(1) Let $F(x, y) = (x^2 + y, y^2 + x)$ and let $L(x, y) = (2x + y, x + 4y)$. The goal of this problem is to prove that the approximation

$$F(1 + h, 2 + k) \approx F(1, 2) + L(h, k)$$

is a good one, in the sense that $L$ is the total derivative of $F$ at $(1, 2)$.

- Write down the limit you have to check to see that $F$ is differentiable at $(x, y) = (1, 2)$ with total derivative $L$.
- Check that the limits you got in the first part are indeed 0.

(2) Total derivative handout: Exercises 4.14, 4.16, 4.17, 5.6, 5.8, 5.9, 6.6, 6.7.

Optional, challenge problems (turn in separately from the homework, if you do them):

(3) Total derivative handout: Challenge Problem 5.10.

(4) Let $B$ be the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Express the entries of $B^n$ in terms of Fibonacci numbers, defined by

$$F_0 = 1$$

$$F_1 = 1$$

$$F_{n+1} = F_n + F_{n-1} \quad n \geq 2.$$  

Prove that your answer is correct.