Practice Midterm I
Professor Thurston

To receive full credit, you must explain your answers (unless otherwise specified).

No calculators of any type are allowed.

This sample midterm adds up to 55 points; the real midterm will be slightly shorter and will have a total of 50 points.

1 (5 points) Answer true or false. For this question only, you need not give full justifications.
(a) If a line is given parametrically by \( x = at + x_0, \ y = bt + y_0, \ z = ct + z_0 \), then the vector \( \langle a, b, c \rangle \) is parallel to the line.
(b) For any two vectors \( \vec{v} \) and \( \vec{w} \), \( |\vec{v} \times \vec{w}| = |\vec{w} \times \vec{v}| \).
(c) For any vectors \( \vec{u}, \vec{v}, \) and \( \vec{w} \), \( \vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \times \vec{w} \).
(d) For any vectors \( \vec{u}, \vec{v}, \) and \( \vec{w} \), \( \vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w}) \).

2 (5 points) Find two unit vectors perpendicular to the line \( x = 2t + 1, \ y = -t - 1 \).

3 (5 points) Use Cramer’s rule to find \( s \) and \( t \) so that
\[
\begin{align*}
  s - t &= 1 \\
  2s + t &= 1
\end{align*}
\]
(4) (10 points) Consider the quadratic surface given by 
\[ x^2 + y^2 - z^2 = 1. \]

(a) Draw at least 3 sections of this surface. As suggestions, you might draw the sections parallel to the \(xy\) plane with \(z = 0\), \(z = 1\), and \(z = 2\).

(b) Which of the following graphs could be a graph of that surface?

\[ \begin{array}{ll}
(1) & \quad \text{[Graph 1]} \\
(2) & \quad \text{[Graph 2]} \\
(3) & \quad \text{[Graph 3]} \\
(4) & \quad \text{[Graph 4]}
\end{array} \]

(5) (10 points)

(a) Find the intersection of the planes \(x + y = 1\) and \(x - 2y + z = -1\).

(b) Find the equation of the plane containing the line you found in part (a) and the point \((1, 1, 1)\). Put your answer in the form \(ax + by + cz + d = 0\).

(6) (10 points) Let \(P_1\) be the plane \(x - z = 0\), let \(P_2\) be the plane \(x + y - z = 1\), and let \(P_3\) be the plane \(-x - y + z = 2\).

(a) Which of these 3 planes are parallel?

(b) Find the acute dihedral angles between all planes that intersect.

(c) Find the distance between the parallel planes.

(7) (10 points)

(a) Find the parametric equations for the tangent line to the curve given parametrically by 
\[ \begin{align*}
x &= t \cos 2t \\
y &= t \sin 2t
\end{align*} \]

at \(t = 0\).

(b) Now find the implicit equation for the line you found in part (a).