House Prices and Interest Rates: A Theoretical Analysis*

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Abstract

We formulate a model of housing market in the presence of heterogenous agents and aggregate shocks. We analytically derive the relation between house prices and interest rates. We show that the distribution of the housing stock is also important for house price dynamics, which may weaken the correlation between interest rates and house prices.

Keywords: House price, mortgage, interest rate, aggregate uncertainty
JEL Codes: D1, D3, D4, D5, R2

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1 Introduction

The recent boom and bust cycle of house prices in U.S. has drawn great attention to the dynamics of house prices. Especially, the influence of housing on business cycle fluctuations makes the analysis of house price dynamics an important phenomena. There has been a surge in understanding the determinants of house prices in the last two decades\(^1\). However, the complexity of housing market together with the mortgage market, the lumpiness in both housing choice and investment, and the presence of transaction costs greatly avoid a thorough analysis of the housing market. Apart from a few studies, there is no theoretical\(^2\) or quantitative\(^3\) work to study the determinants of house price dynamics.

In this paper, we try to fill this gap by theoretically studying the relationship between interest rates and house prices in a two period overlapping generations framework. The existence of aggregate interest rate uncertainty, explicit long-term mortgage contracts and the illiquidity of the housing good distinguish our model from the existing literature. We show that, even though interest rate is the only exogenous process in the model, price of a house is not only a function of interest rates but also depends on the distribution of the housing stock between young and old agents. The joint dynamics of interest rates and the distribution of housing stock determine the interest rate-housing price relationship. Simulations of a plausibly calibrated version of the model show that interest rates and house prices have mostly a negative relation, but the endogenous housing stock can dampen this correlation significantly.

2 The Model

The basic framework studied in this paper is designed to capture the essential components of the housing market as much as possible but yet be able to produce analytical results. It is a two-period overlapping-generations model populated with measure 1 of young and old agents. Individuals face two sources of uncertainty in the economy. On

\(^1\)See Mankiw and Weil (1989), Poterba(1991), Malpezzi (1999), Case and Schiller (2004), Himmelberg et al. (2005), Martin (2005), Quigley and Raphael (2005), Davis and Heathcote (2007) for examples of studies that empirically analyze the dynamics of house prices.

\(^2\)See Ortalo-Magne and Rady (2006) which studies the effect of income fluctuations on house prices.

\(^3\)David and Heathcote (2005), Rios-Rull and Sanchez-Marcos (2008), Piazzesi et al. (2007), Arslan (2008), Chatterjee and Evigungor (2009), Favilukis, Ludvigson and Van Nieuwerburgh (2011) are exceptions of works that study the dynamics of house prices in a quantitative framework.
the aggregate level, the economy faces interest rate uncertainty, and on the individual level, households face a moving shock when they become old. We abstract from the effect of income fluctuations on the housing market and assume a fixed income for both young and old agents. However, this income is not sufficient to buy a house, and young agents have to borrow in the mortgage market to finance their housing purchases. Mortgage payments are made at the end of the period, and agents consume the remaining income.

If an old agent moves from his house, he gets the return from selling the house and pays the remaining mortgage debt at the beginning of the period. Then he borrows in the mortgage market to finance his new house purchase. If he does not move from his house, he makes the second mortgage payment and uses the remaining income to purchase non-housing consumption.

There are risk-neutral profit-maximizing banks which offer mortgage contracts. In equilibrium, banks are indifferent between lending in the bond market and the mortgage market. We further assume that aggregate house supply is fixed and there is no rental market. Lastly, for analytical tractability, we assume quasi-linear preferences, linear in non-housing consumption and quadratic in housing:

\[ u(c, h) = c - \frac{\alpha}{2} (\theta - h)^2 \]

where \( \alpha \) and \( \theta \) are parameters of the model with \( \theta \) greater than the maximum housing size.

