KNOTS WITH SMALL RATIONAL GENUS

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Abstract. If $K$ is a rationally null-homologous knot in a 3-manifold $M$ then there is a compact orientable surface $S$ in the exterior of $K$ whose boundary represents $p[K]$ in $H_1(N(K))$ for some $p > 0$. We define $\|K\|$, the rational genus of $K$, to be the infimum of $-\chi(S)/2p$ over all $S$ and $p$. If $M$ is a homology sphere then this is essentially the genus of $K$. By doing surgery on knots in $S^3$ one can produce knots in 3-manifolds with arbitrarily small rational genus. We show that such knots can be characterized geometrically. More precisely we show that there is a positive constant $C$ such that if $K$ is a knot in a 3-manifold $M$ with $\|K\| < C$ then $(M,K)$ belongs to one of a small number of classes; for example, $M$ is hyperbolic and $K$ is a core of a Margulis tube, $M$ is Seifert fibered and $K$ is a fiber, $K$ lies in a JSJ torus in $M$, etc. Conversely we show that there are pairs $(M,K)$ in each of these classes with $\|K\|$ arbitrarily small. This is joint work with Danny Calegari.