A Theory of Health Investment under Competing Mortality Risks

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Abstract

In this paper we present a theory of health investment when there are multiple causes of death. We analyze the optimal investment rules and the comparative statics. In particular, we examine the conditions under which a cause-specific intervention has a spillover effect. By spillover effect we mean a price reduction in one cause-specific health investment would increase all other investments. We also study the strength of the spillover effect, which is measured in terms of cross price elasticities. We find that, while a cause-specific intervention might not be wasteful, the spillover effect would not be large either.

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Key words: competing risks, health investment, complementarity, quality and quantity of life.

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1 Introduction

The term “competing risks” refers to the various causes of death “competing” to end one’s life. According to the CDC, heart disease and cancer led all other causes of death in 2000. (See Appendix A for the top ten leading causes of death.) Does it make economic sense to spend resources to reduce the mortality caused by a particular disease, given the fact that dying from other causes could be “just around the corner?” The answer to this question has profound policy implications — many activities sponsored by WHO and other health organizations would be deemed wasteful if the answer is negative. Dow, Philipson and Sala-i-Martin (1999) argued that, since the actual lifetime is the Leontief function of cause-specific lifetimes, the consumer has incentives to invest so that the times of death from all causes are equalized. In other words, there is a spillover effect of a cause-specific intervention and resources spent on the intervention are non-wasteful. Their theory, however, directly applies only to deterministic models.

In this paper we present a theory of health investment under competing risks when lifetimes are uncertain. We examine the conditions under which a cause-specific intervention has a spillover effect. Spillover effects exist when any two cause-specific health investments are gross complements, i.e., a decrease in the price of one encourages greater investment in the other. Once we establish the existence of the spillover effect, we explore the strength of such an effect to assess its importance. As in many economic analyses, the effect is best measured in terms of cross price elasticities. From this analysis, we can conclude if and when a cause-specific intervention would be non-wasteful.
and how effective it would be.

The theory of competing risks can be traced back to Bernoulli (1760) who looked into the effect on population mortality of inoculation against smallpox. The central question is this: Would life expectancy be increased as a result of a cause-specific intervention? The answer to the question, however, is only a first step toward a more complete analysis. Given the change in a cause-specific survival function, a consumer would reallocate her resources over her extended lifetime. Life extension, however, implies delayed consumption and lowered average consumption. The increase in quantity (longevity) is thus at the expense of quality. We present a model in which an increase in longevity implies an increase in welfare so that a cause-specific intervention is welfare improving. When all cause-specific lifetimes have a Weibull distribution, there is a closed form representation for life expectancy.

The theory of health investment proposed in this paper is based on an assumption that cause-specific survival functions can be endogenized. There are many ways that a consumer can spend resources to affect survival. Flu shots, measles vaccinations, mammogram screening, and antihypertensive medications are but a few obvious examples. In other words, life expectancy can be regarded as the output of a production function where the inputs are health investments. But health investments are achieved at the expense of consumption of all other goods. Therefore, a consumer would maximize her utility by choosing consumption and cause-specific health investments subject to the budget constraint. By solving this consumer’s problem, we obtain a family of demand functions.

These demand functions have another economic interpretation. Suppose
the market valuation of life is exogenously given. Then the benefit of life extension through health investments can be enumerated. The cost of these investments is the total expenditure. Therefore, the net benefit of health investment measures the net value of life from investments. We show that the optimal health investments derived from maximizing the net value of life are the same as the optimal health investments derived from utility maximization mentioned above.

The demand functions have other interesting properties. Assume different cause-specific health investments are \textit{complements in utility} in the sense that an increase in one health investment raises the marginal utility of another. Under this assumption, we can show that cause-specific health investments are normal goods, satisfy the law of demand, and that different cause-specific health investments are gross complements to one another. Therefore, a lower price for one cause-specific intervention would increase the demand for all others. This establishes the presence of a spillover effect of a cause-specific intervention.

What is the strength of the spillover effect? To assess this, we assume that all cause-specific survival functions have a Weibull distribution, and that the hazard rates are negatively related to the sizes of health investment. Then the overall life expectancy is a CES production function of cause-specific health investments. If the utility function in each time period is of power-function form, then we can obtain closed form representations of the demand functions.

The analysis finds that demand for all health investments are inelastic, which means the incentive to invest in response to a price reduction is small.
The analysis also finds that the absolute value of any cross price elasticity is less than the expenditure share of the cause that is subject to intervention. In other words, a 10% reduction in the price of one cause-specific health investment has the same percentage effect on all other health investments, and that percentage is far less than 10%. Thus, while a cause-specific intervention is non-wasteful, the spillover effect is not large either. We should not expect dramatic improvement in survival and in welfare from any cause-specific intervention.