**Mortgage Market**

Risk-neutral profit-maximizing banks offer fixed-rate 2 period mortgage contracts. We use two conditions to find the mortgage rates and the corresponding payments. The first condition is the *present value condition*, which means that the present value of the payments should be equal to the loan amount:

\[ 1 = \frac{m_t}{1 + r_t^m} + \frac{m_t}{(1 + r_t^m)^2}, \]

\( m_t \) is the mortgage payment corresponding to the mortgage interest rate \( r_t^m \) at time \( t \). The next condition is the *no-arbitrage condition*. In equilibrium, banks should be indifferent between lending in the bond market with net return \( r_t \) and the mortgage market:

\[ 1 = \frac{m_t}{1 + r_t} + (1 - \pi) \frac{m_t}{(1 + r_t)E[(1 + r_{t+1})]} + \pi \frac{1 - m_t + r_t^m}{1 + r_t}. \]
Old Agent’s Problem

If an old agent receives a moving shock he chooses how big a house to purchase depending on the state of the market. If he does not move he stays in his old house. The value of staying in the old house is trivial. The individual consumes the income net of the mortgage payment:

\[ c_{t}^{o,s} = w - \frac{p_{t-1}h_{t-1}^{y}m_{t-1}}{1 + r_{t}}, \]

where \( c_{t}^{o,s} \) is the consumption, \( h_{t}^{y} \) is the current house size for the old staying agent, \( w \) is the fixed income, and \( p \) is the house price.

If the old agent moves, at the beginning of the period, he sells his house, pays off the remaining mortgage debt, and buys a new house. Since at the end of the period the old dies, the interest rate on the new house purchase is the current risk-free interest rate:

\[ c_{t}^{o,m} + (1 - m_{t-1} + r_{t-1}m_{t-1})p_{t-1}h_{t-1}^{y} = w + p_{t}(h_{t-1}^{y} - h_{t}^{o,m}) \] (1)

Solving the mover’s problem results the following housing demand:

\[ h_{t}^{o,m} = \theta - \frac{p_{t}}{\alpha}, \] (2)

which gives the intuitive message that as the house price increases the demand for housing decreases.

Young agent’s problem

For the young agents, the problem is more involved since their current choice affects their next period value. Once a young agent decides about the size of his housing purchase, he gets a mortgage loan from a lender, and makes his first mortgage payment at the end of the period:

\[ c_{t}^{y} = w - \frac{p_{t}h_{t}^{y}m_{t}}{1 + r_{t}}. \]

Solving the young agent’s problem by considering the effects of current choices on the next period variables, we get the following housing demand

\[ h_{t}^{y} = \theta + \beta \pi E[p_{t+1}] - p_{t} \left[ \frac{m_{t}}{1 + r_{t}} + \beta(1 - \pi) \frac{m_{t}}{E(1 + r_{t+1})} + \beta \pi (1 + r_{t}m_{t} - m_{t}) \right] \frac{\alpha(1 + \beta(1 - \pi))}{\alpha(1 + \beta(1 - \pi))}. \] (3)
As it is clear from this equation, the young agents’ housing demand not only depends on current house price and mortgage interest rate but also the expectation of next period house price and interest rate.

**Market Clearing Condition**

There is only one market to clear in this economy, which is the housing market. Since aggregate housing supply is fixed through time, the market clearing condition implies

\[
h_t^y + \pi h_{t}^{o,m} = H - (1 - \pi)h_{t-1}^y, \tag{4}
\]

where \(H\) is the aggregate fixed total supply of housing\(^4\). Since the houses owned by the old agents who do not move, \((1 - \pi)h_{t-1}^y\), will not be traded in the market the effective supply at period \(t\) is \(H - (1 - \pi)h_{t-1}^y\). The total demand for housing at period \(t\) is the sum of housing demand of old agents who move, \(\pi h_{t}^{o,m}\), and the housing demand of the young agents, \(h_t^y\).

**Solution of the Model**

We can solve for the equilibrium house prices by substituting equations (2) and (3) into equation (4), and characterize the equation for house price and the evolution of the housing stock as the following:

**Proposition 1** House price, \(p_t\), depends only on current interest rate, \(r_t\), and housing stock of the young agents in the last period, \(h_{t-1}^y\), and it is linear in \(h_{t-1}^y\):

\[
p_t = k_0^p (r_t) + k_1^p (r_t) h_{t-1}^y \tag{5}
\]

Moreover, the evolution of the housing stock for the young agents is the following:

\[
h_t^y = k_0^h (r_t) + k_1^h (r_t) h_{t-1}^y \tag{6}
\]

where \(k_0^p, k_1^p, k_0^h, \text{ and } k_1^h\) are functions which only depend on \(r_t\).

