Abstract

We present a model of life-insurance purchase that takes into account the age of the beneficiary. The beneficiaries considered herein are young children with no resources whose consumption needs are protected by purchasing life insurance if the breadwinner dies. We show that income transfer grows as the child ages; however, the size of contingent bequest shrinks because the need for protection diminishes. Consequently, among the beneficiaries, the younger one would receive a larger bequest. The aggregate demand for life insurance is positively related to the number of children, their consumption needs, and the length of time to independence.

JEL classification: G22, D91
Key words: Loading factor, birth order, actuarial rate of interest, and the age of independence.
1. Introduction

The standard model of demand for life insurance, e.g., Fischer (1973), assumes that the breadwinner maximizes his expected utility over an uncertain lifetime by choosing the optimal level of consumption and life insurance. The model assumes that beneficiaries will receive bequests if the breadwinner dies and nothing if the breadwinner survives. The demand for life insurance thus obtained is derived from the bequest function alone. Shorrocks (1979, p.415) points out that such a model is unsatisfactory because, among other things, “the bequest function is independent of the number or circumstances of the recipients, the utility associated with a transfer arises purely from the act of donation.” Subsequently, Lewis (1989) presents a model of the demand for life insurance in which the breadwinner maximizes the beneficiaries’ utility, not own utility. He argues that the results of his model are appealing because they simulate the actual calculation of insurance purchase the household makes.

Many questions arise from Lewis’ findings. First, computing the needs of beneficiaries is not the only decision that the breadwinner makes. An altruistic breadwinner chooses his own consumption, decides on the size of bequests, and allocates resources to his heirs while he is alive. Can we incorporate these decisions

\(^{1}\)The emphasis in Fischer is comparative statics. Other theoretical works in the area include, for example, Richard (1975), Campbell (1980), Pissarides (1980), and Karni and Zilcha (1985). Richard extended Merton’s (1971) consumption and portfolio rules to include life insurance. Campbell used the technique of stochastic calculus to derive an explicit demand for life insurance function. Pissarides studied the age-bequest (with life insurance) relationship. Karni and Zilcha studied life insurance and the measures of risk aversion within a state-dependent framework. Also see Borch (1991) for the institutional development of life insurance.
in a single model? Second, the role of child’s age in the breadwinner’s decision making has not been thoroughly studied in the literature. This is an important issue because the breadwinner purchases life insurance to protect his beneficiaries by providing them financial assistance in the event of his death. Since the need for protection changes with a child’s age, so does the decision on contingent bequest. What, then, is the bequest-age relation? Third, we must inquire into the relationship between the age of each child and the distribution of bequests when there are multiple children. Is it based on an equity principle, on primogeniture, or on something else? Last, we have to address Shorrocks’ criticism. What role does the number of children play in deciding income transfer and insurance purchase? This paper provides some answers to these questions.

The approach is to extend Lewis’ model such that life insurance purchase is jointly determined with the breadwinner’s own consumption and with inter vivos income transfer to heirs. To accomplish this, we include in the objective function of the breadwinner’s optimization problem each recipient’s utility function and a bequest function for each child. In other words, we assume the breadwinner is altruistic toward his dependents while he is alive, not just after his death. We also assume that each child will become independent at some point. In the model, each child’s utility function enters the breadwinner’s objective function at birth and exits it after the age of independence; bequests to adult children are an act of donation.

Our proposed theory of intergenerational transfer is different from other models in at least two aspects. First, in other models that emphasize the interaction between generations, beneficiaries have income and engage in strategic actions. Those models apply mainly to adult offspring, and usually exclude young chil-
dren from the analysis. For example, in testing the altruistic theory, Wilhelm (1996) excluded children under the age of 25. In contrast, the beneficiaries in our model are financially constrained young children who have no significant strategic options. Second, we contend that the tender age of the dependent plays an important role in breadwinner’s decision making. Other models simply overlook this issue. The exception is Laitner and Juster (1996, p.895) who recognized the point and presented a three-period model in which the middle period is a time of giving to young offspring.