2 Competing Risks

In this section, we study the allocation of resources by a consumer facing multiple causes of death and the effect of a cause-specific intervention on life expectancy.

2.1 The Basic Model

Assume there are \( n \) causes of death. Let the cause-specific lifetime \( \tau_i, i = 1, 2, \ldots, n \), be the random variable representing the age of death due to cause \( i \) if cause \( i \) is the only risk present. Since one dies only once, the actual lifetime is the random variable

\[
\tau = \min \{ \tau_1, \tau_2, \ldots, \tau_n \}. \tag{1}
\]

To make the model tractable, we assume that both the market interest rate and the subjective discount rate are zero, and that preferences are stable over time in the sense that utility functions in different periods are identical, i.e., \( u_t(c) = u(c) \) for all \( t \geq 0 \). Furthermore, we assume that \( u(c) \) is strictly
increasing and strictly concave in \( c \), satisfying

\[
    u(0) \geq 0 \text{ and } u'(0) = \lim_{x \to 0} u'(x) = \infty.
\]

Given wealth \( W \), the consumer maximizes the expected lifetime utility

\[
    E \int_0^\tau u(c(t)) \, dt,
\]

subject to the lifetime budget constraint

\[
    E \int_0^\tau c(t) \, dt = W. \quad (2)
\]

Let \( F(t) \) be the distribution function of \( \tau \) with density \( f(t) \), i.e., \( F(t) \) represents the probability of dying before or at age \( t \). The survival function is

\[
    S(t) = \Pr(\tau > t) = \Pr\{\tau_1 > t, \tau_2 > t, ..., \tau_n > t\} = 1 - F(t),
\]

satisfying \( S(\infty) = 0 \).\(^1\) Assume \( S(t) \) is integrable. It is well-known that the integral

\[
    \int_0^\infty S(t) \, dt = \bar{\tau} \quad (3)
\]

represents life expectancy. Then the problem is transformed into

\[
    \max_{c(t)} \int_0^\infty u(c(t)) S(t) \, dt, \text{ s.t. } \int_0^\infty c(t) S(t) \, dt = W. \quad (4)
\]

It is obvious that optimal consumption is \( c(t) = W/\bar{\tau} \), which means that a consumer would allocate her resources evenly over her expected lifetime.

\(^1\)We assume that the lifespan lies in the interval \([0, \infty)\) instead of \([0, T^*]\) for some maximal lifespan \( T^* \). The analysis and the results remain unchanged. We choose \( \infty \) so that statistical distributions such as exponential and Weibull distributions can be directly applied. Furthermore, as pointed out in Chang (1991, 2004), \( S(t) \) acts like a discount factor in (4) and the budget equation (2) presumes the annuity market is perfect in the sense that the expected present value of lifetime consumption is equal to the initial wealth.
Any measure that increases the life expectancy would lower the average consumption throughout the lifetime.\(^2\)

Substituting the optimal consumption into the consumer's problem (4), the indirect utility function of (4) becomes

\[
V (W, \tau) = u \left( \frac{W}{\tau} \right) \tau.
\]  

(5)

The indirect utility of the consumer is the product of the quantity of life (measured in terms of the life expectancy \(\tau\)) and the quality of life (measured in terms of the utility of the average lifetime consumption \(u(W/\tau)\)). If, in addition, the utility function is of power-function form, \(u(c) = c^{1-\alpha}\), \(0 < \alpha < 1\), then the indirect utility function

\[
V (W, \tau) = W^{1-\alpha} \tau^\alpha
\]

is of Cobb-Douglas form in wealth and longevity.

The indirect utility function in (5) is well-behaved. It is strictly increasing and strictly concave in wealth \((V_W > 0\) and \(V_{WW} < 0\)) and is also strictly increasing and strictly concave in longevity \((V_\tau > 0\) and \(V_{\tau\tau} < 0\)). In other words, the indirect utility function satisfies the law of diminishing marginal utility of wealth, and of longevity. Furthermore, wealth and longevity are *complements in utility* \((V_{W\tau} > 0)\) in the sense that an increase in longevity \(\tau\) increases the marginal utility of wealth \(V_W\) and an increase in wealth \(W\) increases the marginal utility of longevity \(V_\tau\). These properties follow directly from the strict monotonicity and the strict concavity of \(u(c)\). All but \(V_\tau > 0\) are obvious.