The clear message of the proposition is that house price does not only depend on interest rate but also on the evolution of housing stock. Notice that although total housing stock is fixed in the economy, effective housing supply, which is the total amount of housing available for purchase in a given period is not necessarily fixed.

\(^4\text{Notice that, since the measure of the young and old agents is both 1, and all agents are identical within each generation, individual, aggregate and average values are all the same.}\)
This happens due to the immobility of some young agents when they become old. So, moving probability plays a crucial role in the effectiveness of the housing stock, which allows us to state the following corollary:

**Corollary 1** If $\pi = 1$, house price only depends on interest rate:

$$p_t = k^p_0 (r_t),$$

and $k^h_1 (r_t) = 0$. If $\pi = 0$, then

$$p_t = \frac{\alpha (1 + \beta) (\theta - H + h^y_t)}{\frac{m_t}{1+r_t} + \beta \frac{m_t}{E[1+r_{t+1}]}},$$

$$h^y_t = H - h^y_{t-1}.$$

If we allow the moving probability, $\pi$, to be 1, then it is clear that the effective supply becomes the total supply, which is constant in every period. As a result, housing stock becomes ineffective for the house price dynamics. In the other extreme case, if we set $\pi = 0$, then effective supply oscillates between two values: $h^y_0$, and $H - h^y_0$, i.e. housing stock of young agents becomes independent from interest rate, and initial housing distribution becomes the only determinant of the housing stock forever.

### 3 Numerical Results

To simulate the model, we need to choose values for the parameters in the model. Since the model is a two-period OLG model, total life time is summarized in two periods. Each time period in the model corresponds to 15 years. We assume the stochastic process in the model is the interest rate on a 10-year treasury-bond rate. The average of 10-year treasury bond during the 1962-2009 period is 6.89%. Then we estimate an AR(1) process of these real rates which gives the yearly autocorrelation coefficient, $\rho$, as 0.93 and the coefficient of variation, $\sigma$, as 0.15. Finally, we use the Hussey and Tauchen (1991) method to approximate the AR(1) process with a 100-state Markov chain. We iterate this Markov process 15 times to obtain 15 year transition probabilities.

The second important parameter in our model is the moving probability of old households. PSID data shows that 5.44% of the respondents moves in a year (Cocco (2001)). The corresponding number for 15 years would be 56%. We calibrate $\theta$ to 1 to have risk aversion of 2 for the housing consumption. Aggregate housing supply is
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of a period</td>
<td></td>
<td>15 years</td>
</tr>
<tr>
<td>Number of states in the Markov Process</td>
<td>S</td>
<td>100</td>
</tr>
<tr>
<td>Average of the real interest rates (1975-2006)</td>
<td>µ</td>
<td>6.89%</td>
</tr>
<tr>
<td>Annual autocorrelation of the interest rates (1975-2006)</td>
<td>ρ</td>
<td>0.93</td>
</tr>
<tr>
<td>Coefficient of variation of interest rates (1975-2006)</td>
<td>σ</td>
<td>0.15</td>
</tr>
<tr>
<td>Movement probability</td>
<td>π</td>
<td>0.56</td>
</tr>
<tr>
<td>Housing supply</td>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>Discount factor</td>
<td>β</td>
<td>0.37</td>
</tr>
<tr>
<td>A parameter in the utility function</td>
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</tr>
<tr>
<td>A parameter in the utility function</td>
<td>θ</td>
<td>1</td>
</tr>
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Note. – Coefficient of variation is calculated as standard deviation divided by mean normalized to 1. We set β, the discount factor, to $1/(1 + r)^{15}$, where $r$ is the average yearly interest rate. Since we do not have any prior information about the value of α, and the value of α does not affect our results qualitatively, we set α to 1. Table 2.1 summarizes the parameter values for our benchmark model.

We simulate the model 1,000 times assuming the economy lasts for 1000 periods. The results show that most of the time (around 93%), interest rate and house price are negatively related. However, about 5% of the time, house price and interest rate have a positive comovement. Approximately 2% of the time, when the interest rate does not move, house price still moves, due to the changes in the effective supply. The most effective parameter on the relationship between interest rate and house price is the moving probability, π. As π decreases (from 1 to 0), the influence of housing distribution becomes larger and the percentage of time when interest rate and house price co-move increases in our simulations (from 0% to 7%).