Our findings are the following. First, the inclusion of beneficiaries’ utility functions in the breadwinner’s objective function enables us to draw implications for income transfer to young children. We show that if the actuarial rate of interest is greater than the subjective discount rate, then the income transfer to each child rises as the child ages and that the total expenditure on children is significantly influenced by the number of children and peaks just before the oldest child reaches independence. These results support Espenshade’s (1984) findings in which older children are more expensive because, among other things, they are physically larger and have more activities. Our results are attributed to the breadwinner’s lifetime uncertainty and the unfairness of the insurance market, and thus provide an additional explanation for these findings.

Second, our theory implies that the age profile of bequests to each child falls over time. The intuition is this: Since the purpose of life insurance purchase is to maintain the beneficiary’s standard of living up to the age of independence in the

\[\text{\footnotesize{\cite{Note1}}}\]
event of the breadwinner’s untimely death, such a need for protection declines as the child nears the age of independence.

Third, we show that birth order matters. Our birth-order result is based on equal protection for all children until independence. Since the younger one has further to go before reaching independence, more protection is needed. Therefore, when there are multiple children, the younger child would inherit a larger bequest. This result is not contradictory to primogeniture because primogeniture applies mainly to adult heirs, whereas our model applies to younger children.

Fourth, we address Shorrocks’ point that life insurance purchase must reflect the needs and circumstances of the recipients. We show that, in addition to the standard loading factor effect, the demand for life insurance is positively related to the number, the time to independence, the age differentials, and the standard of living of the beneficiaries.

Finally, our proposed model adds new results to life cycle theory. We show that precautionary saving arises from uncertainty about the breadwinner’s lifetime when there are dependent children in the family. This precautionary saving rises with the breadwinner’s hazard rate and the loading factor of the insurance market. We work out a numerical example using data from a CDC life table to illustrate the significance of this delayed consumption effect.

2. The Model

We assume that the only source of uncertainty in the model is the breadwinner’s lifetime. Let $T$ be the maximal age to which the breadwinner can live and let $p_t$ be the probability that the breadwinner dies at the beginning of period $t \leq T$ conditional on survival to period $t - 1$. By definition, $p_{T+1} = 1$. The decision to
purchase life insurance is made at the end of period \( t - 1 \), or the beginning of period \( t \) before the true state of nature is revealed. The timing of decision making is so modeled that the insurance decision resembles that of a static insurance problem.

This paper departs from Fischer’s model in that we assume the breadwinner supports each child for \( T \) years\(^3\) after which the beneficiary becomes independent. To accomplish this, we include each child’s utility function from birth to age of independence in the breadwinner’s objective function in the expected utility maximization problem. This inclusion provides a structure for ascertaining the size of life insurance, which according to Belth (1985), is determined by “How much and how long are the needs of the beneficiaries?” We also assume the bequest given to each child is optimally allocated so that each child’s utility over the period from breadwinner’s death to independence is maximized. In short, the bequest function can be interpreted as an indirect utility function.

For ease of exposition, the model is developed for only one child. Extension to multiple children is straightforward. Without loss of generality, we assume that the child is born in period \( t = 1 \) and becomes independent in \( t = T + 1 \), where \( T < T \). The type of life insurance considered in this paper is term life insurance that has no cash value and therefore no savings component. Let \( \ell_t (\ell_t \geq 1) \) be the loading factor of period \( t \). Then the price of life insurance is \( \ell_t p_t \) and the insurance premium for face value \( f_t \geq 0 \) is \( \ell_t p_t f_t \). Assume the breadwinner lives through period \( t - 1 \) and accumulates wealth \( w_t \) for period \( t \) from which he pays

\(^3\)In the United States, child support was emphasized in the Family Support Act of 1988, which stipulates that the wages of an absent parent shall be subject to withholding. Altruistic or not, raising one’s dependent children to a certain age seems to be the social norm. I owe this legal reference to Bob Michael.
\( \ell_t p_t f_t \) for a life insurance of face value \( f_t \). Since the decision is made before the state of nature is revealed, the breadwinner enters period \( t \) with wealth \( w_t - \ell_t p_t f_t \).

