IR fixed point pattern of standard model couplings

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Standard model

Out of 17 dimensionless parameters:

\[ \alpha_1, \alpha_2, \alpha_3, y_t, y_b, y_\tau, \lambda_h \]

only 7 couplings are sizable

all others = 0  (in the first approximation)
In the MSSM+1VF

the values of all large couplings:

\[ \alpha_1, \alpha_2, \alpha_3, y_t, y_b, y_\tau, \lambda_h \]

can be understood from the IR fixed point structure of renormalization group equations
MSSM with a complete vectorlike family

We add to the MSSM:

\[ Q, \bar{U}, \bar{D}, L, E + \bar{Q}, U, D, \bar{L}, E \]

or \[ 16 + \bar{16} \] in SO(10) language

We consider:

- **unrelated** gauge couplings at the GUT scale (fundamental scale)
- **unrelated** Yukawa couplings at the GUT scale
- universal Yukawa c. of vectorlike fields at the GUT scale: \( Y_V \)
- common scale for superpartners: \( M_{SUSY} \) (and zero A-terms)
- common scale for vectorlike matter: \( M_V \)

in this talk we identify the two scales: \( M_{SUSY} = M_V \equiv M \)
Big picture

**GUT scale**

$\sim 3 \times 10^{16}$ GeV

**MSSM+1VF**

few TeV

Random unrelated boundary conditions:

$\alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1,0.3]$  
$y_t(M_G), y_b(M_G), y_\tau(M_G), Y_V(M_G) \in [1,3]$  
(larger values of couplings do not affect results significantly)

**Higgs quartic given by gauge couplings at any scale:**

$$\lambda_h(Q) \equiv \frac{g_2^2(Q) + (3/5)g_1^2(Q)}{4} \cos^2 2\beta$$

the plots assume:  $\tan \beta = 40$
Distinctive pattern of couplings emerges.
Big picture

GUT scale
\[ \sim 3 \times 10^{16} \text{ GeV} \]

MSSM+1VF
few TeV

SM

EW scale

GUT: Random boundary conditions

EW: familiar pattern of couplings and masses

\[ Q = 3.5 \times 10^{16} \text{ GeV} \]

GW scale
\[ \sim 3 \times 10^{16} \text{ GeV} \]

EW scale
\[ f_{\text{EW}} \text{ TeV} \]

SM

EW: familiar pattern of couplings and masses

\[ M_Z \]

EW: familiar pattern of couplings and masses
Predicted pattern of gauge couplings

In the MSSM+1VF:

\[ \alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1,0.3] \]

\[ M_G = 3.5 \times 10^{16} \text{ GeV}, \ M = 7 \text{ TeV} \ \text{and} \ \tan \beta = 40 \]

--- universal b.c. \hspace{1cm} \text{M optimized for} \ \alpha_3

\[ \log_{10} E [\text{GeV}] \]

\[ \alpha^{-1}_i(Q) = \frac{b_i}{2\pi} \ln \frac{M_G}{Q} + \alpha^{-1}_i(M_G) \]
Evolution of top, bottom and tau Y.c.

In the MSSM+1VF:

common IR fixed points remain good approximations for a large range of boundary conditions

very effective IR fixed point behavior
Predicted pattern of fermion masses

In the MSSM+1VF:

\[ \alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1, 0.3] \quad y_t(M_G), y_b(M_G), y_\tau(M_G), Y_V(M_G) \in [1, 3] \]

\[ M_G = 3.5 \times 10^{16} \text{ GeV}, \quad M = 7 \text{ TeV} \quad \text{and} \quad \tan \beta = 40 \]

- universal b.c.
- \( Y_V \) optimized for \( m_t \)

SUSY corrections at \( M \) assume all superpartners at the same scale, zero A-terms and \( \mu = -\sqrt{2}M_{SUSY} \)
In the MSSM+1VF