To assess the change in house price more closely, one has to pay attention to the evolution of young agent’s housing stock, $h_t^y$. Figure 1 shows the evolution of young agents’ housing demand as a function of interest rates. The change in the housing demand of the young agents from period $t - 1$ to period $t$ is

$$h_t^y - h_{t-1}^y = k_0^h(r_t) + (k_0^h(r_t) - 1) h_{t-1}^y.$$

As it is clear from the figures, $k_0^h(.)$ is positive and $k_1^h(.) - 1$ is negative. Depending on the magnitude of the last period demand, $h_{t-1}^y$, young agents may increase or decrease
Table 2: Calibrated Parameters

<table>
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</tr>
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<td>$\theta$</td>
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</tr>
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</table>

Note. --Coefficient of variation is calculated as standard deviation divided by mean.

their demand as a response to an increase in interest rates. Thus, it is not possible to definitely know whether young households demand less or more as interest rates increase without knowing the effective supply. This, in turn, influences the effective supply for the next period. Ambiguity in the demand of young agents as a function of interest rates implies an ambiguity in the effective supply for the next period. Note that, the effective supply, $h^e_{t+1}$, for period $t+1$ is

$$h^e_{t+1} = H - (1 - \pi)h^y_t = H - (1 - \pi)(k^h_0(r_t) + k^h_1(r_t)h^y_{t-1}).$$

So the change in the effective supply from period $t$ to period $t + 1$ is

$$dh^e_{t+1} = h^e_{t+1} - h^e_t = -(1 - \pi)k^h_0(r_t) - (1 - \pi)(k^h_1(r_t) - 1)h^y_{t-1} = f(r_t) + g(r_t)h^y_{t-1}.$$  

where $f(r_t) = -(1 - \pi)k^h_0(r_t)$ and $g(r_t) = -(1 - \pi)(k^h_1(r_t) - 1)$. Since $f(.)$ is negative and $g(.)$ is positive, the change in effective supply is ambiguous. So, depending on the magnitude of young agents’ last period housing demand, the effective supply may increase or decrease as interest rates increase. For high values of $h^y_{t-1}$, the effective supply increases while for low values of $h^y_{t-1}$, the effective supply decreases.

Figure 1 shows that both of the coefficients of the house price function decrease as interest rates increase. This implies house prices will decrease as a function of interest rates keeping the housing supply constant. This is intuitive, because as interest rates increase, the demand for housing will decrease since borrowing becomes more expensive.
However, the direction and the magnitude of the shift in the effective supply is not clear because of the reasons we have argued earlier. Depending on the direction and the magnitude of the shift, the equilibrium price may increase or decrease. If the previous period’s house demand of young agents is low enough, then the effective supply will shift to the left sufficiently and the equilibrium price will increase. However, if it is not low enough the equilibrium price level will decrease. Thus, the effect of an interest rate on housing prices is ambiguous and depends on the distribution of the housing stock.

4 Conclusion

In this study we analyze the behavior of housing prices in response to fluctuations in interest rates by using a simple overlapping generations framework. We show that housing prices depend on both interest rates and effective housing supply. The effective housing supply in the market highly depends on the distribution of housing among agents. If young households have the biggest portion of the housing stock, this means, next period’s effective supply will be low (due to the exogenous movement shock), which in turn results in an ambiguous change in housing prices in response to an increase in interest rates. Similarly, if young agents hold a smaller fraction of total housing, next period’s effective supply will be more and housing prices will move ambiguously, if interest rates decrease.

References


Figure 1: Coefficients on house price function
A Proofs

Proof. We can rewrite the market clearing condition in the following way:

$$p_t f(r_t) = B + (1 - \pi)h_{t-1}^y + A\beta\pi E[p_{t+1}]$$ \hspace{1cm} (7)

where

$$A = \frac{1}{\alpha(1+\beta(1-\pi))}, \; f(r_t) = \frac{\pi}{\alpha} + A \left[ m_t \left( \frac{1}{1+r_t} + \frac{\beta(1-\pi)}{E(1+r_{t+1})} \right) + \beta\pi(1 + r_t^m - m_t) \right], \; B = (1 + \pi)\theta - H$$

and the first order condition for the young agents will be:

$$h_t^y = \theta + A\beta\pi E[p_{t+1}] - p_t g(r_t).$$ \hspace{1cm} (8)

where $g(r_t) = f(r_t) - \frac{\pi}{\alpha}$. Equations (7) and (8) yield us to make the following guesses for the functional forms of house price and evolution of young’s housing stock:

$$p_t = k_0^p(r_t) + k_1^p(r_t) h_{t-1}^y$$ \hspace{1cm} (9)

$$h_t^y = k_0^h(r_t) + k_1^h(r_t) h_{t-1}^y$$ \hspace{1cm} (10)