If the breadwinner dies at the beginning of period \( t \), he will leave behind

\[
b_t = w_t - \ell_t p_t f_t + f_t
\]

\( t \) to his heir. This \( b_t \) is Kotlikoff’s (1989) “contingent bequest,” because it is composed of saving, \( w_t - \ell_t p_t f_t \), and the face value of life insurance, \( f_t \). If he survives, the breadwinner will earn income \( y_t \) (exogenously given) and choose consumption \( c_t \) for himself and transfer \( g_t - \ell_t p_t f_t \geq 0 \) to the beneficiary,\(^4\) where \( g_t \) represents gifts \emph{inter vivos}. For simplicity, we assume that there is no human capital investment in the model. Then, the budget equation is

\[
w_{t+1} = (1 + r) (w_t + y_t - c_t - g_t),
\]

given initial wealth \( w_0 \). Thus, for \( 1 \leq t \leq T \), the breadwinner solves the recursive problem

\[
J_t = \max_{c_t, f_t, g_t} \left\{ (1 - p_t) \left[ u_t(c_t) + v_t(g_t - \ell_t p_t f_t) + (1 + \rho)^{-1} J_{t+1} \right] + p_t B_t(b_t) \right\},
\]

subject to (2), where \( u_t(\cdot) \) is breadwinner’s utility function, \( v_t(\cdot) \) is the beneficiary’s utility function, \( \rho \) is the discount rate derived from factors other than lifetime uncertainty, \( J_t \) is the current value of the discounted expected utility function at the beginning of period \( t \), and \( B_t \) is the bequest function of period \( t \). As usual, \( u_t(\cdot), B_t(\cdot) \) and \( v_t(\cdot) \) are assumed strictly increasing and strictly concave.

\(^4\)We deduct insurance premium \( \ell_t p_t f_t \) from gifts \emph{inter vivos} so that the insurance purchase \emph{when viewed from the beneficiary’s perspective} would resemble that in the static model. More precisely, the demand for insurance can be derived by comparing the wealth in the good state, \( g_t - \ell_t p_t f_t \), to that in the bad state, \( b_t = w_t - \ell_t p_t f_t + f_t \).
For \( t \geq T + 1 \), the model is identical to Fischer’s. Specifically, if the breadwinner lives through period \( t \), then he will choose consumption \( c_t \) for himself and accumulates

\[
  w_{t+1} = (1 + r) (w_t - \ell_t p_t f_t - c_t)
\]

for period \( t + 1 \). If he dies at the beginning of \( t \), he will leave behind \( b_t \). Thus, the breadwinner solves the following recursive problem

\[
  J_t = \max_{c_t, f_t} \left\{ (1 - p_t) \left[ u_t(c_t) + (1 + \rho)^{-1} J_{t+1} \right] + p_t B_t(b_t) \right\},
\]

subject to (4).

Assuming interior solutions, the first order conditions for (3) are, for \( 1 \leq t \leq T \),

\[
  \frac{p_t B'_t(b_t)}{(1 - p_t) v'_t(g_t - \ell_t p_t f_t)} = \frac{\ell_t p_t}{1 - \ell_t p_t},
\]

and

\[
  u'_t(c_t) = v'_t(g_t - \ell_t p_t f_t) = \frac{1 + r}{1 + \rho} \frac{\partial J_{t+1}}{\partial w_{t+1}}.
\]

(It should be noted that \( u'_t(c_t) = [(1 + r) / (1 + \rho)] \partial J_{t+1} / \partial w_{t+1} \) is valid for all \( t \leq T \)). Condition (6) says that, in any period, the marginal rate of substitution between bequest and income-transfer is equal to the relative price of insurance. This is a standard (static) insurance equation. Equation (7), on the other hand, says that the marginal utility of current consumption (or that of income transfer) is equal to the discounted expected marginal utility of wealth.

An immediate corollary of (6) is that for \( 1 \leq t \leq T \),

\[
  B'_t(b_t) = \left[ \frac{\ell_t (1 - p_t)}{1 - \ell_t p_t} \right] v'_t(g_t - \ell_t p_t f_t).
\]

It shows that the contingent bequest, \( b_t \), is positively related to the beneficiary’s consumption level if the breadwinner lives, \( g_t - \ell_t p_t f_t \). In this sense, the purchase
of life insurance is related to the beneficiary’s standard of living. In contrast, for \( t > T \), the contingent bequest depends only on the breadwinner’s savings and is an act of donation, because, from (5),

\[
B_t'(b_t) = \frac{\ell_t(1 - p_t) 1 + r \partial J_{t+1}}{1 - \ell_t p_t 1 + \rho \partial w_{t+1}}.
\]