\(^2\)We assume for tractability reasons that the subjective discount rate and the market interest rate are zero. When they are nonzero, consumption is generally not constant over time. For issues related to the effect of life expectancy on consumption over time, the reader is referred to Chang (1991).
To show $V_\tau > 0$, we recall that a strictly concave function $u(c)$ satisfies the following inequality: $u(c) - u(0) > u'(c)c$. Therefore, from the assumption $u(0) \geq 0$, we have

$$V_\tau = u\left(\frac{W}{\tau}\right) - u'\left(\frac{W}{\tau}\right)\left(\frac{W}{\tau}\right) > u(0) \geq 0.$$ 

An immediate implication of $V_\tau > 0$ is that any intervention, cause-specific or otherwise, that prolongs life ($d\tau > 0$) will not be wasteful ($dV > 0$). Thus, the debate on the wastefulness of a cause-specific intervention is closely related to the classic question in survival analysis: What would be the effect on life expectancy if a certain disease were eliminated as a cause of death? See, for example, Bernoulli (1760), David and Moeschberger (1978), Elandt-Johnson and Johnson (1980) and Hougaard (2000).

### 2.2 Weibull Distribution

Whether or not an intervention would extend life depends on the functional form of life expectancy. We consider two of the most commonly employed distributions for survival data: exponential and Weibull distributions. Assume each cause-specific risk has a Weibull distribution,

$$S_i(t) = \exp\{-\lambda_i t^\rho\}, \lambda_i > 0, t > 0, \rho > 0. \quad (6)$$

Clearly, when $\rho = 1$, it is reduced to an exponential distribution. Assume also that all risks are mutually independent. Then the survival function of the actual lifetime $\tau = \min\{\tau_1, ..., \tau_n\}$ is also of Weibull form,

$$S(t) = \exp\left\{-\left(\sum_{i=1}^n \lambda_i\right) t^\rho\right\}, t > 0.$$
Applying integration by parts to (3), we have

\[ \bar{\tau} = E[\tau] = \Gamma \left( 1 + \frac{1}{\rho} \right) \left( \sum_{i=1}^{n} \lambda_i \right)^{-1/\rho}, \]  
(7)

where

\[ \Gamma(\alpha) = \int_{0}^{\infty} x^{\alpha-1} e^{-x} dx \]

is the gamma function, which satisfies

\[ \Gamma(\alpha) = (\alpha - 1) \Gamma(\alpha - 1). \]

See, for example, Hougaard (2000, p.48). This functional form (7) will be quite useful later on when survival functions are endogenized. In the case of exponential distribution, i.e., when \( \rho = 1 \),

\[ \bar{\tau} = E[\tau] = \frac{1}{\sum_{i=1}^{n} \lambda_i}. \]  
(8)

See, also, Hougaard (2000, p.45).

3 A Theory of Health Investment

Now suppose the cause-specific lifetime \( \tau_i \) can be changed through health investment \( x_i \). In so doing, we endogenize \( \tau_i \), and denote it by \( \tau_i(x_i) \). Assume \( \tau_i'(x_i) > 0 \), i.e., an increase in health investment will raise the cause-specific lifetime for any realization of the random variable. We may consider this health investment a form of self-protection in the sense of Erhlich and Becker (1973). Kenkel (1994) called this type of activity which changes the probability of survival preventive medical care. It is distinguished from the type of health investment that changes the health stocks but not the survival probabilities. See, for example, Chang (1996).
3.1 Health Investment as a Consumer Problem

The actual lifetime after health investment \((x_1, x_2, \ldots, x_n)\) is
\[
\tau (x_1, x_2, \ldots, x_n) = \min \{ \tau_1 (x_1), \tau_2 (x_2), \ldots, \tau_n (x_n) \}.
\] (9)

Again, \(\tau (x_1, x_2, \ldots, x_n)\) is a random variable. Let
\[
\bar{\tau} (x_1, x_2, \ldots, x_n) = E [\tau (x_1, x_2, \ldots, x_n)]
\]
be the overall life expectancy. We can regard \(\bar{\tau} (x_1, x_2, \ldots, x_n)\) as the output of a production function with input vector \((x_1, x_2, \ldots, x_n)\). This is because an increase in \(x_i\) delays the cause-specific age of death \(\tau_i (x_i)\) for any realization, which in turns raises the overall life expectancy \(\bar{\tau}\), i.e.,
\[
\bar{\tau}_{x_i} = \frac{\partial \bar{\tau}}{\partial x_i} \geq 0.
\]

Let the price of investment good \(x_i\) be \(p_i\). Given investment \((x_1, x_2, \ldots, x_n)\), the consumer’s wealth is reduced to \(W - \sum_{i=1}^{n} p_i x_i\). Hence, the problem of health investment can be formulated as
\[
\max_{\{x_i\}} V \left( W - \sum_{i=1}^{n} p_i x_i, \bar{\tau} (x_1, x_2, \ldots, x_n) \right)
\] (10)
\[
= \max_{\{x_i\}} \bar{\tau} (x_1, x_2, \ldots, x_n) \left( \frac{W - \sum_{i=1}^{n} p_i x_i}{\bar{\tau} (x_1, x_2, \ldots, x_n)} \right).
\]

In this way the health investment problem under competing mortality risks is formulated as a static consumer problem.

