Problem Set #4
Section 4, Prof. Dylan Thurston
Due Wednesday, October 5, 2005

As a reminder, you are encouraged to work with other students, but please write up your solutions yourself and credit your collaborators.

1 Homework

1. Find the dihedral angle of the tetrahedron. This is the exterior angle between two adjacent faces of the tetrahedron. Recall that you can take the vertices of the tetrahedron to be $(1, 1, 1), (1, -1, -1), (-1, 1, -1), \text{ and } (-1, -1, 1)$. Remember that there is some ambiguity in finding the dihedral angle between planes; in this case, the fact that the dihedral angle of the tetrahedron is bigger than $270^\circ$ removes the ambiguity.

2. Let $L_1$ be the line $\vec{r} = (1, 1, 0) + t(-2, 1, 0)$ and let $L_2$ be the line $\vec{r} = (-1, 0, -1) + t(0, 1, -1)$

   (a) Find the plane containing $L_1$ and parallel to $L_2$. Verify that this plane does not contain $L_2$; $L_1$ and $L_2$ are said to be skew lines.

   (b) Find the distance between the plane you found in part (a) and a point on $L_2$. Does the distance depend on the point on $L_2$ that you picked? In fact, this distance is the closest distance between the lines $L_1$ and $L_2$.

3. Use vectors and the cross product to give a parametric equation for the line of intersection between the planes $x + y + z = 1$ and $x + y - z = 1$.

4. Find the traces (sections) of the surface $x + y^2 + z^2 = 0$ parallel to the $xy$, $yz$, and $xz$ planes. Sketch the resulting surface.

5. Evaluate $(1 + \sqrt{3}i)^3$. Plot $(1 + \sqrt{3}i)$, $(1 + \sqrt{3}i)^2$, and $(1 + \sqrt{3}i)^3$ in the complex plane.

2 Exercises

(These problems are not to be handed in; they are for your practice.)


3. Homework 1 and 2 from the handout on complex numbers.