3. Implications:

3.1. Child Expenditure

For \( 1 \leq t \leq T \), the Euler equation for income transfers to an heir is

\[
v'_{t-1}(g_{t-1} - \ell_{t-1}p_{t-1}f_{t-1}) = R_tv'_t(g_t - \ell_tp_tf_t),
\]

where

\[
R_t = \frac{1 + r 1 - p_t}{1 + \rho 1 - \ell_tp_t}.
\]

Obviously, the age profile of income transfers is rising over time, i.e., child expenditure rises with child’s age, if \( R_t > 1 \) and \( v_t(\cdot) = v(\cdot) \) for all \( t \). The condition for \( R_t > 1 \) is, for all \( 1 \leq t \leq T \)

\[
r + \ell_tp_t > \rho + p_t.
\]

To prove this, we note that, since \( r, \rho, \) and \( p_t \) are small, we can ignore \( r p_t, \rho r, \rho p_t \) and the higher order terms of \( p_t \) and those of \( \rho \) when we approximate \( R_t \):

\[
R_t \approx \frac{1 + r - p_t}{1 + \rho - \ell_tp_t} \approx (1 + r - p_t)(1 + \ell_tp_t - \rho) \approx 1 + (r + \ell_tp_t) - (\rho + p_t).
\]

Condition (10) has a nice economic interpretation. By definition, \( \ell_tp_t \) is the price of life insurance, and therefore \( r + \ell_tp_t \) is the actuarial rate of interest. In other words, in the presence of life insurance, the proper discount rate is not the market interest rate, but the actuarial rate of interest. Similarly, since \( \rho \) is the
discount rate derived from factors other than lifetime uncertainty, \( \rho + p_t \) is the total subjective discount rate. These definitions are standard in uncertain lifetime literature. See, for example, Chang (1991). Condition (10) simply says that the actuarial rate of interest is greater than the total subjective discount rate.

Clearly, if \( \rho < r \), then the rising profile of income transfers can be attributed in part to the discrepancy in the discount rates. What makes our result interesting is that, even if \( \rho \geq r \), we could still have a rising profile provided (10) is satisfied. In this case, \( \ell_t > 1 \), i.e., the insurance market is not actuarially fair. As a result, the recipient would not be fully insured, and there would be delayed transfer to protect the recipient against the breadwinner’s lifetime uncertainty.

Espenshade’s (1984) empirical findings on income transfer show that as children age (up to age 18) they tend to become more expensive. This is often attributed to the fact that older children are physically larger and have more activities. Our model does not take into account any of these factors. Rather, our result is implied by the breadwinner’s lifetime uncertainty, the unfairness of the insurance market, and the way the recipient’s utility enters the breadwinner’s objective function. In other words, our result provides an additional explanation for Espenshade’s findings. The inclusion of college expenditures would only strengthen the result.\(^5\)

3.2. Precautionary Saving

For \( 1 \leq t \leq T \), the Euler equation for breadwinner’s consumption is

\[
u_{t-1} (c_{t-1}) = R_t u_t (c_t). \tag{11}\]

\(^5\)In empirical studies, high school students are generally treated as dependents, but not necessarily college students. For a discussion of whether college expenses should be considered as transfers, the reader is referred to Gale and Scholz (1994).
In the standard life cycle theory with stable preference over time, i.e., $u_t(\cdot) = u(\cdot)$ for all $t$, consumption is constant over time ($R_t = 1$) if $r = \rho$; it is rising over time ($R_t > 1$) if $r > \rho$.

Equation (11) gives a stronger result. Even if $r \leq \rho$, we could have a rising consumption profile provided $\ell_t$ is large enough so that (10) is satisfied. This incentive to save is not derived from the difference in time preferences but from the unfairness of insurance market. Therefore, it is a form of precautionary saving, which rises with the loading factor because $R_t$ is increasing in $\ell_t$.