To ensure that the solution of Problem (10) exists and is unique, we assume that the objective function \(V (W - \sum_{i=1}^{n} p_i x_i, \bar{\tau} (x_1, \ldots, x_n))\) is strictly concave in \((x_1, \ldots, x_n)\). This, among other things, requires that the Hessian
matrix, $H = [h_{ij}]_{n \times n}$, is negative definite, where

$$h_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j} = V_{WW} p_i p_j - V_{W\tilde{\tau}} (p_i \tilde{\tau}_{x_j} + p_j \tilde{\tau}_{x_i}) + V_{\tilde{\tau}\tilde{\tau}} \tilde{\tau}_{x_i} \tilde{\tau}_{x_j} + V_{\tilde{\tau}x_i \tilde{\tau}x_j}. \quad (11)$$

It is easy to verify that $h_{ii} < 0$.

As an illustration, assume all cause-specific lifetimes have a Weibull distribution (6). Health investment affects the distribution by lowering the parameter $\lambda_i$. Assume the effect is in the form of $\lambda_i (x_i) = a_i / x_i^\rho$ for some $a_i > 0$. Then, from (7),

$$\tilde{\tau} (x_1, x_2, ..., x_n) = \Gamma \left(1 + \frac{1}{\rho}\right) \left(\sum_{i=1}^{n} a_i x_i^{-\rho}\right)^{-1/\rho}, \quad (12)$$

which is a CES function with the elasticity of substitution $\sigma = 1 / (1 + \rho)$, $0 < \sigma < 1$.

Assume further that the utility function is of the form $u(c) = c^{1-\alpha}$, $0 < \alpha < 1$. Then the objective function of the consumer’s health investment problem becomes

$$V \left( W - \sum_{i=1}^{n} p_i x_i, \tilde{\tau} (x_1, x_2, ..., x_n) \right) = \left[ \Gamma \left(1 + \frac{1}{\rho}\right) \right]^{\alpha} \left( W - \sum_{i=1}^{n} p_i x_i \right)^{1-\alpha} \left(\sum_{i=1}^{n} a_i x_i^{-\rho}\right)^{-\alpha/\rho}. \quad (13)$$

A direct computation (see Appendix B) shows that the demand for health investment $x_i$, $i = 1, 2, ..., n$, is

$$x_i = \frac{\alpha W}{p_i} \frac{a_i^{\sigma} p_i^{1-\sigma}}{\sum_{k=1}^{n} a_k^{\sigma} p_k^{1-\sigma}}. \quad (13)$$

It follows that

$$\sum_{i=1}^{n} p_i x_i = \alpha W. \quad (14)$$
That is, the total expenditure on all cause-specific health investments is a fixed fraction of the wealth. This particular result is obviously a by-product of the fact that the objective function of the consumer’s problem has the Cobb-Douglas form.

### 3.2 Value of Life

The first-order conditions of (10) are

\[-V_W p_i + V_{\bar{\tau}} \bar{\tau}_{x_i} = 0, \ i = 1, 2, ..., n.\]  

(15)

This is the “marginal benefit equals marginal cost” equation that shows the trade-off between the quantity and the quality of life. Specifically, an additional unit of a cause-specific health investment, $x_i$, increases the life expectancy by $\bar{\tau}_{x_i}$, and hence increases the utility by $V_{\bar{\tau}} \bar{\tau}_{x_i}$ units. This is the marginal benefit of health investment through increased quantity of life. In contrast, an additional unit of $x_i$ costs $p_i$ dollars, which in turns loses $V_W p_i$ units of utility. This is the marginal cost of health investment through decreased quality of life.

Rewrite (15) as

\[\left( \frac{V_{\bar{\tau}}}{V_W} \right) \bar{\tau}_{x_i} = p_i, \ i = 1, 2, ..., n,\]  

(16)

which has an interesting economic interpretation. Let

\[\theta (x_1, ..., x_n) = \frac{V_{\bar{\tau}} (x_1, ..., x_n)}{V_W (x_1, ..., x_n)}.\]

By definition, $\theta$ is the marginal rate of substitution between the quantity of life $\bar{\tau}$ and the quality of life $W$. It measures the willingness to pay for an additional year of statistical life and hence is the marginal valuation of life.
expectancy. See Rosen (1994) for a discussion on this subject. Then the left-hand-side of (16) is the value of the marginal product of health investment, and the right-hand-side is the marginal cost of health investment. At equilibrium \((x_1^*, ..., x_n^*)\), \(\bar{\tau}(x_1^*, ..., x_n^*)\) is the equilibrium life expectancy, and \(\theta^* = \theta(x_1^*, ..., x_n^*)\) is the marginal valuation of the equilibrium life expectancy.

