Hyperbolic Geometry and 2-coordinates

Today, geometric class algorithms (from hyperbolic structures, measured laminations are the temporal limit) construct due to Fein- Guasdue, Grisakias - Shipman - Windshein.

Def: Given $(S, M)$ marked surface as before, not too small.

The Teichmüller space $T(S, M)$ is the space of metrics on $S$ with:

- $\text{hyperbolic : constant curvature } -1$,
- $\text{geodesic boundary at boundary of } S$,
- $\text{max area, ends bounded,}$
- $\text{considered up to diffeomorphism, homeomorphic to identity}$.

\[ T(S, M) \]
Notes
1. Equivalent to conformal structure on $S$ by uniformization theorem.
2. Turn a measured lamination $L$ into a family of metrics:

   ![Diagram]

   As above, lengths in $g$ are dominated by $te$ of times curve crosses $L$.

3. Contrast Teichmüller space with moduli space:
   - Teichmüller space records how metric sits on surface.
   - Teichmüller space is contractible, moduli space is not.
   - Moduli space has finite volume, Teichmüller space does not.

4. $D(S)$ is a manifold of dimension $rac{6g-6+2p+3h}{2}$
Example: Final Project

Sales, N = 35

Time of year: Winter

Needs: Samples, parts

Finishes the project plan

To meet the desired outcome,

Draw the symbols for the

Symbols code for production on

Based on the calculations, the outcome

Revise the drawings once more.
Def. A 

Def. The discretized Teichmüller space \( \mathcal{T}(S, \Gamma) \) is
- a point in \( \mathcal{T}(S, \Gamma) \)
- a choice of homotopy around each cycle from \( M \)

Def (Penner). For an arc \( A \) in \( S \) and \( \hat{z} \in \mathcal{T}(S, \Gamma) \),
the length of \( A \) with respect to \( \hat{z} \) is
\[
\ell_{\hat{z}}(A) = \text{length on geodesic representative of } A \text{ between intersections with horocycles around } \hat{z}.
\]
The length is
\[
\ell_{\hat{z}}(A) = \exp \left( \frac{1}{\mu} \langle A, \hat{z} \rangle_M \right).
\]
All surfaces have only finitely many combinatorial types of triangulations (A theorem). Only finitely many triangulations.

Example: $\mathbb{D}^2$, usual realization:

$G_{(2,n+2)} = \left\{ \begin{array}{ll}
\mathbb{H}^2 & \text{if } n = 1 \\
\mathbb{H}^2 \times \mathbb{R}^2 & \text{if } n > 1
\end{array} \right.$

with spherical boundary.

- $\mathbb{H}^2 \subset \mathbb{R}^3$ on boundary of $\mathbb{H}^2$ (upper half-plane model).
- (end points $-1, 1$) or open ideal $(0,\infty)$-cusp.

For $(n,k)$ also has a scale-a-hyperbolic determinant $\det \left( \begin{array}{cc} n & k \\ -k & n \end{array} \right)$ is $3 \times 3$th.
Suppose we forget the standard and work with an original stochastic process $\tilde{X}(\omega,t)$ and its law $\tilde{P}$. Consider the family of probability measures $\{\tilde{P}(\cdot | \omega, t): \omega \in \Omega, t \geq 0\}$ given by the conditional distribution: $\tilde{P}(A | \omega, t) = \tilde{P}(A | \omega(t))$. Let $\tilde{E}$ denote the expectation with respect to $\tilde{P}$. The sequence of random variables $\tilde{X}(\omega, t)$ is then characterized by the functional equation $\tilde{E}(\tilde{X}(\omega, t) - \tilde{X}(\omega, s)) = \tilde{E}(\tilde{X}(\omega, t) - \tilde{X}(\omega, s))$. The solution to this equation is given by $\tilde{X}(\omega, t) = \tilde{X}(\omega, s) + \tilde{W}(t - s)$, where $\tilde{W}$ is a Brownian motion. The process $\tilde{X}(\omega, t)$ is then a solution to the stochastic differential equation $d\tilde{X}(\omega, t) = \tilde{b}(\tilde{X}(\omega, t)) dt + \tilde{a}(\tilde{X}(\omega, t)) d\tilde{W}(t)$. The initial condition is $\tilde{X}(\omega, 0) = \tilde{X}_0(\omega)$.