How significant is such precautionary saving? To illustrate, we consider isoclastic utility functions $u_t(x) = u(x) = x^{1-\alpha} / (1 - \alpha)$, $\alpha > 0$. Using this functional form, the consumption profile satisfies

$$c_t = R_t^{1/\alpha} c_{t-1},$$

i.e., consumption is growing at the rate of $[(r + \ell_t p_t) - (\rho + p_t)] / \alpha$. In contrast, the growth rate of consumption in the standard life cycle theory is $(r - \rho) / \alpha$. See, for example, Tobin (1967). To compute this additional growth rate, $(\ell_t - 1) p_t / \alpha$, we first note that $p_t$ (the conditional probability of death) rises with the breadwinner’s age:

| Table 1. Conditional probability of death |
|---|---|---|---|---|---|---|---|---|---|
| age | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
| $p_t$ | .6% | .8% | 1.2% | 1.8% | 2.6% | 4.0% | 6.1% | 9.2% | 13.8% |

See Arias (2002, p.38, Table V). Then, if we employ Lewis’s point estimate, $\ell = 2$, and Szpiro’s (1986) estimates that $\alpha$ is between 1.2 and 1.8, the magnitude of this additional growth rate is shown in

| Table 2. Additional growth rate of consumption |
|---|---|---|---|---|---|---|---|---|---|---|
| breadwinner’s age | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 |
| $(\ell_t - 1) p_t / \alpha$ (in %) | 0.3 - 0.5 | 0.4 - 0.7 | 0.7 - 1.0 | 1.0 - 1.5 | 1.4 - 2.2 |
If, however, we employ Fischer’s estimate that $\ell_t$ lies between 1.5 and 3.0, then the growth rate could range from 0.2% to 4.4%.

### 3.3. Contingent Bequest

What can we say about the bequest-age relation? The intent of bequest $b_t$ is to provide the beneficiary with enough resources to survive through certain time periods after the breadwinner’s death. In our model the endpoint of that time period is the age of independence. Under this assumption, if the beneficiary’s subjective discount rate is $r$, then we can formulate the objective function as

$$
\sum_{j=t}^{T} (1 + r)^{t-j} v(x_j),
$$

where $x_j$ are the resources allocated to period $j$, satisfying

$$
\sum_{j=t}^{T} (1 + r)^{t-j} x_j = b_t.
$$

If the breadwinner dies at time $t < T$, then

$$
B_t(b_t) = \max_{\{x_j\}} \sum_{j=t}^{T} (1 + r)^{t-j} v(x_j), s.t. \sum_{j=t}^{T} (1 + r)^{t-j} x_j = b_t.
$$

In other words, the bequest function is the indirect utility function of the beneficiary’s utility maximization problem from breadwinner’s death to the age of independence.

The solution to this problem is standard. The first order condition implies that

$$
x_j = \frac{b_t}{m(T - t + 1)},
$$
where
\[ m(T - t + 1) = \sum_{j=t}^{T} (1 + r)^{t-j}. \]

Therefore,
\[ B_t(b_t) = m(T - t + 1) v \left( \frac{b_t}{m(T - t + 1)} \right), \tag{12} \]

For a given \( r \), \( m(T - t + 1) \) is increasing in \( T - t + 1 \), the length of time to independence. In fact, if \( r = 0 \), then \( m(T - t + 1) = T - t + 1 \), which is exactly the length of time to independence. Equation (12) says that resources will be evenly spread out over the remaining time periods, \( T - t + 1 \) in totality, and that the bequest function is the sum of discounted utilities over the next \( T - t + 1 \) periods.

From (12)
\[ B'_t(b_t) = v' \left( \frac{b_t}{m(T - t + 1)} \right). \tag{13} \]

From (8) and (9), and assuming \( \rho = r \), we have,
\[ \frac{B'_{t-1}(b_{t-1})}{B'_t(b_t)} = \frac{\ell_{t-1}(1 - p_{t-1})}{\ell_t(1 - \ell_{t-1}p_{t-1})}. \]

The ratio \( B'_{t-1}(b_{t-1}) / B'_t(b_t) \) is close to unity because \( p_{t-1} \) is quite small as shown in Table 1 and the loading factor falls in the neighborhood of 2.0. Therefore,
\[ v' \left( \frac{b_t}{m(T - t + 1)} \right) \approx v' \left( \frac{b_{t-1}}{m(T - t + 2)} \right), \]

or
\[ \frac{b_{t-1}}{m(T - s + 1)} \approx \frac{b_t}{m(T - t + 1)}. \]

Since \( m(T - s + 2) > m(T - t + 1) \), we have \( b_{t-1} > b_t \). That is, the age profile of contingent bequests falls as the child ages. This bequest-age relation is similar to Pissarides’ (1980) finding (his \( b_3 \) curve). It captures the intuition that the need for protection diminishes as the child gets closer to the age of independence.
4. Implications: Multiple Children

To focus on effects of the number of children and the age of each child on the allocation of resources, we assume there is no child mortality so that all dependents will reach independence with certainty.