Now assume the market price of statistical life, \(P\), is exogenously given and the consumer in question is the representative consumer. Since the market price of statistical life is the marginal valuation of the equilibrium life expectancy, we have \(\theta^* = P\). For any health investment \((x_1, x_2, ..., x_n)\), the consumer achieves a life expectancy of \(\bar{\tau}(x_1, x_2, ..., x_n)\), which has a market value of 

\[P\bar{\tau}(x_1, x_2, ..., x_n).\]

The cost of this investment is \(\sum_{i=1}^{n} p_i x_i\). The difference between these two terms

\[P\bar{\tau}(x_1, x_2, ..., x_n) - \sum_{i=1}^{n} p_i x_i\]

is the net value of statistical life from health investment \((x_1, x_2, ..., x_n)\). It is easy to verify that the equations in (16) are the first order conditions to the following problem:

\[
\max_{x_1, ..., x_n} \left\{ P\bar{\tau}(x_1, x_2, ..., x_n) - \sum_{i=1}^{n} p_i x_i \right\}.
\]

This proves the following proposition.

**Proposition 1** Given the market price of statistical life, the optimal health investment of the representative consumer under competing risks maximizes the net value of statistical life.
3.3 Wealth Effect

From (15), the wealth effect is given by

$$
\begin{bmatrix}
\frac{\partial x_1}{\partial W} \\
\vdots \\
\frac{\partial x_n}{\partial W}
\end{bmatrix} = [h_{ij}]^{-1} \begin{bmatrix}
V_{WW}p_1 - V_{\bar{W}}\bar{\tau}_1 \\
\vdots \\
V_{WW}p_n - V_{\bar{W}}\bar{\tau}_n
\end{bmatrix}.
$$

(17)

As shown earlier, $V_{WW} < 0$ and $V_{W\bar{\tau}} > 0$. It follows that $V_{WW}p_i - V_{\bar{W}}\bar{\tau}_i < 0$ for all $i$ and that the vector $[V_{WW}p_i - V_{\bar{W}}\bar{\tau}_i]^T$ has only negative entries, where the superscript $T$ stands for the transpose of a matrix. Therefore, the sign of $\frac{\partial x_i}{\partial W}$ (the wealth effect) depends on the inverse matrix $[h_{ij}]^{-1}$, where $h_{ij} = \partial^2 V/\partial x_i \partial x_j$ is defined in (11).

**Proposition 2** All cause-specific health investments are normal goods if they are complements or independent in utility in the sense of $h_{ij} \geq 0$, for all $i \neq j$.

The normal good result is intuitively appealing, indicating that the rich would invest more than the poor to reduce each cause of death. The proposition follows directly from a well-known theorem in the literature of dominant diagonal matrices. Specifically, if the off-diagonal entries are all nonnegative, $h_{ij} \geq 0$, for $i \neq j$, then a negative definite matrix is nonsingular and has an inverse matrix with only nonpositive entries. See, for example, Takayama (1985, Theorem 4.D.3, p. 393). Since all entries in $[h_{ij}]^{-1}$ are nonpositive, we have $\frac{\partial x_i}{\partial W} \geq 0$, for all $i$.

The condition $h_{ij} = V_{x_ix_j} \geq 0$ says that an increase in one cause-specific health investment $x_j$ would never decrease the marginal utility of another $V_{x_i}$. In other words, different cause-specific health investments are either complements in utility ($V_{x_ix_j} > 0$) or independent in utility ($V_{x_ix_j} = 0$). This
seems reasonable. Notice that every term in (11), except $V_x \tau_{x_ix_j}$, is negative. A necessary condition for $h_{ij} \geq 0$ is that $\tau_{x_ix_j} > 0$, i.e., inputs $x_i$ and $x_j$ are complements in producing life expectancy $\tau(x_1, x_2, ..., x_n)$.

It is well-known that inputs are complements in production if the production function is of CES form with the elasticity of substitution less than unity. This suggests that if $S_i(t) = \exp\{-\lambda_i t^\alpha\}$, $\lambda_i(x_i) = a_i/x_i^\rho$ for some $a_i > 0$, then from (12) we have $\tau_{x_ix_j} > 0$. If, in addition, we assume $u(c) = c^{1-\alpha}$, $0 < \alpha < 1$, then, from (13), the income elasticity for any $x_i$ is unitary. This provides an example in which all cause-specific health investments are normal goods.