For large range of b.c. there is a narrow range of $M$ within which all the couplings in the MSSM+1VF meet the corresponding parameters in the SM:

$$M_G = 3.5 \times 10^{16} \text{ GeV}, \quad M = 7 \text{ TeV} \quad \text{and} \quad \tan \beta = 40$$

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Optimizing parameters related to scales

For random unrelated (or unified) parameters:

\[ \alpha_1(M_G), \alpha_2(M_G), \alpha_3(M_G) \in [0.1,0.3] \]
\[ y_t(M_G), y_b(M_G), y_\tau(M_G), Y_V(M_G) \in [1,3] \]

three parameters,

\[ M_G, M, \tan \beta, \]

can be optimized so that none of the seven observables is more than 25% (or 15%) from the measured values.

Further optimizing \( Y_V \) to obtain the required overall size of Yukawa couplings, all 7 observables are within 11% (or 7.5%) from their measured values.
The Electroweak scale

In the MSSM, the EW scale is related to soft SUSY breaking masses and the $\mu$-term, e.g:

$$M_Z^2 \simeq -1.9\mu^2 + 5.9M_3^2 - 1.2m_{H_u}^2 + 1.5m_t^2 - 0.8A_tM_3 + 0.2A_t^2 + \ldots$$

boundary conditions at the GUT scale and $\tan\beta = 10$

Prediction of SUSY:

$$M_Z^2 \leq M_{SUSY}^2$$

includes the EW scale arbitrarily below the SUSY scale.

However, any hierarchy is viewed as unnatural or fine-tuned.

based on intuition that contributions of two parameters precisely cancel only if parameters are carefully chosen/tuned

usually demonstrated by small probability in scans, sensitivity measures…
Model parameter selection

In a model with two parameters: $A, B \sim 1$ contributing to $X$, 

$$X = A - B$$

in order to get $X < 0.001$, for any $A$, the $B$ has to carefully selected:

e.g.: $A = 0.963$

$B = 0.962$

The range that leads to $X < 0.001$

$O(1)$ range of parameter $B$

The range that leads to the desired outcome is not visible!
Model parameter selection

In a model with more $O(1)$ parameters contributing to $X$, 

$$X = A - B + C - D + \ldots$$

in order to get $X < 0.001$, no parameter has to be carefully selected:

- selecting 1 parameter
- 3 parameters
- 6 parameters

effective range that leads to $X < 0.001$ is clearly visible

about $1/3$ of the range of each parameter leads to $X < 0.001$

Making just ~3 random choices for each parameter will necessarily produce an outcome with $X < 0.001$!
Small EW scale is completely ordinary

In SUSY models there are many parameters significantly contributing to the electroweak scale, e.g:

\[ M_Z^2 \simeq -1.9 \mu^2 + 5.9 M_3^2 - 1.2 m_{H_u}^2 + 1.5 m_t^2 - 0.8 A_t M_3 + 0.2 A_t^2 + \ldots \]

boundary conditions at the GUT scale and \( \tan \beta = 10 \)

(这些是额外的隐含参数)

and quite a few even in constrained versions.

Just a few \((n)\) random choices of a handful \((N)\) of SUSY parameters will produce an outcome with the EW scale 1 - 2 orders of magnitude smaller.

No parameter has to be carefully chosen!

What is “special/extreme/unexpected/tuned” based on our intuition, is completely ordinary in more complex models.

for more discussion, see:

Conclusions

In the MSSM+1VF with vectorlike matter and superpartners at a multi-TeV scale:

\[ \alpha_1, \alpha_2, \alpha_3, y_t, y_b, y_\tau, \lambda_h \]

can be understood from the IR fixed point structure of the RGEs

- just one example, similar scenarios might have other interesting features and consequences
- 1st and 2nd generations? \( \rightarrow \) different models for fermion masses
- additional motivation for more complex UV embeddings besides simple SU(5) or SO(10), e.g. Pati-Salam, flipped SU(5), …