4.1. Number and Spacing of Children

The first issue presented with multiple children concerns the number and the spacing of child-bearing. For simplicity, we shall assume that there are no surprises in the family planning in the sense that both the number and the spacing are control variables. Given today’s medical technology, this assumption is reasonable. We also assume that each child will live to the age of independence. Under these assumptions we shall show that the number and the age differentials are endogenously chosen within the framework of expected utility maximization.

Let \( n \) be the number of children and \((d_1, \ldots, d_{n-1})\) be the age differentials between two adjacent children. A choice is thus defined by \((n; d_1, \ldots, d_{n-1})\). Two choices that have the same number of children but different age differentials are different choices. For a given choice, the utility function of each child enters the breadwinner’s objective function at the designated time and exits it after \( T \) periods. The breadwinner then maximizes expected lifetime utility, including children’s, subject to the budget equation. The resulting indirect utility function of a given earning stream represents the maximal value of this choice of number-and-spacing. Given that one’s lifetime is finite, the set of choices is also finite. Therefore, there exists an optimal choice that gives rise to the maximal value. There could be multiple solutions to this discrete choice problem. The actual number and age differentials depend on the model specifications.
4.2. Expenditure on Children

If the choice of number and spacing is such that no two children overlap, then the expenditure on children behaves as if there is only one child at a given time period, i.e., (7) holds in each period $t$. Then, the age profile of each child is increasing over time under the assumption that the actuarial rate of interest is greater than the total subjective discount rate.

That leaves us the interesting case when there are $n \geq 2$ children in the household at time $t$. Similar to (7), the expenditure on each child, $g^i_t - \ell_t p_t f^i_t$, is governed by

\[ u'(c_t) = (v')'(g^i_t - \ell_t p_t f^i_t) = \frac{1 + r}{1 + \rho} \frac{\partial J_{t+1}}{\partial w_{t+1}}, i = 1, \ldots, n. \quad (14) \]

Note that all beneficiaries would receive the same amount of income transfer in any period if they have the same utility function. Similar to the one-child case, the age profile of income transfers to each child rises with age. More important, given initial wealth $w_0$ and earning profile $\{y_t\}$, an increase in the number of children lowers the breadwinner’s consumption profile, and increases the aggregate expenditure on children, $\sum_{i=1}^n (g^i_t - \ell_t p_t f^i_t)$. If age differentials are not very large, the aggregate expenditure on children tends to peak just before the oldest child reaches independence. These results reinforce Espenshade’s (1984) findings that the number of children has a greater impact on parental expenditures than the parents’ socioeconomic status and wife’s employment status.

4.3. Birth Order Effect

The question here is the distribution of bequests. Suppose one child, denoted by $i$, will reach independence at time $T_i$. The other child who is $d$ years younger,
denoted by \( j \), will reach independence at time \( T_i + d \). The division of assets at the breadwinner’s death is denoted by \( k_i \), the percentage of wealth given to beneficiary \( i \). Similar to (1), the contingent bequest to each child at time \( t \) is

\[
b_t^i = k_t^i w_t - \ell_t p_t f_t^i + f_t^i.
\]

From (8), and (14), we have \((B_t^i)' (b_t^i) = (B_t^j)' (b_t^j)\) for all \( i, j \). Therefore, from (13), we have

\[
(v^i)' \left( \frac{b_t^i}{m(T_i - t + 1)} \right) = (v^j)' \left( \frac{b_t^j}{m(T_i + d - t + 1)} \right), \text{ for all } i, j. \tag{15}
\]

The birth order effect is implied by (15). For a fair comparison, we assume that the beneficiaries have identical tastes, i.e., \( v^i(\cdot) = v^j(\cdot) \). Then

\[
\frac{b_t^i}{m(T_i - t + 1)} = \frac{b_t^j}{m(T_i + d - t + 1)}
\]

Since \( m(T_i - t + 1) < m(T_i + d - t + 1) \), the bequest to the elder child is smaller than the bequest to the younger child, i.e., \( b_t^i < b_t^j \). The intuition is that each child would receive “equal protection” from the breadwinner up to the age of independence. The elder child, who has been protected for some time, clearly needs less protection than the younger sibling.