### 3.4 Slutsky Equations

Let $C_{ij}$ be the $(i,j)$-cofactor of the matrix $[h_{ij}]$, and $\det[h_{ij}]$ be the determinant of the matrix $[h_{ij}]$. Then the inverse matrix is given by

$$[h_{ij}]^{-1} = \frac{1}{\det[h_{ij}]} [C_{ji}]_{n \times n}.$$  

Under the assumption of $h_{ij} \geq 0$ for all $i \neq j$, all entries in the inverse matrix $[h_{ij}]^{-1}$ are nonpositive. It follows that, for all $i$ and $j$, either $C_{ji} = 0$ or $C_{ji}$ and $\det[h_{ij}]$ are opposite in sign. This mathematical result is essential in ascertaining the sign of the substitution effect of the Slutsky equation.

From (15), the price effect is given by

$$\left[\begin{array}{c}
\frac{\partial x_1}{\partial p_j} \\
\vdots \\
\frac{\partial x_n}{\partial p_j}
\end{array}\right] = [h_{ij}]^{-1} \left\{ \begin{array}{c}
V_W e_j - x_j \\
V_W \tau - x_j
\end{array} \right\}$$

where $e_j$ is the column vector with 1 in the $j$-th row and 0 elsewhere. Then,
using (17), the Slutsky equations are given by

\[
\frac{\partial x_i}{\partial p_j} = \frac{C_{ji}}{\det[h_{ij}]}V_W - x_j \frac{\partial x_i}{\partial W}, \quad \text{for all } i, j.
\]

The first term on the right-hand-side of (18) is the substitution effect and the second term is the wealth effect.

**Proposition 3** If \( h_{ij} \geq 0 \), for all \( i \neq j \), then \( \partial x_i/\partial p_j \leq 0 \), for all \( i \) and \( j \).

The proof of the proposition is again a direct application of the theory of dominant diagonal matrices. As shown in the previous subsection, all health investments are normal goods. Therefore, the income effect is nonpositive. Under the assumption of \( h_{ij} \geq 0 \), for all \( i \) and \( j \), either \( C_{ji} = 0 \) or \( C_{ji} \) and \( \det[h_{ij}] \) are opposite in sign. Therefore, the substitution effect is either 0 or negative, i.e., all entries of the substitution matrix \([\partial x_i^c/\partial p_j]_{n \times n}\) are nonpositive, where \( x_i^c \) stands for the compensated demand for cause-specific health investment \( x_i \). Combining the income and the substitution effects, we have \( \partial x_i/\partial p_j \leq 0 \), for all \( i \) and \( j \).

**Corollary 4** All cause-specific health investments satisfy the law of demand.

As an illustration, we return to the demand functions obtained in (13). Let \( s_i \) be the expenditure share of investment in \( x_i \), \( p_ix_i \), in the total expenditure of health investments, \( \sum_{i=1}^{n} p_ix_i = \alpha W \), i.e.,

\[
s_i = \frac{p_ix_i}{\alpha W} = \frac{a_i^\sigma p_i^{1-\sigma}}{\sum_{k=1}^{n} a_k^\sigma p_k^{1-\sigma}},
\]

where the last equality is obtained from (13). Then the own price elasticity is

\[
\varepsilon_{ii} = \frac{p_i}{x_i} \frac{\partial x_i}{\partial p_i} = -\sigma - (1 - \sigma) s_i < 0,
\]

(19)
and the law of demand is satisfied. This elasticity in absolute value rises with the expenditure $s_i$ and the elasticity of substitution $\sigma$. Moreover, $|\varepsilon_{ii}| < 1$, i.e., the demand for any health investment $x_i$ is inelastic.

3.5 Spillover Effect

Another corollary of Proposition 3 is this:

**Corollary 5** Under the assumption of Proposition 3, any two cause-specific health investments are gross complements to each other.

The economic implication of this proposition is straightforward. Lowering the price of a cause-specific health investment would increase all health investments. This establishes the spillover effect. It also raises an interesting question: What happens to the total expenditure on all cause-specific health investments? The issue is related to the strength of the spillover effect.

Recall that the lifetime utility of a consumer with health investment $(x_1, x_2, \ldots, x_n)$ is $V (W - \sum_{i=1}^{n} p_i x_i, \bar{r}(x_1, x_2, \ldots, x_n))$ that satisfies $V_{WW} < 0$ and $V_{\bar{r}\bar{r}} < 0$. Given the spillover effect, all health investments are increased and there is a quantity-of-life effect, $V_\bar{r} > 0$. Now assume the total expenditure on health investments $\sum_{i=1}^{n} p_i x_i$ increases so that the consumption of all other goods, $c = W - \sum_{i=1}^{n} p_i x_i$, is reduced. Then there is a reduction in the quality of life. Given $V_{WW} < 0$ and $V_{\bar{r}\bar{r}} < 0$, the loss in the quality of life may be larger than the gain in the quantity of life if $\sum_{i=1}^{n} p_i x_i$ is very large. Such an increase in health investment would not be welfare improving. Since a reduction in the price of a cause-specific health investment is welfare improving, there must be a built-in upper limit on the extent of this spillover effect.
To support the argument that the spillover effect is limited, we return to our example (13). Because of the particular functional form of utility, we arrive at $\sum_{i=1}^{n} p_i x_i = \alpha W$. Even though a consumer increases all cause-specific health investments, the total expenditure of health investments remains unchanged. This shows that the spillover cannot be significant. Indeed, the cross price elasticity

