Dylan P. Thurston: teaching statement

1 Geometry and compassion

My greatest strengths as a teacher and advisor are a strong geometric intuition, empathy with students, and an ability to think on my feet and see to the bottom of students’ questions.

I have taught a wide variety of courses, from small discussion classes to lecture classes of 40 students, from introductory calculus courses to graduate courses. I adopt my teaching style to the circumstances, always striving for student interaction. My teaching continues to evolve and improve as I learn more about the ways that students think and how to communicate effectively with them.

In addition, I have advised many students in a variety of contexts: relatively brief advising about courses, office hours, reading courses, and honors theses. Students seek me out for this purpose, and I find that it is very helpful for my teaching to interact with students one-on-one.

Geometric intuition

I use my geometric intuition to give models for algebraic subject matter. For instance, while teaching Complex Analysis, I spent several lectures on the geometry of simple conformal mappings and just what “conformal” means. This paid off later in the course when we got to the Riemann Mapping Theorem. The students enjoyed this; in response to a question, a student once jumped out of his seat: “Oh! Oh!”

Complex analysis in one variable is one of my favorite subjects to teach because of the intense interplay between the geometry and topology of the complex plane, the analysis of (e.g.) line integrals, and the algebra of (e.g.) Möbius transformations. I learned a great deal from teaching it once, and look forward to teaching it again with further ideas. For instance, I can improve on the discussion of the index of a curve around a point, either by honestly proving that the fundamental group of the punctured plane is $\mathbb{Z}$ or by honestly avoiding it.

I also would love to teach a second-semester course on Riemann surfaces in general and elliptic curves in particular. My point of view would be more geometric and topological than any other treatment I am aware of. There is a great deal of beauty and symmetry to, among other things, the Jacobi elliptic functions, and it can get lost in a maze of formulas. (In my view, they are best understood by concentrating almost entirely on where the poles and zeros lie on the torus.) I taught the second half of such a course at the University of Geneva in 1998, but I have learned a lot since then about the subject and about teaching.

Empathy with students

Empathy with students comes out most when I advise students, whether I am helping students choose courses or guiding them in a thesis project. I enjoy both activities tremendously, and I am good at it. I have advised many math majors on courses; most often what they need is someone to listen sympathetically and push gently from time to time. Many math majors are interested in subjects outside of math, so it helps that I am familiar with many related fields, from computer science to statistics and physics.

I have advised undergraduate theses and conducted one-on-one reading courses, which lets me interact with students in a much more sustained way. This interaction helps me teach, because I can see in detail how students think about mathematics. I have worked with students at very different levels and with very different interests. I suggest topics based on what they are capable of and what they are interested in. For instance, Andrew Chi took my Complex Analysis class and wrote his final paper on the arithmetic-geometric mean. In that paper, his approach was fairly analytic. When
I mentioned that there is a more geometric way to think about the problem (in terms of 2-fold coverings of elliptic curves), he was very interested; I am now advising his senior thesis on the subject. His previous experience with a less intuitive explanation of the result and his own geometric understanding combine to make this a good subject.

Understanding student questions

I think well on my feet, and can quickly understand where student questions and comments come from. I thus adapt a more informal and discussion-oriented style of teaching. Student questions often lead me in unexpected and positive directions. I went the farthest in this direction while teaching a first course in abstract algebra to a small section of 5 students. (Due to a scheduling conflict, the department asked me to help out with these students and I was happy to step in.) Our textbook was good enough and the class small enough that I required the students to read the book before class and hardly lectured at all. Instead, I led discussions on various subtle points, like whether isomorphic groups should be considered “the same”; in more sophisticated language, it is the distinction between isomorphism and canonical isomorphism. In the course of the discussion, I got one student to take both sides of the issue. A senior in my class told me it was the first time she enjoyed a math course. She did a reading course with me on hyperbolic groups and seriously considered graduate school in mathematics as a result.

2 Future courses

I am happy to fill whatever needs the department has. If I get a chance, I would love to teach courses that use my strengths in geometric topology most fully. Besides complex analysis as mentioned above, there are a number of geometry courses I would like to try, from elementary to advanced. All of them might use Escher’s prints, like his Circle Limit IV (on the next page).

- “Geometry and the imagination”, for non-majors, on a variety of elementary topics that can be understood with minimal background, yet get to very deep parts of mathematics. This gives a very different view of what mathematics is all about than the calculus and linear algebra sequence. One topic would be plane symmetry patterns. A hyperbolic symmetry group like that of Circle Limit IV would be something a student might explore in a class project. I have seen this class taught, and have wanted for years to lead a version of it. A syllabus from one version of the course can be found at http://math.dartmouth.edu/~doyle/docs/gi/gi/gi.html.

- “The classical geometries”, for undergraduates. A more rigorous introduction to the different classical geometries: spherical, Euclidean, and hyperbolic. One topic to cover would be the classification of 2-dimensional symmetries in various geometries and the role of the orbifold Euler characteristic in it. In particular, just by looking at the local properties of the Circle Limit IV tiling, you can deduce that it must tile the hyperbolic plane.

- “3-manifold geometry and topology”, for graduate students. A course geared towards fully understanding the Geometrization Conjecture, including key topological lemmas like the Loop and Sphere theorems, and more geometric subjects like Seifert fiber spaces. One example to cover would be the unit tangent bundle to the quotient of the hyperbolic plane by the symmetries of Circle Limit IV, as described in Montesinos’ book *Classical tessellations and three-manifolds*. 

Dylan P. Thurston: teaching statement
Figure 1: Escher’s Circle Limit IV.