This birth order result may appear contradictory to the theory of primogeniture and many empirical findings. But, it is not. On the theoretical front, Chu (1991) shows that in the pursuit for lineal succession, the bequest will go to the eldest child if dependents’ lifetimes are uncertain. Since we assume away child mortality, our result is not at odds with his theory. On the empirical front, most findings of primogeniture are based on the bequests to adult beneficiaries, while our theory applies to the young and financially constrained children.
4.4. Demand for Life Insurance

For the rest of this section we address Shorrocks’ criticisms. From (14),

\[(v^i)' (g_i^t - \ell_t p_t f_i^t) = (v^j)' (g_j^t - \ell_t p_t f_j^t) = N_t,\]

i.e., the marginal utility of income transfer is the same across all beneficiaries. This \(N_t\) can be regarded as an index of consumption need for each beneficiary at time \(t\). Then, from (8), and (13), demand for life insurance for each child satisfies

\[k_i^t w_t + (1 - \ell_t p_t) f_i^t = b_i^t = m (T_i - t + 1) \left[ (v^i)' \right]^{-1} \left\{ \frac{\ell_t (1 - p_t)}{1 - \ell_t p_t} \right\} N_t,\]

and the aggregate demand for life insurance is

\[\sum_{i=1}^{n} f_i^t = \sum_{i=1}^{n} m (T_i - t + 1) \left[ (v^i)' \right]^{-1} \left\{ \frac{\ell_t (1 - p_t)}{1 - \ell_t p_t} \right\} N_t - w_t, \quad (16)\]

In addition to the standard loading factor effect, this demand for life insurance is positively related to the consumption need, \(N_t\), the number, \(n\), of the children, and the time it takes for each child to reach independence, \(m (T_i - t + 1)\). It is negatively related to saving \(w_t\).

5. Concluding Remarks

In this paper we presented a theory of life insurance purchase of an altruistic breadwinner supporting his liquidity-constrained children. The model takes into account the age and the consumption need of the dependent children and derives several results. The key feature is that the breadwinner’s own consumption, the gifts inter vivos, and the purchase of life insurance are jointly determined in our model. We show that, as each child ages, the size of contingent bequest shrinks
because the need for protection diminishes. Among beneficiaries, the younger one would receive a larger bequest. The aggregate demand for life insurance is positively related to the number, age differentials, the living standards, and the time needed for each child to reach adulthood.

While Lewis’ objective is similar to ours, there are major differences between the two papers. By including each dependent’s utility function in the breadwinner’s optimization problem, we have a dynamic model from which emerge several interesting implications that go beyond Lewis’ findings. First, our theory provides new insights into the life cycle theory that there is precautionary saving in a world of unfair insurance. Second, our theory sheds light on the role of the dependent’s age in the breadwinner’s decision making. Because age determines the need for protection it gives rise to birth order effects on the distribution of bequests. Third, we show that the number of children matters in determining expenditure on children. Fourth, we obtain a more detailed demand for life insurance formula that incorporates the number and circumstances of the recipients. Finally, there is a difference in modeling. Lewis assumes that the breadwinner chooses the optimal level of life insurance to maximize the beneficiary’s utility, while assuming his own consumption and the gifts to the heirs are exogenously given. In contrast, in our model all three variables are jointly determined. In this approach the breadwinner still maximizes the child’s expected utility from birth to independence.

To make the model tractable, we have made some simplifying assumptions. Specifically, we have assumed away child mortality, ignored the aspects of human capital investment on children, minimized the role played by the surviving spouse in the purchase of life insurance, assumed only interior solutions to the optimization problems, and discussed only term life insurance. What would happen if
annuities and other types of life insurance are available? Relaxing some or all of these assumptions would test the robustness of the theory developed in this paper. These are for future research.
References