$$
\varepsilon_{ij} = \frac{p_j \partial x_i}{x_i \partial p_j} = -\frac{(1 - \sigma) a_i^\sigma p_j^{1-\sigma}}{\sum_{k=1}^{n} a_k^\sigma p_k^{1-\sigma}} = - (1 - \sigma) s_j, \text{ for all } i \neq j, \quad (20)
$$
satisfying $|\varepsilon_{ij}| < s_j$. That is, these cross elasticities are identical and are less than the expenditure share of the cause that is subject to intervention. For example, if $\sigma = 0.5$ and $s_j = 0.1$, then $|\varepsilon_{ij}| = 0.05$. A 10% decrease in the price of a cause-specific health investment would produce only a 0.5% increase in all other health investments. This example shows that there is indeed a spillover effect and at the same time shows that the effect is not very large.

The spillover effect of health investment implies the spillover effect of survival. This is because a decrease in the price of a cause-specific health investment raises the investment, and hence the survival function, of all others. To illustrate its strength, we again assume all cause-specific lifetimes have a Weibull distribution (6) and $\lambda_i (x_i) = a_i / x_i^\rho$ for some $a_i > 0$. Since

$$
[S_i (t)] (x_i) = \exp \{- a_i x_i^{-\rho t^\rho}\},
$$
the absolute value of the elasticity of the $i$-th survival function with respect to $j$-th price is

$$
- \frac{p_j}{S_i (t)} \frac{\partial S_i (t)}{\partial p_j} = \rho (-\varepsilon_{ij}) [- \log S_i (t)] \text{ for all } i \text{ and } j.
$$
It follows that the total effect on survival of a price change, using (19) and (20), is

\[
- \frac{p_j}{S(t)} \frac{\partial S(t)}{\partial p_j} = - \sum_{i=1}^{n} p_j \frac{S_i(t)}{S(t)} \frac{\partial S_i(t)}{\partial p_j} = \rho \sum_{i=1}^{n} \varepsilon_{ij} [- \log S_i(t)]
\]

\[= \rho \sigma [- \log S_j(t)] + \rho (1 - \sigma) s_j [- \log S(t)].\]

This formula will be useful in testing the theory because it implies that the absolute value of the elasticity of the overall survival function with respect to \(j\)-th price is an increasing function of time \(t\), of expenditure share \(s_j\), and of Weibull parameter \(\rho\). The effect on the survival function of a cause-specific intervention is of interest to research works on the economics of mortality that include Dow, Philipson and Sala-i-Martin (1999), Murphy and Topel (2003), and Becker, Philipson and Soares (2003).

4 Concluding Remarks

We close this paper with some comments on Dow et al (1999) and some thoughts on future research. As mentioned in the Introduction, Dow, Philipson and Sala-i-Martin (1999) presented a theory of competing risks and argued that there is a spillover effect of a cause-specific intervention. Their intuition is built upon the certainty case of (1),

\[T = \min \{T_1, T_2, ..., T_n\},\]

where \(T_i\) is the age of death due to cause \(i, i = 1, 2, ..., n\). With cause-specific health investments, the actual lifetime is

\[T(x_1, x_2, ..., x_n) = \min \{T_1(x_1), T_2(x_2), ..., T_n(x_n)\},\]

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which is the deterministic version of (9). The consumer chooses \((x_1, x_2, \ldots, x_n)\) to maximize the deterministic version of (10):

\[
u \left( \frac{W - \sum_{i=1}^{n} p_i x_i}{T(x_1, x_2, \ldots, x_n)} \right) T(x_1, x_2, \ldots, x_n).
\]

Since one dies only once, this Leontief function of \(T(x_1, x_2, \ldots, x_n)\) implies that a typical consumer will allocate her resources so as to equalize the times of death from all causes, i.e., \(T_i(x_i) = T_j(x_j)\).

