1 Exercises

1. Section 13.3: 1, 3, 9, 13, 21.

2. Section 13.4: 9, 21, 23, 31, 33, 37.

2 Problems

1. Find and prove the optimal angle at which to shoot a projectile so that it travels the greatest horizontal distance before hitting the ground again. Try to explain your reasoning clearly, with complete sentences and not just formulas. Follow the sketch we gave in class:

   (a) Start the projectile at time $t = 0$ at the origin, at an angle of $\theta$ from the horizontal axis.
   (b) Find its position a general time $t$ by integrating the equations of motion $\mathbf{r}'' = \mathbf{a} = (0, -mg)$.
   (c) Find the time at which the projectile hits the ground again, and its position at that time.
   (d) Find the value of $\theta$ for which that horizontal position is as far to the right as possible.

Do the same thing for a projectile fired uphill on a hill with a slope of $20^\circ$.

2. Let a planar curve be defined by setting $y$ to be a function of $x$: $y = f(x)$. Write the curve parametrically, and find the arclength and curvature in terms of the derivatives of $f$.

3. A projectile is fired at an angle of 45 degrees with a speed of 150 m/s. The acceleration due to gravity is 9.8 m/s$^2$ downwards.

   (a) Find the distance along the ground to the point it lands.
   (b) Find an integral expression for the distance the projectile travels (i.e., the arc-length).

   You need not evaluate the integral, although it is possible.

4. Show that if a particle moves with constant speed, its velocity and acceleration vectors are orthogonal. (Remember that the velocity is a vector and the speed is the length of the velocity. If the speed is constant the velocity need not be constant.)