To verify our guess, first substitute (9) into (8):

\[
\begin{align*}
    h_t^y &= \theta + A\beta\pi E[k_0^p(r_{t+1}) + k_1^p(r_{t+1}) h_{t}^y] - g(r_t) \left[ k_0^p(r_t) + k_1^p(r_t) h_{t-1}^y \right] \\
    h_t^y &= \frac{\theta + A\beta\pi E k_0^p(r_{t+1}) - g(r_t) k_0^p(r_t)}{1 - A\beta\pi E k_1^p(r_{t+1})} - \frac{g(r_t) k_1^p(r_t)}{1 - A\beta\pi E k_1^p(r_{t+1})} \frac{h_{t-1}^y}{h_{t-1}^y}
\end{align*}
\]

which results

$$k_0^h(r_t) = \frac{\theta + A\beta\pi E k_0^p(r_{t+1}) - g(r_t) k_0^p(r_t)}{1 - A\beta\pi E k_1^p(r_{t+1})}$$ \hspace{1cm} (11)

$$k_1^h(r_t) = -\frac{g(r_t) k_1^p(r_t)}{1 - A\beta\pi E k_1^p(r_{t+1})}.$$ \hspace{1cm} (12)

Next, substitute (9) and (10) into (7), we get

$$k_0^p(r_t) f(r_t) + k_1^p(r_t) f(r_t) h_{t-1}^y = B + (1 - \pi)h_{t-1}^y + A\beta\pi E \left[ k_0^p(r_{t+1}) + k_1^p(r_{t+1}) (k_0^h(r_t) + k_1^h(r_t) h_{t-1}^y) \right]$$

13
Equalizing the coefficients on $h_{t-1}$, we get

$$
k_p^0(r_t) f(r_t) = B + A \beta \pi E k_p^0(r_{t+1}) + A \beta \pi E k_1^p(r_{t+1}) \kappa^h_0(r_t) \quad (13)
$$

$$
k_p^1(r_t) f(r_t) = 1 - \pi + A \beta \pi E k_p^1(r_{t+1}) \kappa^h_1(r_t) \quad (14)
$$

Notice that equation (14) is a functional equation with $k_p^1$ as the only unknown. Once we solve for that equation, we can solve for $k_p^0$ using equation (13), which only depends on $k_p^0$ and $k_p^1$. Lastly, once we have $k_p^0$ and $k_p^1$, it is straightforward to get $k^h_0$ and $k^h_1$ using equations (11) and (12). This also verifies our guess about the functional forms of house price dynamics and the evolution of housing stock. ■

**Proof.** If $\pi = 1$, then equation (14) becomes

$$k_p^1(r_t) \left[ f(r_t) + \frac{g(r_t) A \beta \pi E k_p^1(r_{t+1})}{1 - A \beta \pi E k_1^p(r_{t+1})} \right] = 0$$

which results $k_p^1(r_t) = 0$. Also using equation (12), we get $k_1^h(r_t) = 0$.

If $\pi = 1$, equation (3) simplifies to the following expression:

$$h_t^y = \theta + \frac{-p_t \left[ \frac{m_t}{1+r_t} + \beta E \left[ \frac{m_t}{1+r_{t+1}} \right] \right]}{\alpha(1+\beta)}.$$  

and $1 = \frac{m_t}{1+r_t} + \frac{m_t}{(1+r_t)(1+r_{t+1})} \Rightarrow m_t = \frac{(1+r_t)E(1+r_{t+1})}{1+E(1+r_{t+1})}$. Note that since agents don’t move, they don’t consider the next period’s housing price. Together with the market clearing condition, the Euler equation becomes,

$$H - h_{t-1}^y = \theta + \frac{-p_t \left[ \frac{m_t}{1+r_t} + \beta E \left[ \frac{m_t}{1+r_{t+1}} \right] \right]}{\alpha(1+\beta)}.$$  

Reorganizing the terms gives the result. ■