Because of the similar formulations in the certainty and the uncertainty cases, they claimed that “the forces towards the equalization of cause-specific lifetimes operate in the more general case as well.” See Dow, Philipson and Sala-i-Martin (1999, p.1361). Unfortunately, this claim is not valid. To extend the equalizing-lifetimes argument to the stochastic case, we must have

\[
\bar{\tau} = \min \{ \bar{\tau}_1, \bar{\tau}_2, \ldots, \bar{\tau}_n \},
\]

where \(\bar{\tau}_i = \int_0^\infty S_i(t) \, dt\) is the life expectancy of cause \(i\). This would not be the case if we use examples (8) and (7), in which \(\bar{\tau}_i = \Gamma (1 + 1/\rho) \lambda_i^{-1/\rho}\).

In contrast, this paper makes the following contributions. First, we present a theory of health investment under competing risks when lifetimes are uncertain. Second, we find a sufficient condition for the existence of the spillover effect of a cause-specific intervention. Third, we illustrate the theory with a carefully worked-out example that has several testable hypotheses. Finally, we assess the strength of the spillover effect.

There are at least two directions for future research. First, we assume, as Dow et al did, that the lifetime wealth is exogenously given, which remains unchanged even if life expectancy is increased. This assumption is subject
to improvement because people do take actions to change their wealth to accommodate for a longer life. Endogenizing lifetime wealth would test the robustness of our results and the policy implications. Second, the health investment as modeled in this paper is a one-time investment that affects the survival function. This setting is also subject to improvement because people do invest in health throughout their lifetimes. It would be interesting to analyze a dynamic model of health investment under competing mortality risks in order to shed light on our understanding of investment behavior under lifetime uncertainty.
Appendix

A. Deaths – Leading Causes

Number and percentage of deaths for leading causes of death

Heart Disease: 700,142 (29.6%)
Cancer: 553,768 (23.0%)
Stroke: 163,538 (7.0%)
Chronic lower respiratory diseases: 123,013 (5.1%)
Accidents (unintentional injuries): 101,537 (4.1%)
Diabetes: 71,372 (2.9%)
Influenza and Pneumonia: 62,034 (2.7%)
Alzheimer’s disease: 53,852 (2.1%)
Nephritis, nephrotic syndrome, and nephrosis: 39,480 (1.5%)
Septicemia: 32,238 (1.3%)
All other causes: 499,283 (20.8%)


B. Derivation of Equation (13):
To derive equation (13), let \( c = W - \sum_{i=1}^{n} p_i x_i \). Then we formulate the problem as:

\[
\max_{c, x_1, \ldots, x_n} \quad \Gamma \left( 1 + \frac{1}{\rho} \right) c^{1-\alpha} \left( \sum_{i=1}^{n} a_i x_i^{-\rho} \right)^{-\alpha/\rho}, \quad \text{s.t.} \quad c + \sum_{i=1}^{n} p_i x_i = W.
\]

Let \( \gamma \) be the Lagrange multiplier of the constrained maximization problem. The first order conditions are

\[
(1 - \alpha) \Gamma \left( 1 + \frac{1}{\rho} \right) c^{-\alpha} \left( \sum_{i=1}^{n} a_i x_i^{-\rho} \right)^{-\alpha/\rho} = \gamma
\]

and

\[
\alpha \Gamma \left( 1 + \frac{1}{\rho} \right) c^{1-\alpha} \left( \sum_{i=1}^{n} a_i x_i^{-\rho} \right)^{-\alpha/\rho - 1} a_i x_i^{-\rho - 1} = \gamma p_i, \text{ for all } i.
\]

It follows that

\[
\alpha \frac{c a_i x_i^{-\rho - 1}}{1 - \alpha \sum_{i=1}^{n} a_i x_i^{-\rho}} = p_i, \text{ for all } i, \quad (21)
\]

and

\[
\frac{a_j x_j^{-\rho}}{a_i x_i^{-\rho}} = \frac{p_j x_j}{p_i x_i}, \text{ for all } i, j. \quad (22)
\]

Equation (22) implies that

\[
\frac{a_j x_j^{-\rho}}{\sum_{i=1}^{n} a_i x_i^{-\rho}} = \frac{p_j x_j}{\sum_{i=1}^{n} p_i x_i} = \frac{p_j x_j}{W - c}, \text{ for all } j. \quad (23)
\]

Substituting (23) into (21), we have

\[
\frac{\alpha}{1 - \alpha} \frac{c}{W - c} = 1,
\]

which implies \( c = (1 - \alpha) W \).

Equations (22) and (14) are the first order conditions for the consumer problem

\[
\max_{x_1, \ldots, x_n} \left( \sum_{i=1}^{n} a_i x_i^{-\rho} \right)^{-1/\rho}, \quad \text{s.t.} \quad \sum_{i=1}^{n} p_i x_i = \alpha W. \quad (24)
\]

The solutions for (24) are well-known and are of the form (13).
References


