAT rate, transfers, and debt when user fees finance all recurrent costs and the return on infrastructure is 20%. The ceiling on the VAT is 17.2% and cuts in transfers are not feasible.
Figure 5a: Paths of the VAT rate, transfers, and debt when user fees finance all recurrent costs and the return on infrastructure is 30%. The ceiling on the VAT is 17.5% and public sector wage increases match private wage increases.

Figure 5b: Paths of the VAT rate, transfers, and debt when user fees finance all recurrent costs and the return on infrastructure is 20%. The ceiling on the VAT is 18.2% and public sector wage increases match private wage increases.
Figure 6: Outcome when the government delays increases in the VAT (Panel A), public investment is inefficient (Panel B), the actual return is less than the expected return (Panel C), and absorptive capacity problems lead to temporary cost overruns of 40 percent on new projects (Panel D). User fees cover all recurrent costs and the return to infrastructure is 20-30 percent. The counterfactual (blue line) comes from Figure 4a when R=.30 (Panels B, C, and D) and from Figure 4b when R=.20 (Panel A).
Figure 7a: Paths of the VAT, transfers, and debt when revenue from user fees increases from 50% to 100% of recurrent costs over a period of ten years. The return on infrastructure is 20-30%, the ceiling on the VAT is 15% and cuts in transfer payments are not feasible.

Figure 7b: Paths of the VAT, transfers, and debt when revenue from user fees increases from 50% to 100% of recurrent costs over a period of ten years. The return on infrastructure is 20-30%, the ceiling on the VAT is 15-15.5% and public sector wage increases match private wage increases.
Figure 8a: Paths of the VAT, transfers, and debt when revenue from user fees increases from 50% to 100% of recurrent costs over a period of ten years and the ceiling on the VAT is 15.5%. The return on infrastructure is 30%, public investment is inefficient ($s = .7$), and cuts in transfer payments are not feasible. In the counterfactual (blue line), revenue from user fees is 50% of recurrent costs.

Figure 8b: Paths of the VAT, transfers, and debt when revenue from user fees increases from 50% to 100% of recurrent costs over a period of ten years and the ceiling on the VAT is 15.5%. The return on infrastructure is 30%, new projects incur temporary cost overruns of 40% ($\phi = 5.19$), and cuts in transfer payments are not feasible. In the counterfactual (blue line), revenue from user fees is 50% of recurrent costs.
Figure 9: Impact on debt accumulation of increasing the VAT a half percentage point. The return on infrastructure is 30%, public sector wage increases match private wage increases, and revenue from user fees increases from 50% to 100% of recurrent costs over a period of ten years.